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Dominator Chromatic Number on Various Classes of Graphs

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ABSTRACT

Let G be a graph. A dominator coloring of G is a proper coloring in which every vertex of G dominates atleast one color class. The dominator chromatic number of G is denoted by χ_d(G) and is defined by the minimum number of colors needed in a dominator coloring of G. In this paper, we obtain dominator chromatic number on various classes of graphs.

Mathematics Subject Classification : 05C15, 05C69

KEYWORDS : Dominator chromatic number, banana graph ,book graph, stacked book graph, dutch wind mill graph, prism graph, crossed prism graph, sunflower graph.

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INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let G = (V, E) be a graph. The open neighborhood N(v) of a vertex v ∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N (v)∪{v}. An induced subgraph G[S], where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S. A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets $V_1$and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice -verse. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and n-1leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H , to be the graph with vertex set V(G)×V(H) and edges between two vertices (u_1 ,v_1) and (u_2,v_2) iff either $u_1=u_2$ and $v_1v_2 \in E(H)$ or $u_1u_2 \in E(G)$ and $v_1=v_2$. A subset S of V is called a dominating set if every vertex in V-S is adjacent to atleast one vertex in S. The dominating set is minimal dominating set if no proper subset of S is a dominating set of G. The domination number γ is the minimum cardinality taken over all minimal dominating set of G. A γ-set is any minimal dominating set with cardinality γ.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ(G). A dominator coloring of G is a proper coloring of G in which every vertex of G dominates atleast one color class. The dominator chromatic number is denoted by χ_d(G) and is defined by the minimum number of colors needed in a dominator coloring of G .This concept was introduced by Ralucca Michelle Gera in 2006[2].

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C^{1} be a minimal dominator coloring of G. We say that a color class $c_i \in C^{1}$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes. A banana graph $B_{m,n}$ is a graph obtained by connecting one leaf of each m copies of an n-star graph with a single root vertex that is distinct from all the stars . The book graph B_m is defined as the graph Cartesian product $P_2 \times K_{1,m-1}$. The stacked book graph $SB_{m,n}$ is the generalization of the book graph to stacked pages . The dutch windmill graph D_m^n is the graph obtained by taking n copies of the cycle graph C_n with a vertex in common .The prism graph Y_n is a graph consisting of a Cartesian product $P_2 \times C_n$,where $P_2$is a path on two vertices and C_n is the cycle graph on n vertices. An n-crossed prism $G_n, n \geq 4$ is a graph obtained by taking two disjoint cycles C_1 and C_2 of order 2n and adding edges $u_i v_{i+1}$ and $u_{i+1} v_i$ for i=1,3,----(n-1). A sunflower graph Sf_n, $n \geq 4$ is a graph obtained from wheel graph $W_n = K_1 + C_n$ with each edge $u_i\, u_{i+1}$ of the cycle C_n can be added to two new edges $u_i v_i$ and $u_{i+1} v_i$.

The dominator chromatic number of paths, cycles were found in [2] and [3].

We have the following observations from [2] and [3].

Theorem A [2] The path P_n of order n≥2 has $\chi_d\,(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 2,3,4,5,7 \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$

Theorem B [3] The cycleC_n has $\chi_d\,(C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n = 4 \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 5 \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$

In this paper, we obtain the least value for dominator chromatic number on various classes of graphs.

Theorem 1 For the banana graph $B_{m,n}$, $\chi_d(B_{m,n})=m+2$

Proof: Let $B_{m,n}$ be the banana graph .The vertex set of the graph $V(B_{m,n}) = \{u\} \cup \left\{ v_{ij} / \begin{smallmatrix} 1 \leq i \leq m \\ 1 \leq j \leq n \end{smallmatrix} \right\}$. That is $B_{m,n}$ consist of one vertex has degree m and m vertices of degree 2 and m vertices of degree (n-1) and m(n-2) vertices of degree 1 respectively. We assign m distinct colors to the vertices of degree (n-1) and the color say (m+1) is assigned to the vertices of degree 1 and degree 2 and the color say (m+2) is assigned to the vertex u. Thus $\chi_d\,(B_{m,n}) = m+2$. □

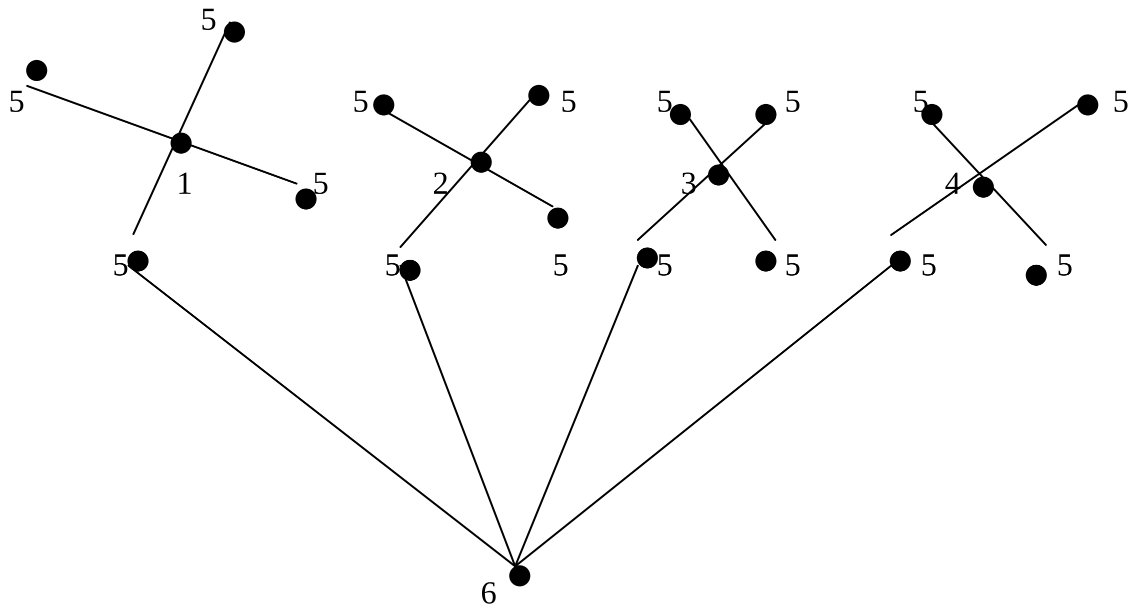

Fig 1 ($B_{4,5}$) $\chi_d(B_{4,5}) = 6$.

Theorem 2 For the book graph B_m, $\chi_d(B_m) = 4$

Proof : Let $P_2 \times K_{1,m}$ be the book graph with vertex set$\{v_1, v_2, v_3, ------, v_{2m}, v_{2m+1}, v_{2m+2}\}$, where (v_1, v_2) and (v_i, v_j) i=3,5,7------,2m+1 and j=4,6,8,------,2m+2 form the pages of B_m. We assign colors 1 and 2 to v_1 and v_2 repectively, assign the colors 3 and 4 to the set of vertices $\{v_3, v_5, v_7, --------, v_{2m+1}\}$ and the set of vertices $\{v_4, v_6, v_8, ----, v_{2m+2}\}$ respectively. Thus $\chi_d(B_m) = 4$. □

Consider B_4

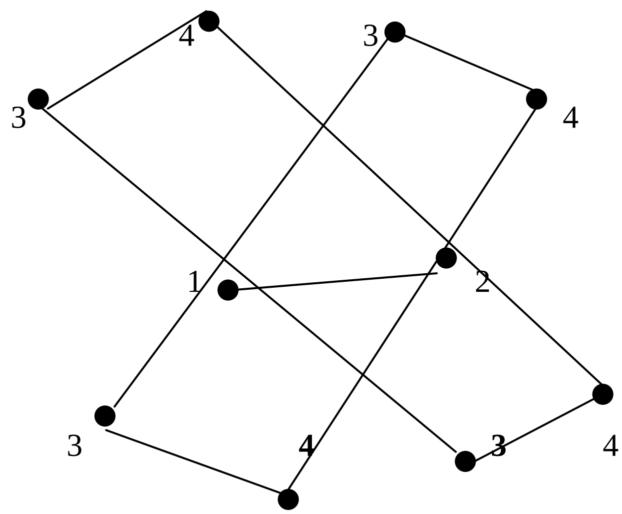

Fig 2.(B_4) $\chi_d(B_4) = 4$

Theorem 3 For any stacked book graph $SB_{m,n}$, $\chi_d(SB_{m,n})=n+2$

Proof: Let $SB_{m,n} = P_n \times K_{1,m}$ be the stacked book graph and let $V(SB_{m,n})=\left\{v_{ij}/ {}_{1\le j\le m+1}^{1\le i\le n}\right\}$ such that B_i isomorphic to the vertex induced subgrarph v_{1i}, v_{2i}, v_{3i},---------,v_{ni}. We assign n distinct colors 1,2,3,--------,n to v_{11}, v_{21}, v_{31},---------,v_{n1} and colors n+1 and n+2 to the set of vertices v_{ij}, $1\le j \le m+1$ and i=1,3,5-----,n if n is odd and the set of vertices $v_{ij}, 1\le j \le m+1$ and i=2,4,6-----,n if n is even respective□Thus $\chi_d(SB_{m,n})=n+2$.

Consider $SB_{3,4}$

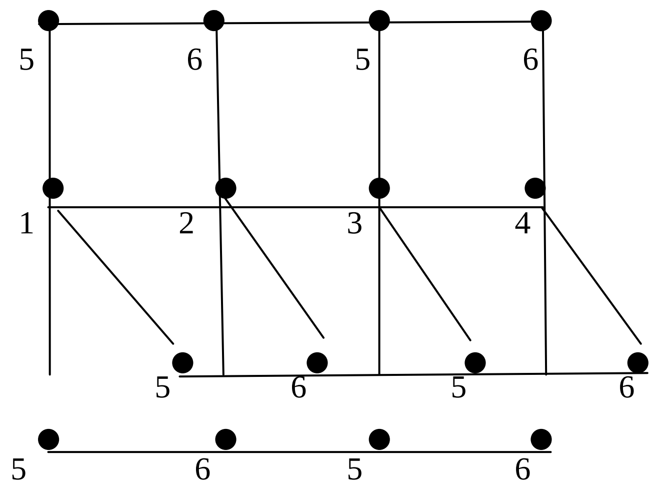

Fig 3.($SB_{3,4}$

$\chi_d(SB_{3,4})=6$

Theorem 4 For the dutch wind mill graph D_m^n, $\chi_d(D_m^n)=n\left\lceil\frac{m-3}{3}\right\rceil+3$

Proof: Consider D_m^n formed by n copies of the cycle c_m with $V(D_m^n)=\{v_{ij}/ {}_{j=1,2,3,-----m}^{i=1,2,3,-----n}\}$. For each i=1,2,3,---,n $\{v_{i1}, v_{i2}, v_{i3}$,---------,$v_{im}\}$ be the vertices of i- th copy of cycle C_m and $v_{11}=v_{21}=v_{31}$=--------- = v_{n1} is a common vertex. We assign color 1 and 2 to a common vertex v_{11} and the set of vertices $\{v_{i2}, v_{im}\}$, i=1,2,3----,n and we assign n $\chi_d(C_{m-3})$ distinct colors to remaining vertices $\{v_{i3}, v_{i4}, v_{i5}$,---------,$v_{i\,m-1}\}$,i=1,2,3,------,n. Finally we need n $\chi_d(C_{m-3})$ +1 colors to need dominator coloring. so $\chi_{td}(D_m^n)$=n $\chi_d(C_{m-3})+1= n\left\lceil\frac{m-3}{3}\right\rceil+2+1=n\left\lceil\frac{m-3}{3}\right\rceil+3$. □

Thus $\chi_d(D_m^n)=n\left\lceil\frac{m-3}{3}\right\rceil+3$.

Consider D_{12}^{3}

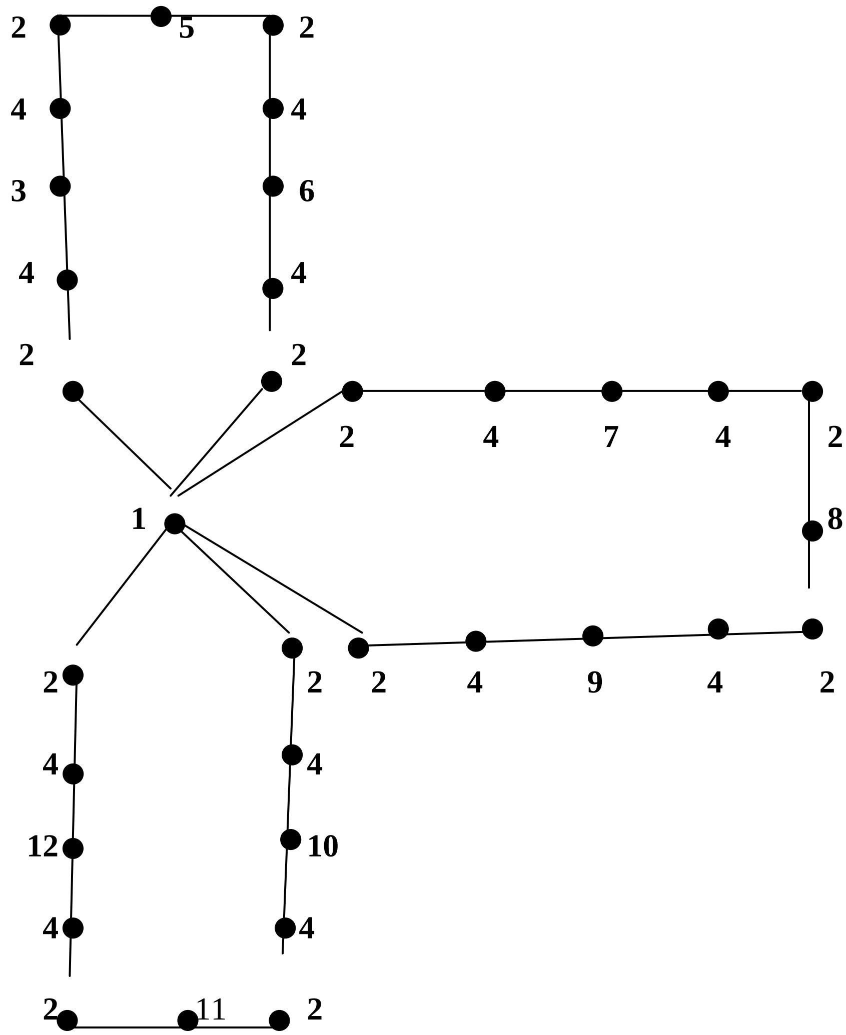

Fig 4 (D_{12}^{3})

$\chi_{td}(D_{12}^{3})=12$

Theorem 5 For the prism graph Y_n, $\chi_d(Y_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 2 & if\ n \equiv 0,3\ (mod 4) \\ \left\lceil \frac{n}{2} \right\rceil + 3 & if\ n \equiv 1,2\ (mod\ 4) \end{cases}$

Proof: Let Y_n be a prism graph and $V(Y_n) = \{u_1, u_2, u_3, \text{-------}, u_n, v_1, v_2, v_3, \text{-------}, v_n\}$. We consider two cases.

Case (1) When $n \equiv 0\ ,3(mod4)$. We have two subcases.

Subcase(1.1) When $n \equiv 0\ (mod4)$,since $N(u_i) \cap N(v_{i+2}) = \emptyset$--------(*) for every i=1,2,----n,

$i \equiv 0\ (mod\ n)$ and $\sum d(v_i) \equiv 0\ (mod4)$ .Let D={ u_1,u_5,u_9,----u_{n-3},v_3,v_7,----v_{n-1}} be the vertices and satisfies equation $(*)$ and $|D|=\frac{n}{2}$,we assign $\frac{n}{2}$ distinct colors to the vertices in D and assigned two repeated colors say $\frac{n}{2}+1$ and $\frac{n}{2}+2$ to the remaining vertices such that adjacent vertices receives different colors. So $\chi_d(Y_n)= \frac{n}{2}+2$.

Subcase (1.2) When $n \equiv 3\ (mod4)$, since $\sum d(v_i) \equiv 1(mod4)$, $\frac{n}{2}$ vertices of $V(Y_n)$ satisfying equation$(*)$ and the vertices v_{n-1},v_n does not satisfies equation $(*)$. Assign $\left\lceil\frac{n}{2}\right\rceil$ distinct colors to the vertices satisfying equation $(*)$ and v_{n-1} and by subcase(1.1), we get $\left\lceil\frac{n}{2}\right\rceil+2$. So $\chi_d(Y_n)= \left\lceil\frac{n}{2}\right\rceil+2$.

Case (2) When $\equiv 1\ ,2(mod4)$. We have two subcases.

Subcase(2.1) When $n \equiv 1\ (mod4)$,since $\sum d(v_i) \equiv 3\ (mod4)$, and subcase(1.2) $\left\lceil\frac{n}{2}\right\rceil$ vertices satisfying equation $(*)$ and two vertices say u_{n-1} and v_n does not satisfies equation $(*)$. By applying the same coloring as in subcase (1.2) ,we get a proper coloring except the vertices u_{n-1} and v_n .So we use two distinct colors say $\left\lceil\frac{n}{2}\right\rceil$ and $\left\lceil\frac{n}{2}\right\rceil+1$ to the vertices u_{n-1} and v_n respectively and we assigned two repeated colors say ($\frac{n}{2}+2$) and ($\frac{n}{2}+3$) to the remaining vertices such that adjacent vertices receives different colors .So $\chi_d(Y_n) = \left\lceil\frac{n}{2}\right\rceil+3$.

Subcase(2.2) When $n \equiv 2\ (mod4)$, since $\sum d(v_i) \equiv 2\ (mod4)$, ($\frac{n}{2}$-1)vertices satisfying equation $(*)$ and 4 vertices does not satisfies equation $(*)$. Among the 4 vertices u_{n-1},u_{n-2},v_n, $v_{n-1,}$two vertices say, u_{n-1},v_{n-1} receive two distinct colors and remaining two vertices u_{n-2},v_n have received the already used repeated colors. So $\chi_d(Y_n) = \left\lceil\frac{n}{2}\right\rceil+3$.

Thus $$\chi_d(Y_n) = \begin{cases} \left\lceil\frac{n}{2}\right\rceil+2 & if\ n \equiv 0,3\ (mod4) \\ \left\lceil\frac{n}{2}\right\rceil+3 & if\ n \equiv 1,2\ (mod\ 4) \end{cases}$$

Consider Y_{10}

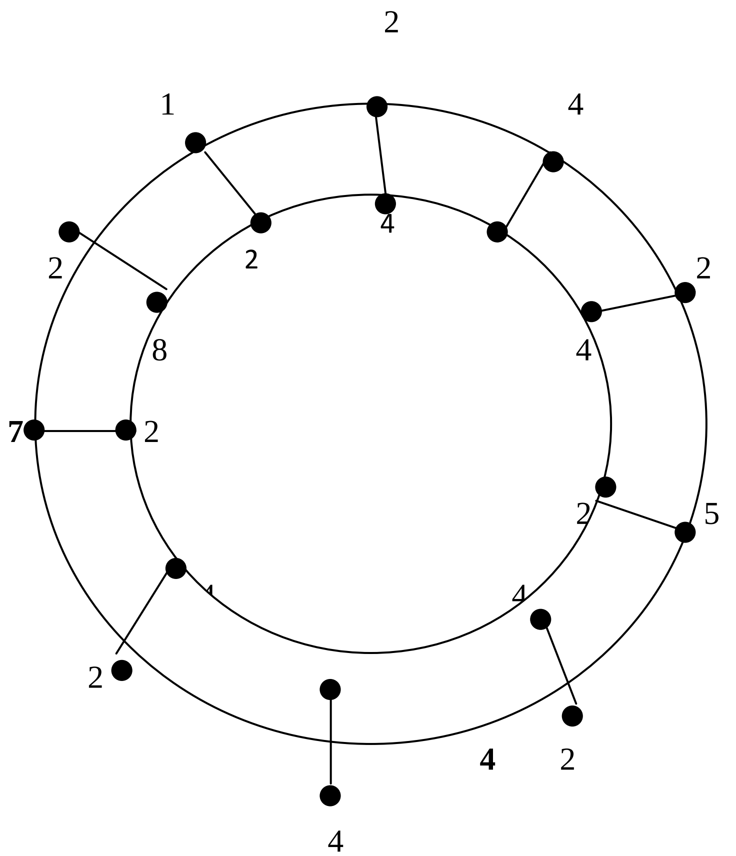

Fig 5 (Y_{10})

$\chi_d(Y_{10})=8$

Theorem 6 For n-crossed prism graph G_n , $\chi_d(G_n) = \begin{cases} n+2 & \text{if } n \text{ is even} \\ n+3 & \text{if } n \text{ is odd} \end{cases}$

Proof: Let G_n be an n-crossed prism graph and it is a graph obtained by taking two disjoint cycles C_1 and C_2 of order 2n and adding edges u_iv_{i+1} and $u_{i+1}v_i$ for i=1,3,----(n-1). let $V(C_1)=\{u_1,u_2,u_3,\text{-------},u_{2n}\}$ and $V(C_2)=\{v_1,v_2,v_3,\text{-------},v_{2n}\}$ .We consider two cases.

Case(1)When n is even.We assign n distinct colors to the vertices $\{ v_1,v_5,v_9,\text{-------},v_{2n-3},u_2,u_6,u_{10},\text{-----------},u_{2n-2}\}$ and assigned two repeated colours say n+1 and n+2 to the remaining vertices such that adjacent vertices received different colors. So $\chi_d(G_n)$ =n+2.

Case(2) When n is odd. We assign n+1 distinct colors to the vertices $\{ v_1,v_5,v_9,\text{-------},v_{2n-1},u_2,u_6,u_{10},\text{-------},u_{2n}\}$ and assigned two repeated colours say n+2 and n+3 to the remaining vertices such that adjacent vertices received different colors. So $\chi_d(G_n)$ =n+3.

Thus $\chi_d(G_n) = \begin{cases} n+2 & \text{if } n \text{ is even} \\ n+3 & \text{if } n \text{ is odd} \end{cases}$ □

Consider G_7

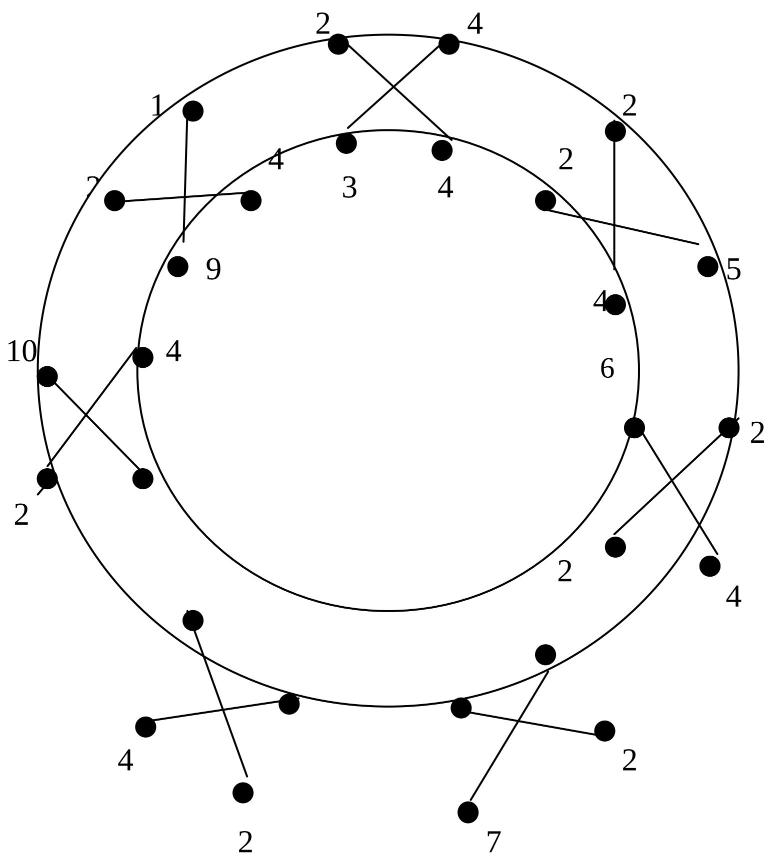

Fig 6 (G_7)

$\chi_d(G_7)$=10.

Theorem 7 Any sunflower graph Sf_n , $\chi_{td}(Sf_n) = \left\lceil \frac{n}{2} \right\rceil + 2$

Proof: Let Sf_n, $n \geq 4$ be a sunflower graph and it is a graph obtained from wheel graph $W_n = K_1 + C_n$ with each edge $u_i u_{i+1}$ of the cycle C_n can be added to two new edges $u_i v_i$ and $u_{i+1} v_i$. Let $V(Sf_n)$={ $u, u_1, u_2, u_3,$ ------------, u_n, $v_1, v_2, v_3,$ -----------, v_n },where deg u=n, deg u_i=5 and deg v_i=2 for all i=1,2,3----,n. We consider two cases.

Case(1) When n is even. We allot $\frac{n}{2}$ distinct colors to the vertices { $u_1, u_3, u_5, \text{-------}, u_{n-1}$ } ,the color $\frac{n}{2}+1$ to the vetices u and v_i , i=1,2,---,n and $\frac{n}{2}+2$ to the vertices { $u_2, u_4, u_6, \text{-------}, u_n$ } ,we got dominator coloring. So $\chi_d(Sf_n) = \frac{n}{2}+2$.

Case(2) When n is odd. We allot $\left\lceil\frac{n}{2}\right\rceil$ distinct colors to the vertices { $u_1, u_3, u_5, \text{-------}, u_{n-2}, u_{n-1}$ } ,and the remaining coloring as in case(1) we got a dominator coloring. So $\chi_d(Sf_n) = \left\lceil\frac{n}{2}\right\rceil + 2$. □

Consider Sf_8

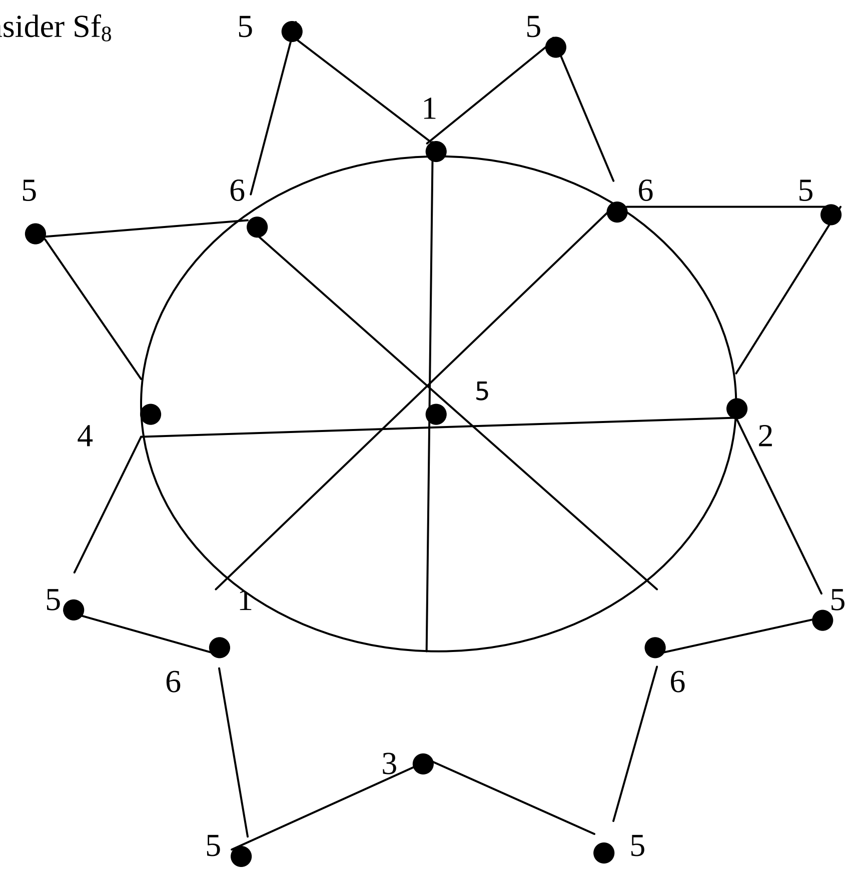

Fig 7 (Sf_8)

$\chi_{td}(Sf_8) = 6$

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