Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Infinite Single Integral Representation for the Polynomial Set $R_n(x_1, x_2, x_3)$

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ABSTRACT

In the present paper, an attempt has been made to express a Infinite Single Integral Representation for the polynomial set $R_n(x_1, x_2, x_3)$. Many interesting new results may be obtained as particular cases on separating the parameter.

AMS Subject Classification: Special function-33

KEYWORD: Appell Function, Generalized Hyper geometric Polynomial, Integral Representation, Lauricella function.

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1. INTRODUCTION

We defined the generalized hypergeometric polynomial set $R_n(x_1, x_2, x_3)$ by means of generating relation,

$$(1-\nu t)^{-\lambda}\left(1-\mu_1 x_2^{r_1} t^{r_1}\right)^{-\lambda_1}$$

$$F\left[\begin{matrix}(A_p);(C_u);(E_h)(G_m) \\ (B_q);(D_v);(F_k);(H_w)\end{matrix}\ \mu x_1^{r_1} t, \mu_2 x_2^{-r_2} t^{r_2}, \mu_3 x_3^{-r_3} t^{r_3}\right]$$

$$= \sum_{n=0}^{\infty} R_{n,r;r_1;r_2;r_3;r_4;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\lambda_2;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1, x_2, x_3)\, t^n \quad \ldots (1.1)$$

whereν, λ, λ_1, λ_2, $\lambda_3$are real and r, $r_1$are non-negative integer ande_2, e_3 are natural numbers.

The left hand side of (1.1) contains the product of generalized hyper geometric function and Lauricella function in the notation of Burchanall and Chaundy[1].

The polynomial set contains number of parameters, for simplicity we shall denote

$$R_{n,r;r_1;r_2;r_3;r_4;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\lambda_2;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1, x_2, x_3)$$

By$R_n(x_1, x_2, x_3)$.

where n denotes the order of the polynomial set.

After little simplification (1.1) gives

$$R_n(x_1, x_2, x_3) = \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{r_2=0}^{\left[\frac{n-r-r_1 s_1}{e_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1 - r_2 s_2}{e_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1 s_1-(r_2-1)s_1-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1 s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[(C_u)\right]_{n-r-r_1 s_1-r_2 s_2-r_3 s_3}\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s(\lambda_1)_{s_1}\nu^s \mu_1^{s_1}\mu_2^{s_2}\mu_3^{s_3}}{\left[(D_v)\right]_{n-r-r_1 s_1-r_2 s_2-r_3 s_3}\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3} s!\, s_1!\, s_2!\, s_3!}$$

$$\times \frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1 s_1-r_2 s_2-r_3 s_3} x_2^{r_1 s_1+r_2 s_2}}{(n-r-r_1 s_1-r_2 s_2-r_3 s_3)!\, x_3^{r_3 s_3}} \quad \ldots (1.2)$$

2. NOTATIONS

I.(i)(n) = 1, 2, 3, ……… n – 1, n.

(ii) $(a_p) = a_1, a_2, a_3,$ ……… a_p.

(iii)$(a_p\ ;i) = a_1, a_2,$ …… a_{i-1}, a_{i+1} ……… a_p.

II.(i) $[(a_p)] = a_1 \cdot a_2 \cdot a_3,$……… a_p.

(ii)$\left[(a_p)\right]_n = \prod_{i=1}^{p}(a_i)_n = (a_1)_n\,(a_2)_n \ldots\ldots (a_p)_n$.

III.(i)$\Delta(a,b) = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots + \frac{b+a-1}{a}$.

(ii)$\Delta_k(a;b) = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \ldots\ldots \left(\frac{b+a-1}{a}\right)_k$

$= \prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k$

(iii)$\Delta\left[m;(a_p)\right] = \prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k$.

IV.(i)$\Gamma\left[(a_p)\right] = \prod_{i=1}^{p}\Gamma(a_i)$.

(ii)$\Gamma\left[(a_p);s\right] = \prod_{i=s+1}^{p}\Gamma(a_i)$.

(iii)$\Gamma\left[a+\frac{(m)}{m}\right] = \prod_{r=1}^{m}\Gamma\left(a+\frac{r}{m}\right)$.

(iv)$\Gamma\left[\Delta(a;b)\right] = \prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right)$.

V.(i)$\Gamma_*\left(a \pm b\right) = \Gamma\left(a+b\right)\Gamma\left(a-b\right)$.

(ii) $\Gamma_*(a+b) = \Gamma(a+b)\Gamma(a-b)$.

VI.(i)$M_1 = \frac{\left[(A_p)\right]_n \left[(C_u)\right]_n \left(\mu x_1^{r_4}\right)^n}{\left[(B_q)\right]_n \left[(D_v)\right]_n n!}$

3. THEOREM

For $r_2 > 1$, and $r_3 > 1$, we have

$$R_n(x_1,x_2,x_3) = \frac{s^a\ \Gamma(d-c)\Gamma(d-a)\ M_1}{\Gamma(a)\Gamma(d)\Gamma(d-c-a)} \int_0^\infty e^{-ax} x^{a-1} \begin{bmatrix} c; & \\ & a^x \\ d; & \end{bmatrix}$$

$$\times F^{1+q+v\ :\ h\ :\ m\ :\ 1}_{p+u\ :\ k\ :\ w} \left[\begin{matrix} [-n:r,r_1,r_2,r_3],\left[1-\left(B_q\right)-n:r,r_1,r_2-1,r_3-1\right] \\ \text{———————}, \quad \left[1-\left(A_p\right)-n:r,r_1,r_2-1,r_3-1\right] \end{matrix}\right.$$

$$\left[(1-(D_v)-n):r,r_1,r_2,r_3\right],\left[(E_h):1\right]\left[(G_m):1\right][\lambda:1],[\lambda_1:1],\left[(a):1\right],\left[d-(-a):1\right]$$

$$\left[(1-(C_u)-n):r,r_1,r_2,r_3\right],\left[(F_k):1\right]\left[(H_w):1\right],[d-a;1],\text{——};$$

$$\frac{\nu(-1)^{r(p+q+u+\nu+1)}}{\left(\mu x_1^{r_4}\right)^{r}}, \frac{\mu_1 x_2^{r_1}(-1)^{r_1(p+q+u+\nu+1)}}{\left(\mu x_1^{r_4}\right)^{r_1} \quad s},$$

$$\left.\frac{\mu_2 x_2^{r_2}(-1)^{r_2(p+q+u+\nu+1)+p+q}}{\left(\mu x_1^{r_4}\right)^{r_2}}, \frac{\mu_3(-1)^{r_3(p+q+u+\nu+1)+p+q}}{\mu\left(x_1^{r_4} x_3\right)^{r_3}}\right] dx \qquad \ldots (3.1)$$

Proof : we have

$$I_5 = \int_0^\infty \rho^{-sx} x_1^{a-1} \; {}_1F_1\left[\begin{matrix} c; \\ \quad sx_1 \\ d; \end{matrix}\right] \times \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(C_u)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(D_\nu)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[(E_h)\right]_{s_2} \left[(G_m)\right]_{s_3} (\lambda)_s \, \nu^s \, (\lambda_1)_{s_1} \, \mu_1^{s_1} \, x_2^{r_1s_1+r_2s_2} \, \mu_2^{s_2} \, \mu_3^{s_3}}{\left[(F_k)\right]_{s_2} \left[(H_w)\right]_{s_3} \quad s! \quad s_1! \quad s_2! \quad s_3! \; x_3^{r_3s_3}}$$

$$\times \frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3} s^{s_1} (d-a)_{s_1}}{(n-r-r_1s_1-r_2s_2-r_3s_3)! \;\; (\alpha)_{s_1} \;\; (d-c-a)_{s_1}}$$

$$= \int_0^\infty \rho^{-sx} x_1^{a+s_1-1} F\left[\begin{matrix} c; \\ \quad sx \\ d; \end{matrix}\right] \times \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(E_h)\right]_{s_2}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(D_\nu)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3} \left[(F_k)\right]_{s_2}}$$

$$\times \frac{\left[(G_m)\right]_{s_3} \;\; (\lambda)_s \;\; \nu^s \;\; (\lambda_1)_{s_1} \;\; \nu_1^{s_1} \;\; x_2^{r_1s_1+r_2s_2} \;\; \mu_2^{s_2} \;\; \mu_3^{s_3}}{\left[(H_w)\right]_{s_3} \quad s! \quad s_1! \quad s_2! \quad s_3! \; x_3^{r_3s_3} (n-r-r_1s_1-r_2s_2-r_3s_3)!}$$

$$\times \frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3} s^{s_1} (d-a)_{s_1} \Gamma(a+s_1)\Gamma(d)\Gamma(d-c-a-s_1)}{(a)_{s_1} \quad (d-c-a)_{s_1} \quad s^{a+s_1} \quad \Gamma(d-c) \quad \Gamma(d-a-s_1)}$$

$$= M_1 \frac{\Gamma(a)\Gamma(d)\Gamma(d-c-a)}{s^a \Gamma(d-c)\Gamma(d-a)} \times \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{\left[1-(B_q)-n\right]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3} \left[1-(D_\nu)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}}{\left[1-(A_p)-n\right]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3} \left[1-(C_u)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(-1)^{r(p+q+u+v+1)}\,\mu_1^{s_1}\,(-1)^{r_1(p+q+u+v+1)s_1}\,x_2^{r_1 s_1}\,(\lambda_1)_{s_1}}{[(F_k)]_{s_2}\,[(H_w)]_{s_3}\quad s!\quad (\mu x_1^{r_4})^{r+r_1s_1+r_2s_2+r_3s_3}\quad s_1!}$$

$$\times \frac{\mu_2^{s_2}\; x_2^{r_2 s_2}\;(-1)^{r_2(p+q+u+v+1)+p+q\}s_2}\;\mu_3^{s_3}}{s_2!\qquad s^{s_1}} \quad \dots (3.2)$$

The single terminating factor $(-n)_{r+r_1s_1+r_2s_2+r_3s_3}$ makes all summation in (3.2), runs upto∞.

$$= \frac{M_1\,\Gamma(a)\Gamma(d)\Gamma(d-c-a)}{s^a\quad \Gamma(d-c)\,\Gamma(d-a)} \sum_{s=0}^{\infty}\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{\infty}\sum_{s_3=0}^{\infty}$$

$$\times \frac{[1-(B_q)-n]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}\;[1-(D_v)-n]_{r+r_1s_1+r_2s_2+r_3s_3}}{[1-(A_p)-n]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}\;[1-(C_u)-n]_{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(-1)^{r(p+q+u+v+1)}\,(\lambda_1)_{s_1}\,\mu_1^{s_1}\,x_2^{r_1s_1}}{[(F_k)]_{s_2}\,[(H_w)]_{s_3}\quad s!\quad (\mu x_1^{r_4})^{r}\quad s^{s_1}\; s_1!\;(\mu x_1^{r_4})^{r_1s_1}}$$

$$\times \frac{(-1)^{r_1(p+q+u+v+1)s_1}\,(-1)^{r_2(p+q+u+v+1)+p+q\}s_2}\;x_2^{r_2s_2}\;\mu_2^{s_2}}{(\mu x_1^{r_4})^{r_2s_2}}$$

$$\times \frac{\mu_3^{s_3}\,(-1)^{r_3(p+q+u+v+1)+p+q\}s_3}}{(\mu x_1^{r_4} x_3)^{r_3s_3}}$$

$$= M_1 \frac{\Gamma(a)\Gamma(d)\Gamma(d-c-a)}{s^a\;\Gamma(d-c)\;\Gamma(d-a)} R_n(x_1,x_2,x_3)$$

On using[2]

$$\int_0^\infty e^{-st} t^{a-1}\; {}_1F_1\left[\begin{matrix} c; \\ \\ d; \end{matrix}\quad st\right] dt = \frac{\Gamma(a)\Gamma(d)\Gamma(d-c-a)}{s^a\;\Gamma(d-c)\;\Gamma(d-a)}$$

Where $R_e(a)$, $R_e(s) > 0$, $R_e(d) > R_e(c+a)$.

PARTICULAR CASES OF (3.1)

On specializing the values of the parameters involved in Lauricella form, a number of known and unknown polynomials can be obtained as the particular cases of polynomial set $R_n(x_1,x_2,x_3)$. Some of them, which are well known, are listed below.

I. GegenbouerPolynomials

On taking $p = 0 = q = u = h = s$; $v = 0 = k = r = r_2 = r_4 = \lambda = \nu = x_2$; $D_1 = \lambda + \frac{1}{2} = F_1$; $\mu = \frac{1}{2} = \mu_2$

andwriting $\frac{x+1}{x-1}$ for x_1 in (3.1), we get

$$C_n^{\lambda}(x)=\frac{s^a\ \Gamma(d-a)\Gamma(d-c)(2\lambda)_n}{\Gamma(a)\ \Gamma(d)\ \Gamma(d-c-a)n!}\left(\frac{x-1}{2}\right)^n\int_0^{\infty}e^{-sx}t^{a-1}F\begin{bmatrix}c; & \\ & sx\\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix}-n,\frac{1}{2}-\lambda-n\ (a:1),(d-c-a:1); & \\ & \frac{x-1}{x+1}\times\frac{1}{s}\\ \lambda+\frac{1}{2},(d-a:1); & \end{bmatrix}dx$$

where $C_n^{\lambda}(x)$ are the Gegenbouer Polynomials.

II. Gegenbouer Polynomials

On making the substitution $p = 0 = q = u = s;\ u = v = 1 = r = r_4 = x_2 = \lambda = \nu = \mu = \mu_2;$

$F_1=\lambda+\frac{1}{2}$, $r_2 = 2$ and writing $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.1),we get

$$C_n^{\lambda}(x)=\frac{s^a\ \Gamma(d-a)\Gamma(d-c)(2\lambda)_n\ x^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)}\int_0^{\infty}e^{-sx}t^{a-1}F\begin{bmatrix}c; & \\ & sx\\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix}\frac{-n}{2},\frac{-n}{2}+\frac{1}{2},(s:1),(d-c-a:1); & \\ & \frac{x^2-1}{sx^2}\\ \lambda+\frac{1}{2},(d-a:1); & \end{bmatrix}dx$$

where $C_n^{\lambda}(x)$ are the Gegenbouer Polynomials.

III. Legendre Polynomials

If we put $p = 0 = q = u = v = m;\ w = 1 = r = r_4 = \nu = \lambda = \mu = \mu_3 = r_3;\ H_1 = 1,\ r_3 = 2$

and $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.1), we get

$$P_n(x)=\frac{s^a\ \Gamma(d-a)\Gamma(d-c)x^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)}\int_0^{\infty}e^{-sx}x^{a-1}F\begin{bmatrix}c; & \\ & sx\\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix}\frac{-n}{2},\frac{-n}{2}+\frac{1}{2},(s:1),(d-c-a:1); & \\ & \frac{x^2-1}{sx^2}\\ 1,(d-a:1); & \end{bmatrix}dx$$

where $P_n(x)$ are the Legendre Polynomials

IV. Panda, Rekha Polynomials

On making the substitution $p = 0 = q = u = v$; $r = 1 = r_4 = \nu = \lambda = x_2$; $\mu_3 = \mu = \nu$, $r_3 = m$, $x_1 = x$ and replacing (G_m) by (a_r) and (H_W) by (b_S) in (3.1), we arrive at

$$A_n(x) = \frac{s^a\ \Gamma(d-a)\Gamma(d-c)(\nu x)^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)} \int_0^\infty e^{-sx} x^{a-1} F\begin{bmatrix} c; & \\ & sx \\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix} \Delta(m;-n)(a_r),(s:1),(d-c-a;1); & \\ & \frac{\mu}{s}\left(\frac{-m}{\nu x}\right)^m \\ (b_s),(d-a;1); & \end{bmatrix} dx$$

where $A_n(x)$ are the generalized polynomials defined by Panda[3]

V. Brafman Polynomials:

On putting $p = 0 = q = u = v = s$; $r = 1 = r_4 = \lambda = \nu = \mu = \mu_3 = x_1 = x_2$; $r_3 = p = \mu$; $\mu_3 = x$ in (3.1) and $G_m = \alpha_u$; $H_n = \beta_V$; we get

$$B_n^p(x) = \frac{s^a\ \Gamma(d-a)\Gamma(d-c)(-p)^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)} \int_0^\infty e^{-sx} x^{a-1} F\begin{bmatrix} c; & \\ & sx \\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix} \Delta(p;-n),\alpha_1,\alpha_2,\ldots\ldots\alpha_u,(s;1),(d-c-a:1); & \\ & \frac{x}{s} \\ (d-a;1),\beta_1,\beta_2\ldots\ldots\beta_v; & \end{bmatrix} dx$$

where $B_n^p(x)$ are the generalization of Hermite polynomials by Brafman[4]

VI. Lommel Polynomials

$q = 0 = v = m = s$; $p = 1 = u = w = \lambda = \nu = x = r_4$; $\mu_3 = -1$, $r_3 = 2 = \mu$; $A_1 = 1$, $C_1 = \nu$, $H_1 = \nu$ and z for x_1 in (3.1), we achieve

$$R_{n,\nu}\left(\frac{1}{z}\right) = \frac{s^a\ \Gamma(d-a)\Gamma(d-c)(\nu)_n(2z)^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)} \int_0^\infty e^{-sx} x^{a-1} F\begin{bmatrix} c; & \\ & sx \\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix} -\frac{n}{2},-\frac{n}{2}+\frac{1}{2},(s;1),(d-c-a;1); & \\ & \frac{-1}{sz^2} \\ \nu,-n,1-\nu-n,(d-a:1); & \end{bmatrix} dx$$

where $R_{n,\nu}\left(\frac{1}{z}\right)$ are the Lommel Polynomials[5]

VII. Khanna, I.K Polynomials

On taking $p = 0 = q = s$; $u = 1 = v = m = w = \lambda = \nu = r = r_4 = x_2 = y$, $r_3 = m$, $\mu_3 = \mu$; $\mu = \nu$ and instead $(D_u) = (\beta_q)$; $x_1 = x$ $(C_u) = (\alpha_p)$, $(G_m) = (a_r)$ and $(H_w) = (b_s)$ in (3.1), we arrive at

$$B_n(x,y) = \frac{s^a\ \Gamma(d-a)\Gamma(d-c)\left[(\alpha_p)\right]_n (\nu x)^n}{n!\ \Gamma(a)\ \Gamma(d)\ \Gamma(d-a-c)\left[(\beta_q)\right]_n} \int_0^\infty e^{-sx} x^{a-1} F\begin{bmatrix} c; & \\ & sx \\ d; & \end{bmatrix}$$

$$\times F\begin{bmatrix} \Delta(m;-n), \Delta\left(m, 1-(\beta_q)-n\right), (a_r), \\ (s;1), (d-c-a:1); \\ \qquad\qquad \dfrac{\mu(-m)^{m(q-p+1)}}{s(\nu xy)^m} \\ \Delta\left(m; 1-(\alpha_p)-n\right), (b_s), (d-a:1); \end{bmatrix} dx$$

where $B_n(x, y)$ are the Polynomials defined by Khanna I.K.[6]

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