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Super Harmonic Mean Labeling of Some Graphs

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ABSTRACT

Let G be a graph with p-vertices and q-edges. Let $f : V(G) \rightarrow \{1, 2,\ldots, p + q\}$ be a injective function. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by $f^*(e) = \left\lceil \frac{2f(u)\,f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)\,f(v)}{f(u)+f(v)} \right\rfloor$. Then f is called a Super Harmonic mean labeling if $f(V(G)) \cup \{f(e) / e \in E(G)\} = \{1, 2, \ldots, p+q\}$. A graph which admits super harmonic mean labeling is called Super Harmonic mean graph. In this paper, we investigate super harmonic mean labeling of some standard graphs.

KEYWORDS: Graph, Super harmonic mean labeling, Super harmonic mean graphs.

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INTRODUCTION

We begin with simple, finite, connected and undirected graph G = (V, E) with p-vertices and q-edges. For a detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary [2]. S. Somasundaram and R.Ponraj introduced mean labeling of graphs in [3]. R.Ponraj and D. Ramya introduced super mean labeling of graphs in[4]. S. Somasundaram and S.S. Sandhya introduced the concept Harmonic mean labeling in [5] and studied their behaviour in [6, 7, 8]. S. Sandhya and C.David Raj introduced super harmonic mean labeling in [9]. In this paper we investigate super harmonic mean labeling of Ladder graph attached with pendant vertex, comb graph attached with pendant vertex, Middle graph, Double Triangular snakes attached with pendent vertex and Alternate Double Triangular snakes. We now give the following definitions which are useful for the present investigation.

Definition 1.1:

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Most of the graph labeling problems have following three common characteristics.

1. a set of numbers for assignment of vertex labels.
2. a rule that assigns a label to each edge.
3. Some conditions there labels must satisfy.

Definition 1.2:

A function f is called a mean labeling of graph G if f : V(G) → {0, 1, 2,…,q} is injective and the induced edge function f^*: E(G) → {1, 2,…,q} defined as follows is bijective.

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2}, & f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2}, & f(u)+f(v) \text{ is odd.} \end{cases}$$

The graph which admits mean labeling is called a mean graph.

Definition 1.3:

A function f is called a harmonic mean labeling of graph G if f :V(G)→{1,2,…q+1} is injective and the induced edge function f^*: E(G)→{1,2, ...,q} defined as $f^*(e = uv) = \left\lceil \dfrac{2f(u)\,f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \dfrac{2f(u)\,f(v)}{f(u)+f(v)} \right\rfloor$ is bijective.

The graph which admits harmonic mean labeling is called a harmonic mean graph.

Definition 1.4:

Let $f : V(G) \rightarrow \{1, 2, \ldots p + q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^{*}(e = uv)$ is defined by $f^{*}(e) = \left\lceil \frac{2f(u)\,f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)\,f(v)}{f(u)+f(v)} \right\rfloor$. Then f is called a super harmonic mean labeling if $f(V(G)) \cup \{f(e) / e \in E(G)\} = \{1, 2, \ldots, p + q\}$. A graph which admits super harmonic mean labeling is called super harmonic mean graph.

Definition 1.5:

The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition 1.6:

The graph $P_n \odot K_1$ is called comb.

Definition 1.7:

The graph $C_n \odot K_1$ is called crown.

Definition 1.8:

The Ladder L_n, $n \geq 2$, is the product graph $P_n \times P_2$ and contains 2n vertices and $3n - 2$ edges.

Definition 1.19:

The Middle graph M(G) of a graph is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent iff either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 1.10:

A Double Triangular Snake $D(T_n)$ consists of two triangular Snakes that have a common path.

Definition 1.11:

An Alternate Double Triangular Snake $A(D(T_n))$ consists of two Alternate Triangular Snakes that have a common path.

Example 1.12:

A Super Harmonic Mean labeling of a graph G is shown below.

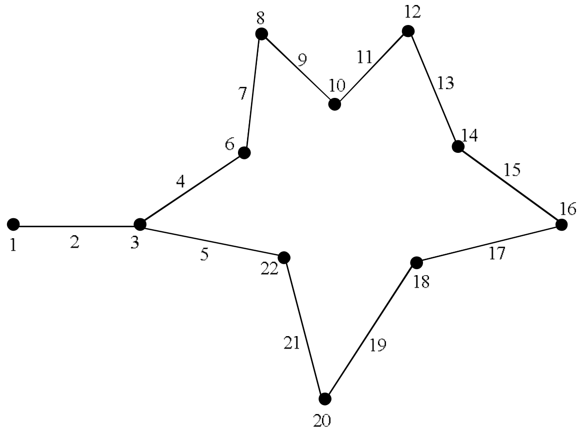

Figure 1

Remark 1.13:

In a Super Harmonic mean labeling, the labels of vertices and edges are together form{1, 2, 3, …, p + q}.Now we shall use the following theorems for reference.

Theorem 1.14: [10]

Crowns are Super Harmonic Mean graphs.

Theorem 1.15: [10]

Comb is a Super Harmonic Mean graph.

Theorem 1.16: [11]

Double Triangular Snakes and Alternate Double Triangular Snakes are Super Geometric Mean graphs.

2. MAIN RESULTS

Theorem 2.1:

Let G be a graph obtained from a Ladder L_n, $n \geq 2$ by joining a pendant vertex with a vertex of degree two on both sides of upper and lower path of the Ladder. Then G is a super Harmonic mean graph.

Proof:

Let $L_n = P_n \times P_2$ be a Ladder. Let G be a graph obtained from a Ladder by joining pendant vertices u, w, x, z with v_1, v_n, u_1, u_n (vertices of degree 2) respectively on both sides of upper and lower path of the Ladder. The graph is displayed below

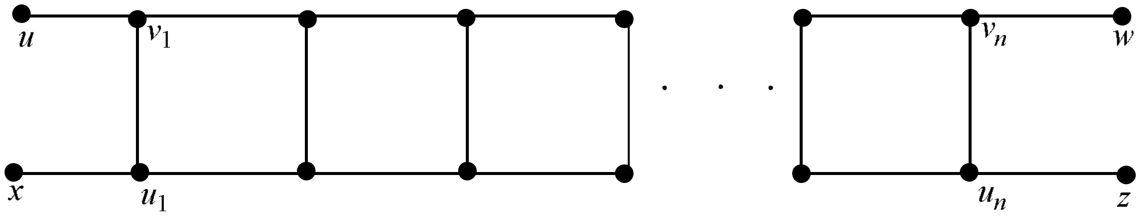

Figure 2

Here p + q = 5n + 6.Define a function f : V(G) → {1, 2, …, p + q} by f(u) = 1, $f(v_1)$ = 5,$f(v_i)$ = 5i, 2 ≤ i ≤ n,f(w) = 5n + 5,f(*x*) = 3, $f(u_i)$ = 5i + 3, 1 ≤ i ≤ n except for i = 4m – 1, m = 1, 2, 3, … In this case, $f(u_i)$ = 5i + 2.Also, f(z) = 5n + 6.Edges are labeled with, $f(v_iv_{i+1})$ = 5i +2, 1 ≤ i ≤ n – 1 except for the case i = 4m – 1, m = 1, 2, 3, … . In this case, $f(v_iv_{i+1})$ = 5i + 3.Also, $f(uv_1)$ = 2,$f(v_n\, w)$ = 5n + 2, $f(x\, u_1)$ = 4, $f(u_iu_{i+1})$ = 5i + 4, i ≤ i ≤ n – 1,$f(u_n\, z)$ = 5n + 4,$f(v_i\, u_i)$ = 5i +1,1 ≤ i ≤ n. Therefore, f(V(G)) ∪ {f(e) : e ∈ E(G)} = {1, 2,…, p + q}. Hence, the edge labels are distinct.Hence, G is a super harmonic mean graph.

Example 2.2:

A Super Harmonic Mean Labeling of G when n = 5 is shown below.

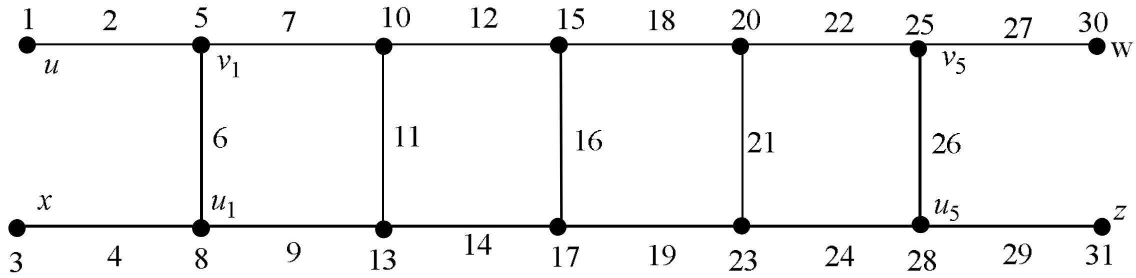

Figure 3

Theorem 2.3:

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two of a comb graph. Then G is a super harmonic mean graph.

Proof :

Comb (P_n ⊙ K_1) is a graph obtained from a path $P_n = v_1v_2 \dots v_n$ by joining a vertexu_i to v_i, 1 ≤ i ≤ n. Let G be a graph obtained by joining a pendant vertex w to v_n (a vertex of degree 2) . The graph is displayed below.

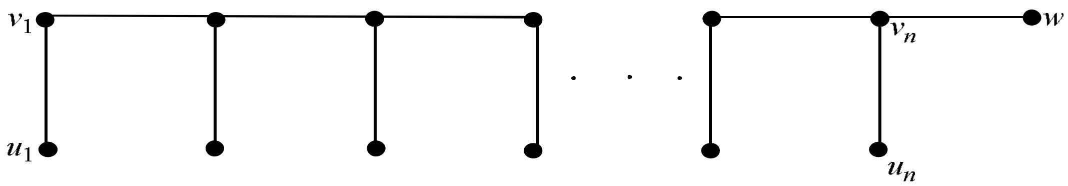

Figure 4

Here p + q = 4n + 1.Define a function f : V(G) → {1, 2, …, p + q}by $f(v_i) = 4i - 1$, $1 \le i \le n$, $f(w) = 4n + 1$,$f(u_i) = 4i - 3$, $1 \le i \le n$. Edges are labeled with $f(v_i\, v_{i+1}) = 4i$, $1 \le i \le n - 1$,$f(v_n\, w) = 4n$, $f(v_iu_i) = 4i - 2$, $1 \le i \le n$.Therefore f(V(G)) ∪ {f(e) : e ∈ E(G)} = {1, 2,…, p + q}.Then the edge labels are distinct. Hence, G is super harmonic mean graph.

Example 2.4:

A Super Harmonic Mean labeling G when n = 6 is shown below.

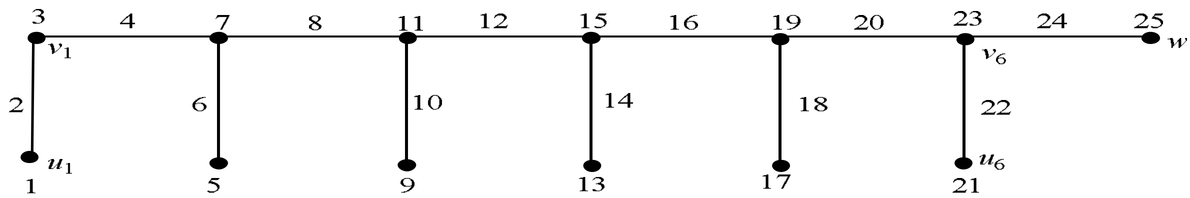

Figure 5

Theorem 2.5:

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a comb graph. Then G is a super Harmonic mean graph.

Proof :

Comb (P_n ⊙ K_1) is a graph obtained from a path $P_n = v_1v_2\ldots v_n$ by joining a vertex u_i to v_i, $1 \le i \le$ n.

Let G be a graph obtained by joining pendant vertices w and z to v_1 and v_n respectively. The graph is displayed below.

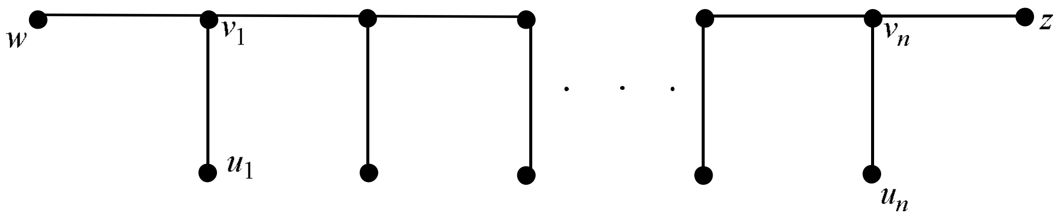

Figure 6

Here p + q = 4n + 3.Define a function f : V(G) → {1, 2, …, p + q} by f(w) = 1, $f(v_1) = 3$, $f(v_i) = 4i + 1$, $2 \le i \le n$,f(z) = 4n + 3, $f(u_1) = 6$,$f(u_i) = 4i - 1$, $2 \le i \le n$,Edges are labeled with $f(wv_1) = 2$, $f(v_1\, v_2) = 4i + 1$,$f(v_iv_{i+1}) = 4i + 2$, $2 \le i \le n - 1$,$f(v_n\, z) = 4n + 2$,$f(v_i\, u_i) = 4i$, $1 \le i \le n$, Therefore, f(V(G)) ∪ {f(e) : e ∈ E(G)} = {1, 2,…, p + q}. Hence, the edge labels are distinct. Thus f is a super harmonic mean graph.

Example 2.6:

A super harmonic mean labeling of G when n = 5 is given below.

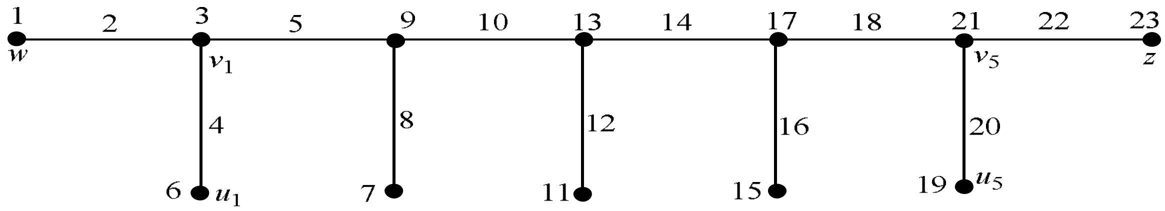

Figure 7

Theorem 2.7:

The middle graph of a path is a super harmonic mean graph.

Proof :

The middle graph M(G) of a graph G is the graph whose vertex set is {v : v ∈V(G)} ∪ {u : u ∈U(G)} and the edge set is $\{u_iu_{i+1} : u_i \in$ U(G) and u_i and u_{i+1} are adjacent edges of G} ∪ $\{v_iu_i : v_i \in V(G), u_i \in U(G)$ and v_i is incident with $u_i\}$.Then join each u_1 and u_m by a pendant vertex namely w and z respectively. Let G be a graph with vertex set V(G) ∪ U(G). Here the vertex set consists of the vertices namely, V ={w, v_1, v_2, …, v_{n-2}, z} and U = {u_1, u_2, …, u_m}. The graph is displayed below.

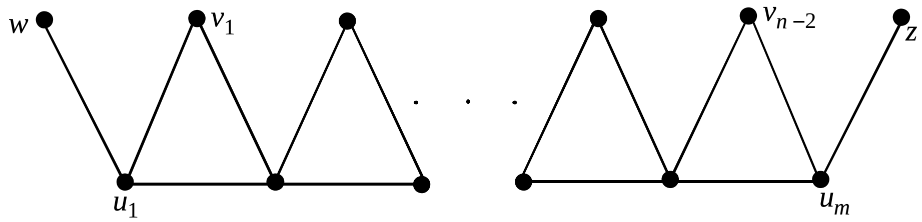

Figure 8

Here p + q =2n +3m -2, Define a function f : {v(G) ∪ U(G)} → {1, 2, …., p + q} by f(w) = 1 $f(v_i) = 5i + 1$, $1 \le i \le n-2$, $f(z) = 5(n-1)$,$f(u_i) = 5i-2$, $1 \le i \le n-1$.The edges are labeled with, $f(wu_1) = 2$,$f(v_iu_i) = 5i-1$, $1 \le i \le n-2$,$f(v_iu_{i+1}) = 5i+2$, $1 \le i \le n-2$,$f(z\,u_m) = 5m-1$,$f(u_i\,u_{i+1}) = 5i$, $1 \le i \le n-2$.Therefore, f(V(G)) ∪ {f(e) : e ∈ E(G)} = {1, 2,…, p + q}.Then the edge labels are distinct. Therefore, M(G) is a super harmonic mean graph.

Example 2.8 :

The super harmonic mean labeling of the middle graph M(G) when n = 7 and m = 6 is displayed below.

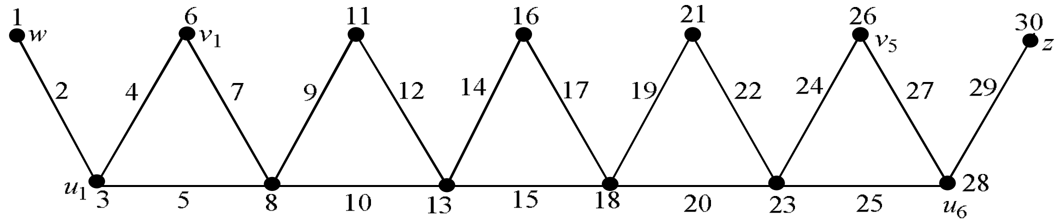

Figure 9

Theorem 2.9:

A Double Triangular Snake $D(T_n)$ attached with one pendant vertex is a super harmonic mean graph.

Proof:

Let $D(T_n)$ be the Double Triangular snake. Consider a path x, u_1, u_2, …., u_n. Join $x\, u_1$ and u_iu_{i+1} with two new vertices v_i and w_i, $1 \leq i \leq n - 1$. Let G be a graph obtained by attaching one pendant vertex with $D(T_n)$. The graph is displayed below.

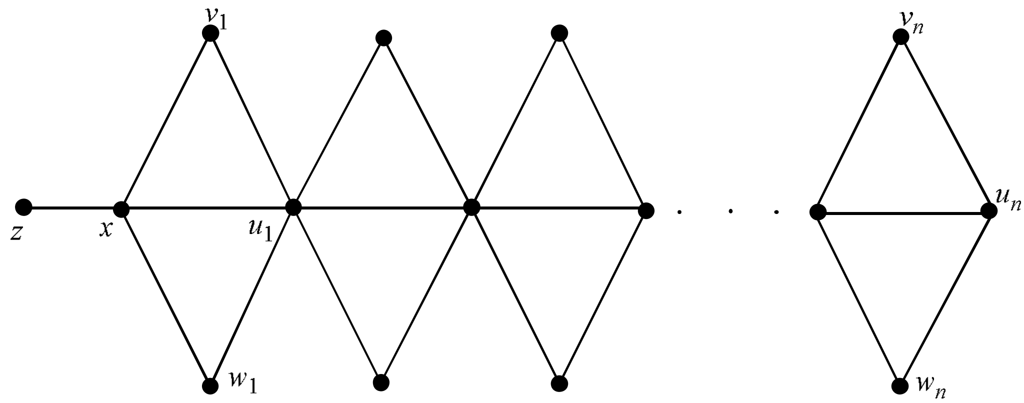

Figure 10

Here p + q = 8n + 3. Define a function $f : V(G) \rightarrow \{1, 2, \ldots, p + q\}$ by $f(z) = 1$, $f(x) = 6$, $f(u_i) = 8i + 3$, $1 \leq i \leq n$, $f(v_1) = 3$, $f(v_i) = 8i - 1$, $2 \leq i \leq n$, $f(w_i) = 8i + 1$, $1 \leq i \leq n$ Edges are labeled with, $f(zx) = 2$, $f(x\, u_1) = 8$, $f(u_iu_{i+1}) = 8j - 2$ for all $1 \leq i \leq n - 1$ and $2 \leq j \leq n$, $f(x\, v_1) = 4$, $f(v_1\, u_1) = 5$, $f(v_iu_i) = 8i$, $2 \leq i \leq n$, $f(x\, w_1) = 7$, $f(u_i\, w_i) = 8i + 2$, $1 \leq i \leq n$, $f(u_i\, v_{i+1}) = 8i + 4$, $1 \leq i \leq n - 1$, $f(u_i\, w_{i+1}) = 8i + 5$, $1 \leq i \leq n - 1$. Hence , $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p + q\}$. Then the edge labels are distinct. Therefore, G is a super harmonic mean graph.

Example 2.10:

Super harmonic mean labeling of $D(T_5)$ attached with one pendant vertex is shown below.

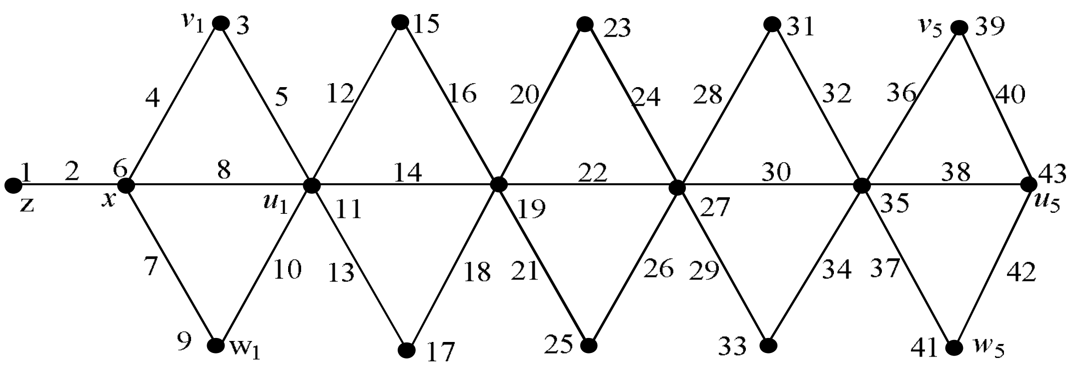

Figure 10

Theorem 2.11:

Alternate Double Triangular snake $A(D(T_n))$ is a super harmonic mean graph.

Proof:

Let G be a graph $A(D(T_n))$.Consider the path $u_1u_2 \ldots u_m$.To construct G, join u_i and u_{i+1} (alternatively) with two new vertices v_i and w_i, $1\leq i \leq n$. If $A(D(T_n))$ starts from u_2, we consider two cases.

Case 1:

In this case m = 2n+1. The graph is displayed below.

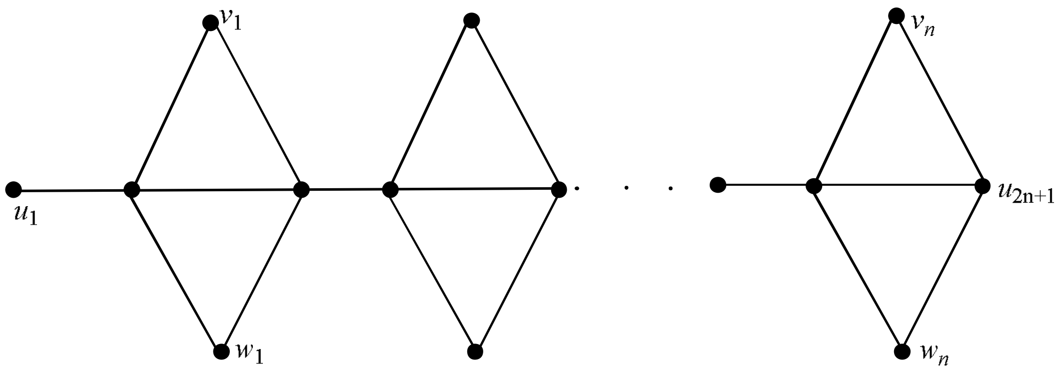

Figure 12

Here p + q = 10 n+1. Define a function $f : V(G) \to \{1, 2, \ldots, p + q\}$ by$f(u_2) = 6$, $f(u_{2i-1}) = 10i - 9$, $1 \leq i \leq n + 1$, $f(u_{2i}) = 10j + 3$, for $2 \leq i \leq n$ and $1 \leq j \leq n - 1$, $f(v_1) = 3$, $f(v_i) = 10j + 7$, $2 \leq i \leq n$, $1 \leq j \leq n - 1$, $f(w_i) = 10i - 1$, $1 \leq i \leq n$. The edges are labeled with $f(u_{2i-1}\, u_{2i}) = 10i - 8$, $1 \leq i \leq n$,$f(u_2\, u_3) = 8$, $f(u_{2i}\, u_{2i+1}) = 10i - 4$, $2 \leq i \leq n$,$f(v_i\, u_{2i}) = 10i - 6$, $1 \leq i \leq n$,$f(v_1u_3) = 5$,$f(v_i\, u_{2i+1}) = 10i - 2$, $2 \leq i \leq n$,$f(u_2\, w_1) = 7$, $f(u_{2i}\, w_i) = 10i - 5$, $2 \leq i \leq n$,$f(u_{2i+1}\, w_i) = 10i$, $1 \leq i \leq n$ .Hence $f(V(G)) \cup \{f(e) / e \in E(G)\} = \{1, 2,\ldots, p + q\}$. Therefore, we get distinct edge labels.The labeling pattern of $A(D(T_4))$ is shown below.

Example 2.12:

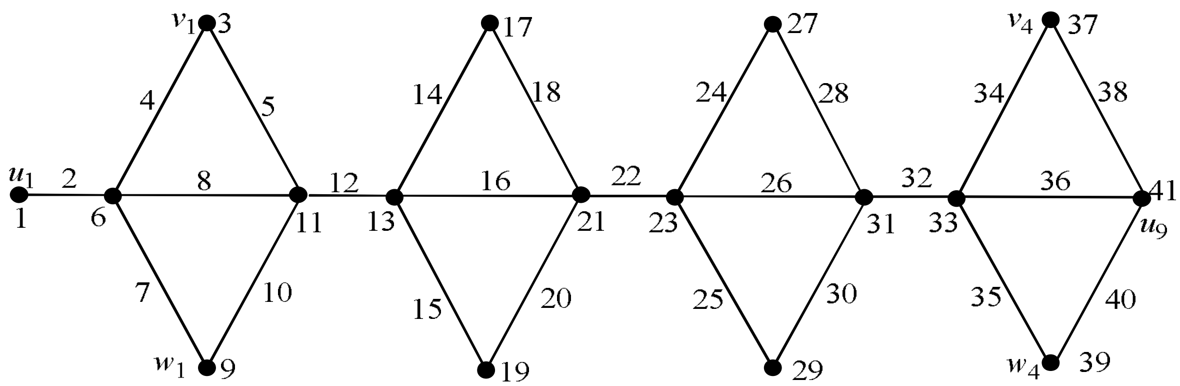

Figure 13

In this case f provides a super harmonic mean labeling of G.

Case 2 :

In this case, $m = 2n + 2$,The graph is displayed below

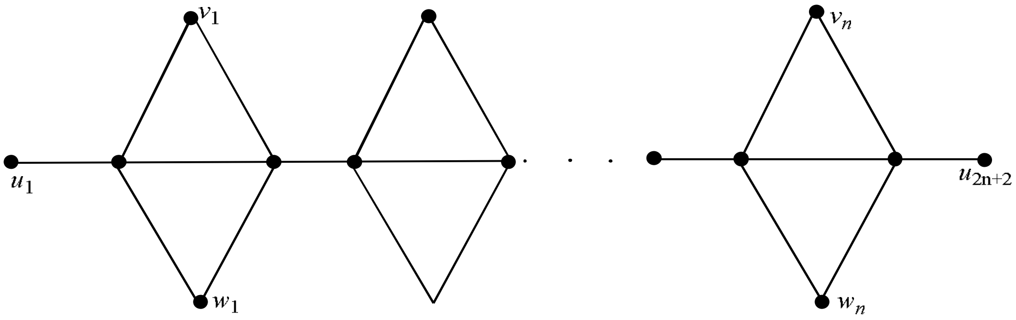

Figure 14

Here $p+q = 10n + 3$.Define a function $f : V(G) \rightarrow \{1, 2,\ldots, p + q\}$ by,$f(u_2) = 6$,$f(u_{2i-1}) = 10i - 9$, $1 \leq i \leq n + 1$, $f(u_{2i}) = 10j + 3$, $2 \leq i \leq n + 1$and $1 \leq j \leq n$,$f(v_1) = 3$,$f(v_i) = 10j + 7$, $2 \leq i \leq n$ and $1 \leq j \leq n - 1$, $f(w_i) = 10i - 1$, $1 \leq i \leq n$. The edges are labeled with $f(u_{2i-1} u_{2i}) = 10i - 8$, $1 \leq i \leq n + 1$, $f(u_2 u_3) = 8$,$f(u_{2i} u_{2i+1}) = 10i - 4$, $2 \leq i \leq n$,$f(v_i u_{2i}) = 10i - 6$, $1 \leq i \leq n$,$f(v_1 u_3) = 5$, $f(v_i u_{2i+1}) = 10i - 2$, $2 \leq i \leq n$,$f(u_2 w_1) = 7$, $f(u_{2i} w_i) = 10i - 5$, $2 \leq i \leq n$,$f(u_{2i+1} w_i) = 10i$, $1 \leq i \leq n$.Hence , $f(V(G)) \cup \{f(e) / e \in E(G)\} = \{1, 2,\ldots, p + q\}$. Hence the edge labels are distinct. The labeling pattern of $A(D(T_4))$ is shown below.

Example 2.13:

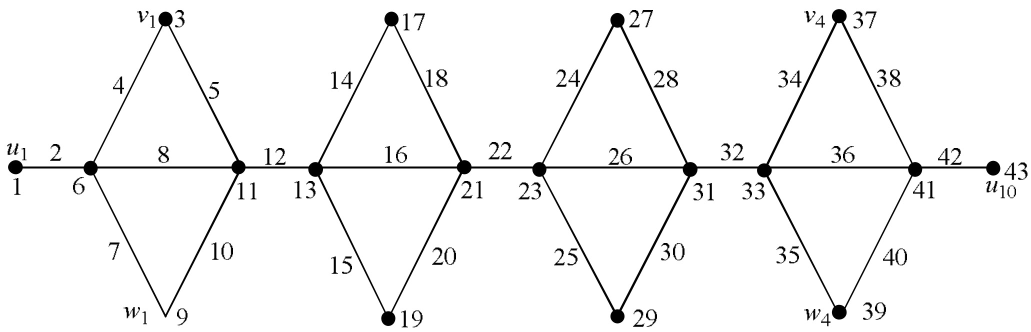

Figure 15

In this case also, f provides a super harmonic mean labeling of G. Therefore, In both cases, $A(D(T_n))$ is a super harmonic mean graph.

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