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Triple Infinite Integral Representation For The Polynomial Set $R_n(x_1, x_2, x_3)$

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ABSTRACT

In the present paper, an attempt has been made to express a Triple Infinite Integral Representation for the polynomial set $R_n(x_1, x_2, x_3)$. Many interesting new results may be obtained as particular cases on separating the parameter.

KEYWORDS: Appell Function, Generalized Hyper geometric Polynomial, Integral Representation, Lauricella function.

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1. INTRODUCTION

We defined the generalized hyper geometric polynomial set $R_n(x_1, x_2, x_3)$ by means of generating relation,

$$(1-\nu t)^{-\lambda}\left(1\text{-}\mu_1 x_2^{r_1} t^{r_1}\right)^{-\lambda_1}$$

$$F\left[\begin{matrix}(A_p);(C_u);(E_h)(G_m) \\ \\ (B_q);(D_v);(F_k);(H_w)\end{matrix}\ \mu x_1^{r_1} t, \mu_2 x_2^{-r_2} t^{r_2}, \mu_3 x_3^{-r_3} t^{r_3}\right]$$

$$=\sum_{n=0}^{\infty} R_{n,r;r_1;r_2;r_3;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\mu;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1,x_2,x_3)t^n \qquad \ldots (1.1)$$

Whereν, λ, λ_1, μ, μ_1, μ_2, $\mu_3$are real and r, $r_1$are non-negative integer and r_2, $r_3$are natural numbers.

The left hand side of (1.1) contains the product of generalized hyper geometric function and Lauricella function in the notation of Burchanall and Chaundy[1].

The polynomial set contains number of parameters, for simplicity we shall denote

$$R_{n,r;r_1;r_2;r_3;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\mu;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1,x_2,x_3)$$

by R_n (x_1, x_2, x_3).

where n denotes the order of the polynomial set.

After little simplification (1.1) gives

$$R_n(x_1,x_2,x_3)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{r_2=0}^{\left[\frac{n-r-r_1s_1}{e_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{e_3}\right]}$$

$$\times\frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_1-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times\frac{\left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s(\lambda_1)_{s_1}\,\nu^s\mu_1^{s_1}\mu_2^{s_2}\mu_3^{s_3}}{\left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}\,s!s_1!s_2!s_3!}$$

$$\times \frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3} \quad x_2^{r_1s_1+r_2s_2}}{\left(n-r-r_1s_1-r_2s_2-r_3s_3\right)!\, x_3^{r_3s_3}} \quad \text{.. (1.2)}$$

2. NOTATIONS

I.(i) $(n) = 1, 2, 3, \ldots\ldots\ldots n-1, n.$

(ii) $(a_p) = a_1, a_2, a_3, \ldots\ldots\ldots a_p.$

(iii) $(a_p\,; i) = a_1, a_2, \ldots\ldots a_{i-1}, a_{i+1} \ldots\ldots\ldots a_p.$

II.(i) $[(a_p)] = a_1 \cdot a_2 \cdot a_3, \ldots\ldots\ldots a_p.$

(ii) $\left[(a_p)\right]_n = \prod_{i=1}^{p} (a_i)_n = (a_1)_n (a_2)_n \ldots\ldots (a_p)_n.$

III.(i) $\Delta(a, b) = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots + \frac{b+a-1}{a}.$

(ii) $\Delta_k(a; b) = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \ldots\ldots \left(\frac{b+a-1}{a}\right)_k$

$$= \prod_{r=1}^{a} \left(\frac{b+r-1}{a}\right)_k$$

(iii) $\Delta\left[m; (a_p)\right] = \prod_{i=1}^{p} \prod_{r=1}^{m} \left(\frac{a_i + r - 1}{m}\right)_k.$

IV.(i) $\Gamma\left[(a_p)\right] = \prod_{i=1}^{p} \Gamma(a_i).$

(ii) $\Gamma\left[(a_p); s\right] = \prod_{i=s+1}^{p} \Gamma(a_i).$

(iii) $\Gamma\left[a + \frac{(m)}{m}\right] = \prod_{r=1}^{m} \Gamma\left(a + \frac{r}{m}\right).$

(iv) $\Gamma\left[\Delta(a; b)\right] = \prod_{r=1}^{a} \Gamma\left(\frac{b+r-1}{a}\right).$

V.(i)

(ii) $\Gamma_*(a+b)=\Gamma(a+b)\Gamma(a-b).$

VI.(i) $M_1=\dfrac{\left[(A_p)\right]_n\left[(C_u)\right]_n\left(\mu x_1^{r_4}\right)^n}{\left[(B_q)\right]_n\left[(D_v)\right]_n n!}$

3. THEOREM

For $r_2 > 1$ and $r_3 > 1$

$$R_n(x_1,x_2,x_3)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)M_1}{\sqrt{\pi}i^n\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times F_{p+u\,:\,k\,:\,w}^{1+q+v\,:\,h\,:\,m}\left[\begin{matrix}[-n:r,r_1,r_2,r_3],\left[1-(B_q)-n:r,r_1,r_2-1,r_3-1\right]\\ \rule{3cm}{0.4pt},\quad\left[1-(A_h)-n:r,r_1,r_2-1,r_3-1\right]\end{matrix}\right.$$

$$\left[(1-(D_v)-n):r,r_1,r_2,r_3\right],\left[(E_h):1\right]\left[(G_m):1\right][\lambda:1],[\lambda_1:1],\left[\left(1+\frac{\nu}{2}+u\right):1\right]$$

$$\left[(1-(C_u)-n):r,r_1,r_2,r_3\right],\left[(F_k):1\right]\left[(H_w):1\right],\left[\left(\frac{1}{2}+\frac{\nu}{2}\right):1\right]\left[\left(1+\frac{\nu}{2}\right):1\right]$$

$$\frac{\nu(-1)^{r(p+q+u+v+1)}}{\left(\mu x_1^{r_4}\right)^r},\frac{\mu_1x_2^{r_1}(-1)^{r_1(p+q+u+v+1)}}{\left(\mu x_1^{r_4}\right)^{r_1}},$$

$$\left.\frac{\mu_2x_2^{r_2}(-1)^{r_2(p+q+u+v+1)+p+q}}{\left(\mu x_1^{r_4}\right)^{r_2}},\frac{\mu_3(-1)^{r_3(p+q+u+v+1)+p+q}}{\left(\mu x_1^{r_4}x_3\right)^{r_3}}\right]dxdydz \qquad \ldots(3.1)$$

Proof : we have

$$I=\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1 s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1-r_2 s_2}{r_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(E_h)\right]_{s_2}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(F_k)\right]_{s_2}}$$

$$\times \frac{\left[(G_m)\right]_{s_3} (\lambda)_s \, \nu^s \, (\lambda_1)_{s_1} \, \nu_1^{s_1} \, x_2^{r_1 s_1 + r_2 s_2} \, \mu_3^{s_3} \left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3}}{\left[(H_w)\right]_{s_3} \; s! \; s_1! \; s_2! \; x_3^{r_3 s_3} (n-r-r_1s_1-r_2s_2-r_3s_3)!}$$

$$\times \frac{\left(1+\frac{\nu}{2}\pm\mu\right)_{s_1}}{\left(\frac{1}{2}+\frac{\nu}{2}\right)_{s_1} \left(1+\frac{\nu}{2}\right)_{s_1}}$$

$$= \int_0^\infty \int_0^\infty \int_0^\infty (x^2+y^2)^{\frac{\nu+2s_1-1}{2}} (x^2+y^2+z^2)^{-\frac{\nu-2s_1}{2}} \exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times \cos\left[2\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right] f\left(x^2+y^2+z^2\right)$$

$$\times \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1 s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1-r_2 s_2}{r_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}}$$

$$\times \frac{\left[(E_h)\right]_{s_2} \left[(G_m)\right]_{s_3} (\lambda)_s \, \nu^s \, (\lambda_1)_{s_1} \, \nu^{s_1} \, x_2^{r_1 s_1 + r_2 s_2} \, \mu_2^{s_2} \, \mu_3^{s_3}}{\left[(F_k)\right]_{s_2} \left[(H_w)\right]_{s_3} \; s! \; s_1! \; s_2! \; x_3^{r_3 s_3} \; s_3!}$$

$$\times \frac{\left(\lambda x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3} \left(1+\frac{\nu}{2}\pm\mu\right)_{s_1} dxdydz}{(n-r-r_1s_1-r_2s_2-r_3s_3)! \left(\frac{1}{2}+\frac{\nu}{2}\right)_{s_1} \left(1-\frac{\nu}{2}\right)_{s_1}}$$

$$= \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1 s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1-r_2 s_2}{r_3}\right]} \frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(E_h)\right]_{s_2} \left[(G_m)\right]_{s_3} (\lambda)_s \, \nu^s \, (\lambda_1)_{s_1} \, \mu_1^{s_1} x_2^{r_1 s_1 + r_2 s_2}}{\left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(F_k)\right]_{s_2} \left[(H_w)\right]_{s_3} \; s! \; s_1! \; s_2!}$$

$$\times \frac{\mu_2^{s_2} \, \mu_3^{s_3} \left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3} \left(1+\frac{\nu}{2}\pm\mu\right)_{s_1}}{x_3^{r_3 s_3} \; s_3! \; (n-r-r_1s_1-r_2s_2-r_3s_3)! \left(\frac{1}{2}+\frac{\nu}{2}\right)_{s_1} \left(1-\frac{\nu}{2}\right)_{s_1}}$$

$$\times \frac{i^n \sqrt{\pi}\ \Gamma\left(\frac{1}{2} \pm \mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}+s_1\right)\Gamma\left(1+\frac{\nu}{2}+s_1\right)}{\left(1+\frac{\nu}{2}\right)_{s_1}\ 8\ \Gamma\left(1+\frac{\nu}{2}\pm\mu+s_1\right)} \times \int_0^\infty f(t) J_n(t)\,dt$$

$$= \frac{i\sqrt{\pi}\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}{8\ \Gamma\left(1+\frac{\nu}{2}\pm\mu\right)}$$

$$\times M_1 \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1 s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1-r_2 s_2}{r_3}\right]}$$

$$\times \frac{\left[1-(B_q)-n\right]_{r+r_1 s_1+(r_2-1)s_2+(r_3-1)s_3}\ \left[1-(D_\nu)-n\right]_{r+r_1 s_1+r_2 s_2+r_3 s_3}}{\left[1-(A_p)-n\right]_{r+r_1 s_1+(r_2-1)s_2+(r_3-1)s_3}\ \left[1-(C_u)-n\right]_{r+r_1 s_1+r_2 s_2+r_3 s_3}}$$

$$\times \frac{\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}\,\nu_1^{s_1}\ (-1)^{r(p+q+u+\nu+1)}\,(-1)^{r_2(p+q+u+\nu+1)s_1}}{\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}\quad s!\quad s_1!\ \left(\mu x_1^{r_4}\right)^{r_1}\ \left(\mu x_1^{r_4}\right)^{r_1 s_1}\ s_2!}$$

$$\times \frac{x_2^{r_1 s_1}\ \mu_2^{s_2} x_2^{r_2 s_2}\,(-1)^{\{r_2(p+q+u+\nu+1)+p+q\}s_2}\,(-n)_{r+r_1 s_1+r_2 s_2+r_3 s_3}}{\left(\mu x_1^{r_4}\right)^{r_2 s_2}\quad s_3!\quad x_3^{r_3 s_3}}$$

$$\times \frac{\mu_3^{s_3}\,(-1)^{\{r_3(p+q+u+\nu+1)+p+q\}s_3}}{\left(\mu x_1^{r_4} x_3\right)^{r_3 s_3}} \int_0^\infty f(t)\ J_n(t)\,dt \qquad \ldots (3.2)$$

The single terminating factor makes all summation in (3.2) run upto ∞, and we finally achieve

$$= \frac{i^n\sqrt{\pi}\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)} R_n(x_1,x_2,x_3)\int_0^\infty f(t) J_n(t)\,dt$$

on using[2]

$$\int_0^\infty\int_0^\infty\int_0^\infty (x^2+y^2)^{\frac{\nu-1}{2}}(x^2+y^2+z^2)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right] f\left(x^2+y^2+z^2\right)dxdydz$$

$$= \frac{i^n\sqrt{\pi}\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1+\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)}$$

PARTICULAR CASES(3.1)

On specializing the values of the parameters involved in Lauricella form, a number of known and unknown polynomials can be obtained as the particular cases of polynomial set $R_n(x_1,x_2,x_3)$. Some of them, which are well known, are listed below.

(I) Hermite Polynomials

On setting $p = 0 = q = u = v = s$; $\lambda = 1 = \nu = \lambda_1 = r = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$

and writing x for x_1 in (3.1) we achieve

$$H_n(x) = \frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(2x)^n}{\sqrt{\pi}\, i^n\, \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)\, J_n(t)\,dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty \left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right] f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, \left[\left(1+\frac{\nu}{2}\pm\mu\right):1\right]; \\ \\ \left(\frac{1}{2}+\frac{\nu}{2}:1\right), \left(1+\frac{\nu}{2}:1\right); \end{matrix} \; -\frac{1}{x^2}\right] dxdydz$$

where $H_n(x)$ are the Hermite Polynomials.

(II) Bedient Polynomials[3]

If we set $q = 0 = s$; $p = 1 = u = v = \lambda = \lambda_1 = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$, $A_1 = \alpha$,

$C_1 = \beta$, $D_1 = \alpha + \beta$ and writing x_1 for x in (3.1), we get

$$G_m(\alpha,\beta,x) = \frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(\alpha)_n(\beta)_n(2x)^n}{(\alpha+\beta)_n\sqrt{\pi}\, n!\, i^n\, \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)\, J_n(t)\,dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty \left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right] f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, 1-\alpha-\beta-n, \left[\left(1+\frac{\nu}{2}\pm\mu\right):1\right]; \\ \\ 1-\alpha-n, 1-\beta-n, \left(\frac{1}{2}+\frac{\nu}{2}:1\right), \left(1+\frac{\nu}{2}:1\right); \end{matrix} \; -\frac{1}{x^2}\right] dxdydz$$

Where $G_m(\alpha, \beta, x)$ are Bedient Polynomials.

(III). Sylvester Polynomials[4]

If we put $p = 0 = q = u = v = s$; $\lambda = 1 = r_1 = \nu$; $= \mu_1 = \mu = r$; $\lambda_1 = x$ and writing

x for x_1 in (3.1) we arrive at

$$\phi_n(x) = \frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)\ x^n}{\sqrt{\pi}\ i^n\ n!\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)\ J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty \left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right] f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix} -n, x, \left(1+\frac{\nu}{2}\pm\mu:1\right); \\ \qquad\qquad\qquad \frac{1}{x} \\ \left[\left(\frac{1}{2}+\frac{\nu}{2}\right):1\right], \left(1+\frac{\nu}{2}:1\right); \end{matrix}\right] dxdydz$$

where $\Phi_n(x)$ are the Sylvester Polynomials.

(IV) Laguerre Polynomials

On putting $p = 0 = q = u = v = h = s$; $k = 1 = \lambda = \nu = \mu_2 = \mu = r_4 = r_2 = x_1$; $x_2 = y, F_1 = 1 + \alpha$ in (3.1), we obtained

$$L_n^{(\alpha)}(x) = \frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(1+\alpha)_n}{n!\sqrt{\pi}\ i^n\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)\ J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty \left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right] f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix} -n, x, \left(1+\frac{\nu}{2}\pm\mu\right):1; \\ \qquad\qquad\qquad y \\ 1+\alpha, \left(\frac{1}{2}+\frac{\nu}{2}:1\right), \left(1+\frac{\nu}{2}:1\right); \end{matrix}\right] dxdydz$$

where $L_n^{(\alpha)}(y)$ are the Laguerre Polynomials.

(V) Jacobi Polynomials

On making the substitution $p = 0 = q = u = h = s = r$; $r_2 = v = 1 = k = \lambda = \nu = \mu_2 = r_4$; $\mu = 1 = \mu_1$; $D_1 = 1 + \alpha$; $F_1 = 1 + \beta$; and instead of $x_2 = \frac{x-1}{x+1}$ in (3.1), we get

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\beta)_n\ 8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(x-1)^n}{n!\tau^n\sqrt{\pi}\ i^n\ \Gamma\left(\frac{1}{2}\pm\frac{\mu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)\ J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty \left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix} -n,-\alpha-n,\left(1+\frac{\nu}{2}\pm\mu\right):1; \\ \frac{x+1}{x-1} \\ 1+\beta,\left(\frac{1}{2}+\frac{\nu}{2}:1\right),\left(1+\frac{\nu}{2}:1\right); \end{matrix}\right]dxdydz$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

4. CONCLUSION

These polynomials are of at most important for scientists, engineers and physical sciences, because these occurs in the solution of integral equation and analytic function, which describe physical problems. The newly defined polynomials may be immense use in new phase of Mathematics relevant to Physics, Chemistry, Engineering and Social Sciences.

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