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Assessing Adequacy of Probability Distribution for Development of IDF Relationships for Mandla and Jabalpur

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ABSTRACT

Rainfall intensities of various frequencies and durations are the important parameters for the hydrologic design of storm sewers, culverts and other hydraulic structures. This can be achieved by rainfall intensity-duration-frequency (IDF) relationship, which is determined through rainfall frequency analysis. This paper exemplifies the use of Gumbel and Frechet distributions for modelling annual n-hourly maximum rainfall for different duration of ‘n’ such as 1-hour (hr), 2-hr, 3-hr, 6-hr, 12-hr, 18-hr, 24-hr, 48-hr and 72-hr recorded at Mandla and Jabalpur rain-gauge stations. Order Statistics Approach is applied for determination of distributional parameters for estimation of rainfall and development of IDF relationships for different return periods. Goodness-of-Fit (GoF) test involving Kolmogorov-Smirnov test is used for checking the adequacy of fitting of distributions to the recorded data. Model Performance Indicators (MPIs) such as root mean square error and coefficient of determination are used to analyse the performance of IDF relationships given by Gumbel and Frechet distributions. Based on GoF test results and MPIs values, the study suggested that the developed IDF relationships by Gumbel distribution are better suited for estimation of rainfall intensity at Mandla and Jabalpur.

KEYWORDS: Frechet, Gumbel, Mean square error, Order statistics, Rainfall

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INTRODUCTION

Rainfall intensities of various frequencies and durations are the important parameters for the hydrologic design of storm sewers, culverts and many other structures. This can be achieved by developing of rainfall intensity-duration-frequency (IDF) relationship through rainfall frequency analysis (RFA) that is used to estimate rainfall depth at a point (called as point analysis) for a specified exceedance probability and duration. A point analysis is based either on annual maximum series or on partial duration (based on peak over threshold) series. In the present study, annual maximum rainfall series is considered for RFA. Rainfall in a region can be characterised if the intensity, duration and frequency of the diverse storms occurring at that place are known[1-3]. The frequency-data for storms of various durations, so obtained, can be represented by IDF curves, which give a plot of rainfall intensity versus rainfall duration and recurrence interval.

Raiford et al[4] have updated the existing IDF curves in a region and obtained these curves at ungauged sites in the region using the newly developed RFA techniques based on product moment and L-moment methods. They have also developed IDF curves and isopluvial maps for the region encompassing South Carolina, North Carolina and Georgia. Kim et al[5] improved the accuracy of IDF curves by using long and short duration separation technique. They derived IDF curves by using cumulative distribution function (CDF) for the site under consideration using multi-objective genetic algorithm. Ben-Zvi[6] proposed a procedure for development of IDF curves on partial duration series which are substantially larger than those commonly used for this purpose. He concluded that the proposed procedure superior to the current ones where the use of large samples would reduce the sensitivity of predicted intensities to sampling variations. Bara et al[7] applied the simple scaling theory to the IDF characteristics of short duration rainfall. They have concluded that the IDF relationships, which were deduced from daily rainfall showed acceptable results in comparison with the IDF curves obtained from at-site short duration rainfall data. Okonkwo and Mbajiorgu[8] have developed IDF curves for south-eastern Nigeria using graphical and statistical methods and the results were compared. They have found that IDF curves developed based on statistical methods had a close match for the lower return periods of 2-year (yr) to 10-yr and differ for higher return periods of 50-yr to 100-yr, but the difference was not significant at 5% level. Khaled et al[9] applied L-moments and generalised least squares regression methods for estimation of design rainfall depths and development of IDF relationships. Rashid et al[10] applied Pearson Type-III distribution for modelling of short duration rainfall and development of IDF relationships for Sylhet City in Bangladesh. Antigha and Ogarekpe[11] applied Gumbel distribution for development of IDF curves for prediction of rainfall intensities for Calabar Metropolis, Nigeria.

In probability theory, extreme value distributions namely Gumbel, Frechet and Weibull are generally considered for frequency analysis of meteorological variables. On the other hand, Atomic Energy Regulatory Board (AERB)[12] guidelines described that the Order Statistics Approach (OSA) is the most appropriate method for determination of parameters of Gumbel and Frechet distributions. Though number of methods is available for parameter estimation, OSA estimators are popular owing to less bias and minimum variance. AERB guidelines also described that the Mean+SE (where Mean denotes the estimated rainfall and SE the Standard Error) is generally adopted for arriving at a design parameter. In this context, an attempt has been made to estimate the rainfall for different return periods for different durations of ‘n’ such as 1-hr, 2-hr, 3-hr, 6-hr, 12-hr, 18-hr, 24-hr, 48-hr and 72-hr adopting Gumbel and Frechet distributions (using OSA) for development of IDF relationships for Mandla and Jabalpur. In the present study, Weibull distribution is not considered for RFA because of non-existence of OSA for determination of distributional parameters. Goodness-of-Fit (GoF) test involving Kolmogorov-Smirnov (KS) statistic is applied for checking the adequacy of fitting of distributions to the recorded data. Model performance indicators (MPIs) such as root mean square error (RMSE) and coefficient of determination (R^2) are used to analyse the performance of the developed IDF relationships by Gumbel and Frechet distributions for estimation of rainfall intensity for the stations under study. The methodology adopted for development of IDF relationships using Gumbel and Frechet distributions, determination of KS test statistic and D-index, and computations of MPIs are briefly described in the following sections.

METHODOLOGY

Probability Distributions

The CDFs [F(R)] of Gumbel and Frechet distributions are given by:

$$F(R) = e^{-e^{-\left(\frac{R_G - \alpha_G}{\beta_G}\right)}}, \ \alpha_G, \beta_G > 0 \qquad \ldots (1)$$

$$F(R) = e^{-\left(\frac{R_F}{\beta_F}\right)^{(-\lambda_F)}}, \ \lambda_F, \beta_F > 0 \qquad \ldots (2)$$

where α_G and β_G are location and scale parameters of Gumbel distribution[13]. The parameters of the distribution are computed by OSA and used for estimation of rainfall (R_G) for different return periods (T) are computed from $R_G=\alpha_G+Y_T\beta_G$ with $Y_T=-\ln(-\ln(1-(1/T))$. Based on extreme value theory, Frechet distribution can be transformed to Gumbel distribution through logarithmic transformation using natural logarithm of the actual variable. Under this transformation, the scale (β_F) and shape (λ_F) parameters of Frechet distribution are determined by OSA for estimation of rainfall (R_F) using $R_F=\mathrm{Exp}(R_G)$, $\beta_F=\mathrm{Exp}(\alpha_G)$ and $\lambda_F=1/\beta_G$. Here, R_G and R_F are the estimated rainfalls by Gumbel and Frechet distributions respectively.

Order Statistics Approach

OSA is based on the assumption that the set of extreme values constitutes a statistically independent series of observations. The OSA estimators of Gumbel distribution are given by:

$$\alpha_G = r^*\alpha_M^* + r'\alpha_M' \text{ and } \beta_G = r^*\beta_M^* + r'\beta_M' \quad \ldots (3)$$

where r^* and r' are proportionality factors, which can be obtained from the selected values of k, n and n' using the relations as follows:

$$r^* = kn/N \text{ and } r' = n'/N \quad \ldots (4)$$

Here N is the sample size containing the basic data that are divided into k sub groups of n elements each leaving n′ remainders. α_M^* and β_M^* are the distribution parameters of the groups, and α_M' and β_M' are the parameters of the remainders, if any. These can be computed from the following equations:

$$\alpha_M^* = (1/k)\sum_{i=1}^{n}\alpha_{ni}S_i, \quad \alpha_M' = \sum_{i=1}^{n'}\alpha_{n'i}R_i,$$

$$\beta_M^* = (1/k)\sum_{i=1}^{n}\beta_{ni}S_i \text{ and } \beta_M' = \sum_{i=1}^{n'}\beta_{n'i}R_i \quad \ldots (5)$$

where $S_i = \sum_{i=1}^{n} R_{ij}$, j=1,2,3,..,k. Here, R_i is the i^{th} observation in the remainder group having n' elements, R_{ij} is the i^{th} observation in the j^{th} group having n elements. Table 1 gives the weights of α_{ni} and β_{ni} used in determination of OSA estimators of Gumbel and Frechet distributions.

Table 1: Weights α_{ni} and β_{ni} for Determination of OSA Estimators of Gumbel and Frechet Distributions

α_{ni} / β_{ni}	i					
	1	**2**	**3**	**4**	**5**	**6**
α_{2i}	0.91637	0.08363				
α_{3i}	0.65632	0.25571	0.08797			
α_{4i}	0.51099	0.26394	0.15368	0.07138		
α_{5i}	0.41893	0.24628	0.16761	0.10882	0.05835	
α_{6i}	0.35545	0.22549	0.16562	0.12105	0.08352	0.04887
β_{2i}	-0.72135	0.72135				
β_{3i}	-0.63054	0.25582	0.37473			
β_{4i}	-0.55862	0.08590	0.22392	0.24879		
β_{5i}	-0.50313	0.00653	0.13046	0.18166	0.18448	
β_{6i}	-0.45927	-0.03599	0.07319	0.12672	0.14953	0.14581

The SE on the estimated rainfall by OSA can be obtained from

$$SE = \left(r^* R_n + r' R_{n'} \right)^{1/2} \qquad \ldots (6)$$

where $r^* = (1/k)(kn/N)^2$ and $r' = \left(n'/N \right)^2$. R_n and $R_{n'}$ are defined by the general form as $R_n = \left(A_n Y_T^2 + B_n Y_T + C_n \right) \beta_G^2$. The values of A_n, B_n, and C_n used in computing the SE are given in Table 2.

Table 2: Variance determinators for R_n

n	**A_n**	**B_n**	**C_n**
2	0.71186	-0.12864	0.65955
3	0.34472	0.04954	0.40286
4	0.22528	0.06938	0.29346
5	0.16665	0.06798	0.23140
6	0.13196	0.06275	0.19117

Kolmogorov-Smirnov Test

The KS statistic is defined by

$$KS = \underset{i=1}{\overset{N}{Max}} \left(F_e(R_i) - F_D(R_i) \right) \qquad \ldots (7)$$

Here $F_e(R_i)$=(m-0.44)/(N+0.12) is the empirical CDF of R_i and $F_D(R_i)$ is the computed CDF of R_i. In $F_e(R_i)$ formula, m is the rank assigned to the observations arranged in ascending order and N is the number of observations. If the computed value of KS statistic (using probability distribution) is less than that of theoretical value at the desired significance level, then the distribution is considered to be suitable for modelling rainfall data[14].

Procedure for Development of IDF Relationship

IDF is a mathematical relationship between the rainfall intensity, duration, and return period. Intensity is defined as the time rate of rainfall, which is the depth per unit time (mm/ hr, or mm/ day as the case may be), which is generally termed as average intensity over the duration of rainfall. Theoretically, the intensity of storm in a region varies with duration in such a way that high intensity generally corresponds to short duration, and low intensity to longer duration[15-18]. The general form of empirical equation used in development of IDF relationship is expressed by:

$$I = A * (D_T)^{-B} \qquad \dots (8)$$

where I = P/D_T is the rainfall intensity (mm/hr), D_T is the rainfall duration (hr) corresponding to return period (T), and the terms A and B are model parameters. Here, P is defined as Mean+SE (where Mean denotes the estimated rainfall and SE the Standard Error) obtained from either Gumbel or Frechet (R_G and R_F). Method of least squares is applied to compute the parameters of the IDF empirical formula. By applying logarithm on both sides of Eq. (8), we get Log (I) = Log(A)-Blog(D_T) $\Rightarrow$ Y=a-BX. The parameters A and B are computed from Eqs. (9-10) and are expressed by:

$$B = \frac{\sum_{i=1}^{N} Y_i \sum_{i=1}^{N} X_i - N \sum_{i=1}^{N} X_i Y_i}{N \sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i \right)^2} \qquad \dots (9)$$

$$A = \mathrm{Exp}\left(\overline{Y} + B\overline{X}\right) \qquad \dots (10)$$

Model Performance Indicators

The performance of IDF relationships given by Gumbel and Frechet distributions are evaluated by RMSE and R^2. Theoretical descriptions of RMSE and R^2 are expressed by:

$$RMSE = \left(\frac{1}{N}\sum_{i=1}^{N}\left(I_i - I_i^*\right)^2\right)^{0.5} \quad \ldots (11)$$

$$R^2 = \left(\frac{\sum_{i=1}^{N}\left(I_i - \bar{I}\right)\left(I_i^* - \overline{I^*}\right)}{\sqrt{\left(\sum_{i=1}^{N}\left(I_i - \bar{I}\right)^2\right)\left(\sum_{i=1}^{N}\left(I_i^* - \overline{I^*}\right)^2\right)}}\right)^2 \quad \ldots (12)$$

where I_i is the recorded rainfall intensity of i^{th} observation, I_i^* is the estimated rainfall intensity of i^{th} observation, $\bar{I}$ is the average recorded rainfall intensity and $\overline{I^*}$ is the average estimated rainfall intensity[19].

APPLICATION

An attempt has been made to develop IDF curves for different return periods from 2-yr to 1000-yr for Mandla and Jabalpur. Mandla station is located between the latitude 22° 36' N and longitude 80° 23' E in the east central part of Madhya Pradesh. Similarly, Jabalpur station is located between the latitude 23° 12' N and longitude 79° 57' E in Madhya Pradesh. Hourly rainfall data[20] recorded at Mandla for the period 1969-1991 and Jabalpur for the period 1969-1994 are used to compute the series of annual n-hourly maximum rainfall for different durations of ‘n’ such as 1-hr, 2-hr, 3-hr, 6-hr, 12-hr, 18-hr, 24-hr, 48-hr and 72-hr. The series were further used to compute the rainfall estimates for different return periods using OSA estimators of Gumbel and Frechet distributions. The estimated rainfalls are considered as a base values for development of IDF relationships using Eq. (8).

RESULTS AND DISCUSSIONS

Estimation of Rainfall using Probability Distributions

By applying the procedures described above, a computer program was developed and used to fit the recorded rainfall data at Mandla and Jabalpur stations. The program computes the distributional parameters, rainfall estimates for different return periods from 2-yr to 1000-yr for different durations, KS statistic, and model parameters of IDF curves together with RMSE and R^2. Tables 3-6 give the rainfall estimates together with standard error (SE) for different return periods by Gumbel and Frechet distributions (using OSA) for the stations under study. From the results, it may be noted that the estimated rainfalls for return periods from 5-yr to 1000-yr by Frechet distribution are consistently higher when compared to Gumbel for Mandla and Jabalpur.

Table 3 (a): Rainfall estimates together with SE for different return periods by Gumbel distribution (using OSA) for Mandla

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of									
	1-hr		2-hr		3-hr		6-hr		12-hr	
	R_G	SE	R_G	SE	R_G	SE	R_G	SE	R_G	SE
2	51.1	4.2	68.6	6.7	78.9	8.3	93.2	8.4	109.0	7.8
5	70.3	6.6	99.5	10.6	117.2	13.2	132.1	13.4	145.1	12.4
10	83.0	8.7	119.9	14.0	142.6	17.3	157.9	17.6	168.9	16.3
20	95.2	10.8	139.5	17.3	166.9	21.5	182.6	21.9	191.8	20.2
50	111.0	13.6	164.9	21.8	198.4	27.1	214.6	27.5	221.5	25.5
100	122.9	15.7	184.0	25.3	221.9	31.3	238.6	31.9	243.7	29.5
200	134.6	17.9	202.9	28.7	245.4	35.6	262.5	36.2	265.8	33.5
500	150.2	20.7	227.9	33.3	276.4	41.3	294.0	42.0	295.0	38.9
1000	162.0	22.9	246.8	36.8	299.9	45.6	317.9	46.4	317.1	42.9

R_G: Estimated rainfall by Gumbel distribution; SE: Standard Error

Table 3 (b): Rainfall estimates together with SE for different return periods by Gumbel distribution (using OSA) for Mandla

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of							
	18-hr		24-hr		48-hr		72-hr	
	R_G	SE	R_G	SE	R_G	SE	R_G	SE
2	124.5	8.3	134.1	8.8	169.4	12.8	196.9	16.8
5	162.6	13.1	174.6	13.9	228.4	20.3	274.3	26.7
10	187.8	17.2	201.3	18.3	267.5	26.7	325.5	35.0
20	212.0	21.4	227.0	22.7	305.0	33.1	374.7	43.4
50	243.4	27.0	260.2	28.6	353.5	41.7	438.3	54.7
100	266.9	31.2	285.1	33.1	389.8	48.3	486.0	63.3
200	290.3	35.5	309.9	37.6	426.1	54.9	533.5	72.0
500	321.1	41.1	342.6	43.6	473.8	63.6	596.2	83.5
1000	344.4	45.4	367.4	48.1	510.0	70.3	643.5	92.2

Table 4 (a): Rainfall estimates together with SE for different return periods by Frechet distribution (using OSA) for Mandla

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of									
	1-hr		2-hr		3-hr		6-hr		12-hr	
	R_F	SE	R_F	SE	R_F	SE	R_F	SE	R_F	SE
2	47.7	4.1	62.9	6.7	71.6	8.3	86.2	8.3	103.9	8.0
5	69.6	9.7	100.1	17.4	119.0	22.7	131.5	20.6	146.2	18.2
10	89.4	16.6	136.1	31.8	166.5	43.0	173.9	36.6	183.3	30.6
20	113.7	26.9	182.8	54.4	229.9	75.9	227.4	60.8	227.6	48.1
50	155.1	47.5	267.8	104.1	349.1	151.0	321.7	112.0	301.4	82.3
100	195.7	70.9	356.5	164.8	477.5	246.2	417.3	172.2	372.0	119.9
200	246.8	104.0	474.0	256.1	652.2	394.0	540.7	260.1	458.7	171.5
500	335.2	168.6	690.4	448.7	984.1	718.1	761.1	438.9	604.8	269.2
1000	422.4	240.1	917.3	677.4	1343.1	1116.5	985.5	643.9	745.3	374.0
R_F: Estimated rainfall by Frechet distribution; SE: Standard Error										

Table 4 (b): Rainfall estimates together with SE for different return periods by Frechet distribution (using OSA) for Mandla

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of							
	18-hr		24-hr		48-hr		72-hr	
	R_F	SE	R_F	SE	R_F	SE	R_F	SE
2	119.3	8.4	129.0	9.2	161.1	13.2	184.3	16.6
5	163.2	18.6	176.9	20.3	231.7	30.9	274.3	40.3
10	200.9	30.6	218.1	33.5	294.7	52.6	357.0	70.3
20	245.2	47.2	266.5	51.7	371.2	84.0	459.6	115.0
50	317.3	78.8	345.6	86.5	500.5	146.7	637.4	207.1
100	384.9	112.7	419.8	123.9	626.1	216.9	814.4	313.4
200	466.6	158.1	509.6	174.1	782.7	314.9	1039.7	465.4
500	601.6	242.1	658.2	267.2	1050.5	504.1	1434.8	768.6
1000	728.8	330.1	798.6	364.7	1312.2	710.8	1830.3	1109.1

Table 5 (a): Rainfall estimates together with SE for different return periods by Gumbel distribution (using OSA) for Jabalpur

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of									
	1-hr		2-hr		3-hr		6-hr		12-hr	
	R_G	SE	R_G	SE	R_G	SE	R_G	SE	R_G	SE
2	54.2	3.6	76.0	5.0	88.9	5.8	120.1	8.9	150.1	10.8
5	71.8	5.7	100.6	8.0	117.6	9.3	163.9	14.3	203.0	17.2
10	83.5	7.6	116.8	10.5	136.5	12.3	192.8	18.8	238.0	22.7
20	94.6	9.4	132.4	13.1	154.7	15.3	220.6	23.4	271.6	28.3
50	109.1	11.9	152.6	16.6	178.2	19.3	256.6	29.6	315.1	35.7
100	120.0	13.8	167.7	19.2	195.9	22.4	283.6	34.2	347.6	41.4
200	130.8	15.7	182.8	21.9	213.4	25.5	310.4	39.0	380.1	47.1
500	145.1	18.2	202.7	25.4	236.6	29.6	345.9	45.2	422.9	54.7
1000	155.8	20.1	217.7	28.0	254.1	32.7	372.7	50.0	455.3	60.4

Table 5 (b): Rainfall estimates together with SE for different return periods by Gumbel distribution (using OSA) for Jabalpur

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of							
	18-hr		24-hr		48-hr		72-hr	
	R_G	SE	R_G	SE	R_G	SE	R_G	SE
2	168.4	12.6	179.0	13.9	218.2	17.3	260.7	19.0
5	229.8	20.0	247.1	22.2	303.0	27.6	353.7	30.3
10	270.5	26.4	292.3	29.3	359.1	36.4	415.3	39.9
20	309.5	32.9	335.5	36.4	413.0	45.3	474.4	49.7
50	360.0	41.5	391.6	46.0	482.7	57.3	550.9	62.8
100	397.9	48.1	433.6	53.3	534.9	66.4	608.2	72.8
200	435.6	54.7	475.4	60.7	587.0	75.5	665.3	82.8
500	485.4	63.5	530.6	70.4	655.7	87.6	740.7	96.1
1000	523.0	70.1	572.3	77.8	707.6	96.8	797.6	106.2

Table 6 (a): Rainfall estimates together with SE for different return periods by Frechet distribution (using OSA) for Jabalpur

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of									
	1-hr		2-hr		3-hr		6-hr		12-hr	
	R_F	SE	R_F	SE	R_F	SE	R_F	SE	R_F	SE
2	50.5	3.7	70.1	5.4	81.8	6.1	105.4	8.6	132.3	10.2
5	71.7	8.7	100.7	12.6	116.1	14.0	154.8	20.7	190.2	23.8
10	90.4	14.7	128.0	21.5	146.3	23.7	199.8	35.9	241.7	40.7
20	113.0	23.3	161.0	34.4	182.7	37.6	255.1	58.3	304.3	65.1
50	150.7	40.3	216.8	60.0	243.6	65.0	350.0	103.9	410.0	113.7
100	187.1	59.0	270.9	88.7	302.2	95.2	443.7	155.9	512.6	168.1
200	232.0	84.9	338.3	128.5	374.7	136.9	561.9	229.6	640.3	243.8
500	308.1	134.5	453.4	205.5	497.4	216.6	767.4	374.7	858.7	390.1
1000	381.8	187.9	565.8	289.3	616.2	302.6	971.2	535.9	1072.0	549.5

Table 6 (b): Rainfall estimates together with SE for different return periods by Frechet distribution (using OSA) for Jabalpur

Return period (yr)	Estimated rainfall (mm) with SE (mm) for the series of							
	18-hr		24-hr		48-hr		72-hr	
	R_F	SE	R_F	SE	R_F	SE	R_F	SE
2	150.2	12.4	160.4	14.2	195.6	17.7	234.8	18.8
5	221.5	29.9	243.1	35.3	298.6	44.1	342.3	44.7
10	286.5	52.0	320.2	62.6	395.1	78.7	439.3	77.1
20	366.7	84.7	417.0	103.8	517.0	131.2	558.0	124.6
50	504.8	151.5	586.9	190.3	732.0	242.1	760.7	220.5
100	641.4	228.0	758.2	291.5	949.9	372.9	959.5	329.1
200	814.1	336.6	978.6	438.4	1231.6	563.3	1209.2	482.0
500	1115.3	551.0	1370.4	735.3	1734.9	951.0	1640.7	781.0
1000	1414.8	790.0	1767.5	1073.6	2247.6	1395.5	2066.4	1110.7

Analysis Based on GoF Test

Based on the annual n-hourly maximum rainfall series recorded at Mandla and Jabalpur, KS statistic was computed by Gumbel and Frechet distributions (using OSA) from Eq. (7), and given in Table 7.

Table 7: Computed Values of KS Statistic by Gumbel and Frechet distributions (using OSA) for Mandla and Jabalpur

Rainfall data series	Computed values of KS statistic for			
	Mandla		Jabalpur	
	Gumbel	Frechet	Gumbel	Frechet
1-hr	0.141	0.176	0.144	0.153
2-hr	0.098	0.154	0.128	0.127
3-hr	0.089	0.142	0.096	0.129
6-hr	0.113	0.157	0.118	0.125
12-hr	0.114	0.159	0.119	0.100
18-hr	0.133	0.158	0.121	0.160
24-hr	0.155	0.203	0.129	0.106
48-hr	0.121	0.171	0.130	0.113
72-hr	0.143	0.142	0.124	0.130

From Table 7, it may be noted that the computed values of KS statistic by Gumbel and Frechet distributions are less than that of theoretical values (0.284 for Mandla and 0.267 for Jabalpur) at 5% significance level, and at this level, both distributions are acceptable for modelling rainfall data recorded at Mandla and Jabalpur stations.

Development of IDF Relationships

The Mean+SE (referred as P) values given in Tables 3-6 were used to compute the rainfall intensity using $I=P/D_T$. These values are used to develop IDF relationships for different return periods for Mandla and Jabalpur. The parameters (A and B) of the IDF empirical equations were determined from Eqs. (9-10) for Mandla and Jabalpur, and given in Tables 8 and 9. The values of RMSE and R^2 given by the developed IDF relationships were computed from Eqs. (11-12) and also given in Tables 8 and 9.

Table 8: Parameters of IDF Relationships together with R^2 and RMSE (mm/ hr) for Different Return Periods adopting Gumbel and Frechet distributions for Mandla

Return period (yr)	MPIs and parameters of IDF relationships given by							
	Gumbel				Frechet			
	Model parameters		MPIs		Model parameters		MPIs	
	A	B	RMSE	R^2	A	B	RMSE	R^2
2	58.948	0.708	1.5	0.998	54.569	0.700	1.1	0.999
5	85.041	0.724	3.5	0.998	89.773	0.726	4.5	0.992
10	102.860	0.731	4.8	0.992	126.060	0.744	8.8	0.984
20	120.050	0.735	6.1	0.989	175.200	0.761	15.4	0.974
50	142.420	0.739	7.8	0.988	268.610	0.784	30.2	0.959
100	159.250	0.742	9.0	0.987	370.290	0.800	48.3	0.946
200	175.950	0.744	10.3	0.986	510.170	0.817	75.8	0.933
500	198.080	0.746	11.9	0.985	778.810	0.839	134.2	0.915
1000	214.860	0.747	13.2	0.984	1072.500	0.856	204.2	0.901

Table 9: Parameters of IDF Relationships together with R^2 and RMSE (mm/ hr) for Different Return Periods adopting Gumbel and Frechet distributions for Jabalpur

Return period (yr)	MPIs and parameters of IDF relationships given by							
	Gumbel				Frechet			
	Model parameters		MPIs		Model parameters		MPIs	
	A	B	RMSE	R^2	A	B	RMSE	R^2
2	63.022	0.645	1.9	0.997	58.427	0.653	1.5	0.998
5	84.381	0.635	2.5	0.997	86.516	0.639	2.1	0.998
10	98.970	0.631	2.8	0.996	113.050	0.629	2.9	0.997
20	113.010	0.627	3.3	0.996	146.470	0.619	3.8	0.996
50	131.360	0.625	3.9	0.996	205.080	0.606	5.6	0.994
100	145.150	0.623	4.3	0.996	264.080	0.597	7.4	0.992
200	158.900	0.622	4.7	0.996	339.830	0.587	10.0	0.989
500	177.050	0.620	5.2	0.996	474.180	0.575	15.2	0.984
1000	190.710	0.619	5.6	0.996	610.00	0.565	20.8	0.980

Performance Analysis on IDF Relationships

From Tables 8 and 9, it may be noted that the RMSE values on the estimated rainfall intensity by Gumbel distribution are lesser than those obtained with Frechet for various return periods ranging from 10-yr to 1000-yr for both the stations under study. Also, From Table 8, it may be noted that the R^2 values given by developed IDF relationships adopting Gumbel and Frechet distributions are varied from 0.984 to 0.998 and 0.901 to 0.999 respectively for Mandla. For Jabalpur, the R^2 values on the developed IDF relationships by Gumbel and Frechet are nearer to the perfect correlation value of 1. Based on RMSE values, it is suggested that the developed IDF relationships for different return periods by Gumbel may be considered for estimation of rainfall intensity at Mandla and Jabalpur stations. The plots of IDF curves for different return periods using Gumbel distribution for Mandla and Jabalpur stations are developed and delineated in Figures 1 and 2 respectively.

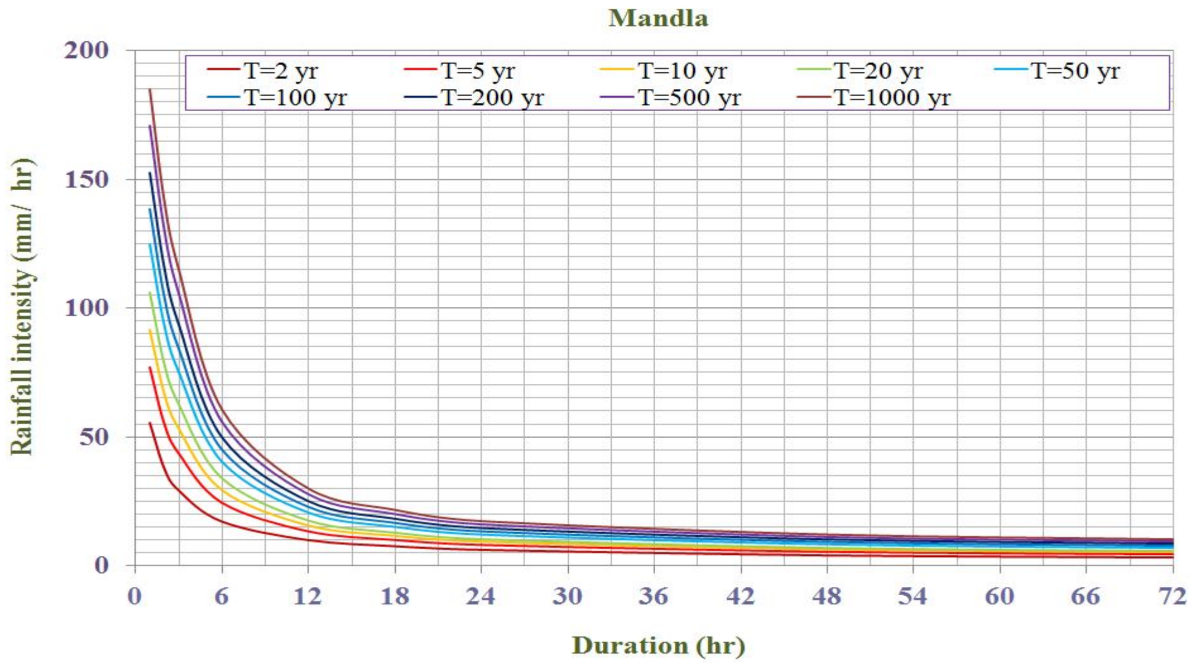

Figure 1: IDF Curves for Different Return Periods Using Gumbel Distribution for Mandla

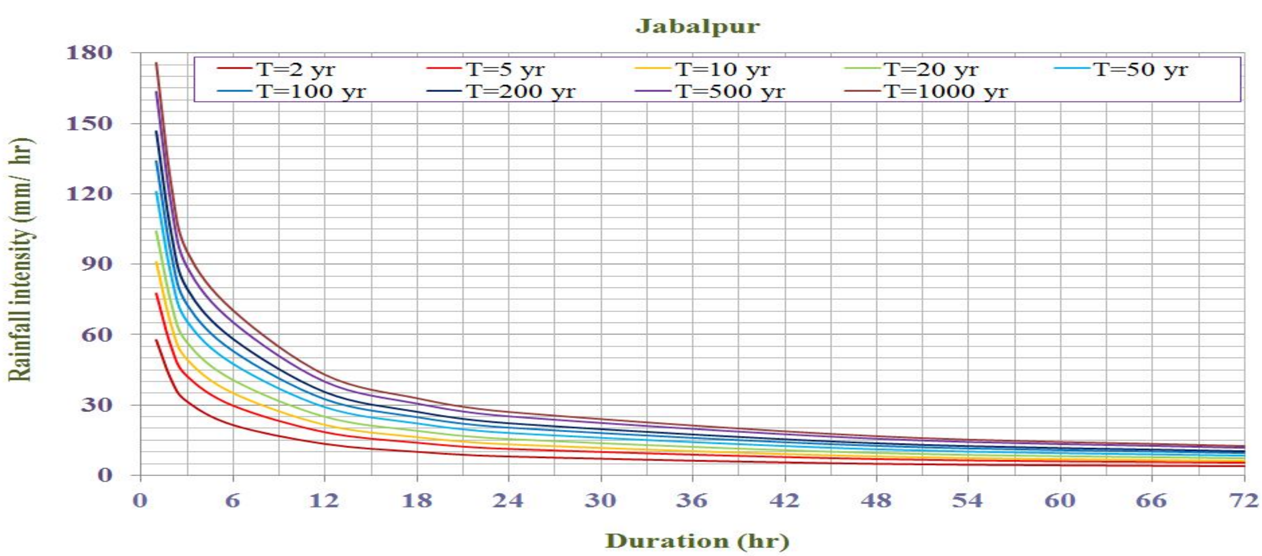

Figure 2: IDF Curves for Different Return Periods Using Gumbel Distribution for Jabalpur

CONCLUSIONS

The paper presented a computer aided procedure for modelling hourly rainfall data recorded at Mandla and Jabalpur rain-gauge stations. From the results of the data analysis, the following conclusions are drawn from the study.

- KS test results supported the use of Gumbel and Frechet distributions for modelling hourly rainfall data recorded at Mandla and Jabalpur.
- Mean+SE (where Mean denotes the estimated rainfall and SE the Standard Error) values given by Gumbel and Frechet distributions (using OSA) for different durations of ‘n’ such as 1-hr, 2-hr, 3-hr, 6-hr, 12-hr, 18-hr, 24-hr, 48-hr and 72-hr are used for development of IDF relationships for different return periods.
- RMSE values obtained from the developed IDF relationships using Gumbel distribution are lesser than the corresponding values of Frechet for different return periods from 10-yr to 1000-yr for both the stations.
- Based on RMSE values, Gumbel distribution is identified as better suited for modelling hourly rainfall data for development of IDF relationships for Mandla and Jabalpur.
- R^2 obtained from developed IDF relationships for different return periods adopting Gumbel distribution are varied from 0.984 to 0.998 for Mandla. For Jabalpur, it is noted that the R^2 values based on developed IDF relationships given by Gumbel are nearer to the perfect correlation value of 1.
- IDF relationships given by Gumbel may be useful for decision makers to estimate the rainfall intensity for any specific return period in a short time as also for planning and designing of any water resources projects at Mandla and Jabalpur.

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