Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Dufour and Soret Effects on MHD Free Convection Heat and Mass Transfer Flow in Porous Medium

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ABSTRACT

In this study, we observed the Dufour and Soret effect in a rotating system of free convection MHD heat and mass transfer flow. The fluid is assumed to be electrically conducting, viscous and incompressible and past an infinite vertical plate embedded in a porous medium in a rotating system. A magnetic field is assumed normal to the plate. Finite Difference Method has been used to study the physical significance of Dufour number, Soret number, Prandtl number and heat source parameter in the velocity, temperature and concentration profiles along with in the rate of heat transfer, the rate of mass transfer and skin friction. The observed results were depicted in graphs and tables.

KEYWORDS: Rotating system, infinite vertical plate, Finite Difference Method, viscous fluid, incompressible fluid

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INTRODUCTION

Combined heat and mass transfer problem in a free convection flow is a necessary area of research due to its significant applications in petroleum industries, agricultural industries and also in mechanical and aeronautical engineering. In a heat and mass transfer flow, the effect of energy flux and mass flux becomes noticeable if a density difference occurs in the flow. The mass flux or the thermal diffusion effect on mass transfer flow was first investigated by Charles Soret in 1879. The energy flux or the diffusion thermo effect was first observed by Swiss scientist L Dufour in 1873. In comparison with the effect described by Furrier and Fick, Soret and Dufour effect are of smaller in order. But they have the significance in the areas like Geophysics, hydrology.

In 1952, Chapman and Cowling[1] mentioned in their book the influence of Dufour and Soret effect in heat and mass transfer problems. In 1972, Ekart and Darke[2] established the facts that the Soret effect has an influence on isotope separation and in the mixture between the gases of very low molecular weight (He, H_2) and medium molecular weight (H_2, air) and the magnitude of Dufour effect cannot be neglected. Further, the significance of this effect in heat and mass transfer problems were developed by Dursunkaya and Worek[3] in 1992, Sattar and Alam[4] in 1994 and so on. In 2009, the study of Shariful Alam and MM Rahman[5] led to the conclusion that in free convection heat and mass transfer flow through the vertical plate in a porous media for a medium molecular weight fluid, Dufour and Soret effect cannot be ignored. In 2010, T Hayat et al.[6] showed the same conclusion for mixed convection flow past a vertical stressing plate embedded in a porous medium. In 2010, N Ahmed et al.[7] investigated the effect of Hall current, thermo diffusion and heat source in an unsteady MHD free convection flow. In 2014, Ali J Chamkha et al.[8] investigated the effect of chemical reaction parameter with Soret and Dufour number in MHD mixed convection from a vertical cone. In 2016, P Sudarsana Reddy et al.[9] investigated Soret and Dufour effect on MHD convective flow of nanofluids past a porous medium with the heat source. In 2016, D Sarma et al.[10] investigated the Soret effect along with Hall current and rotation on MHD natural convection flow past an accelerated vertical plate in a porous medium. Recently in 2017, Imran Ullah et al.[11] investigated the Soret and Dufour effects on mixed convection slip flow in a Casson fluid.

In this study we consider an unsteady, MHD free convection flow past an accelerated infinite vertical plate embedded in a porous medium. A magnetic field is applied normal to the plate. The considered fluid is electrically conducting, viscous and incompressible. We observed the influence of Soret and Dufour effect in velocity profiles, temperature profiles, concentration profiles, Nusselt number, Sherwood number and Skin Friction. The effect of heat source, Prandtl number and porosity parameter was also observed. Results were depicted in graphs and tables.

MATHEMATICAL FORMULATION

We considered an electrically conducting, viscous and incompressible fluid. It is assumed that the fluid past through an accelerated infinite vertical plate embedded in a porous medium. Along x' axis ,t he infinite vertical plate is assumed and the fluid distance from the plate is assumed along y'. A magnetic field of strength B_0 is applied normal to the plate. Initially, the plate and the fluid are at rest and the temperature and the concentration of each element of the fluid and the plate are

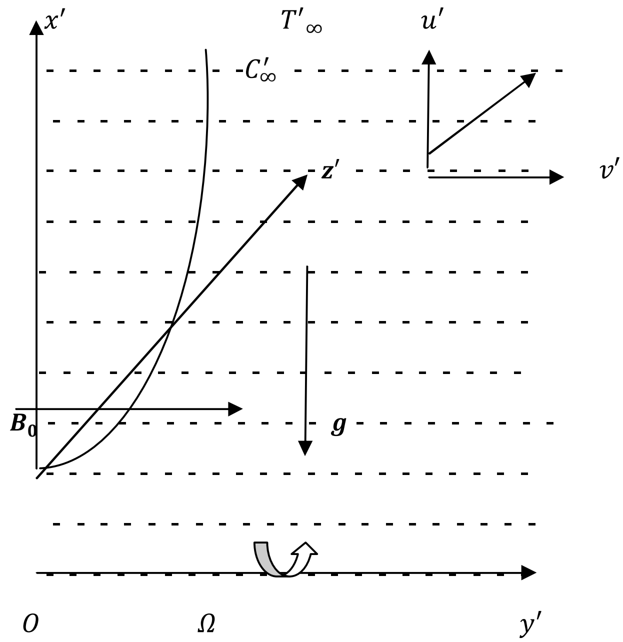

Figure 1: Formulation of the Problem

maintained uniformly at T'_∞ and C'_∞ respectively. At the time t>0, the system i.e. the fluid along with the plate is assumed to be rotate with uniform angular velocity Ω' about y' axis. The plate starts to accelerate along x' axis with velocity u'. The temperature and concentration at the surface of the plate are raised to uniform temperature T'_w and uniform species concentration C'_w respectively and maintained thereafter. All the quantities are depending on the time and the fluid distance from the plate. Considering Bossiness approximation, the governing equations are,

Conservation of momentum:

$$\frac{\partial u'}{\partial t'} + 2\Omega w' = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma}{\rho}\frac{B_0^{\;2}}{(1+m^2)}(u' + mw') + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) - \frac{\nu u'}{k'_1} \qquad (1)$$

$$\frac{\partial w'}{\partial t'} - 2\Omega u' = \nu \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma}{\rho}\frac{B_0^{\;2}}{(1+m^2)}(mu' - w') - \nu \frac{w'}{k_1'} \qquad (2)$$

Conservation of energy:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho C_p}(T' - T'_\infty) + \frac{D_M K_T}{C_s C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

Conservation of species concentration:

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + \frac{D_M K_T}{T_M}\frac{\partial^2 T'}{\partial y'^2} - Kr'(C' - C'_\infty) \tag{4}$$

Initial and boundary conditions are,

$u'=w'=0,\ T'=T'_\infty,\ C'=C'_\infty$ for all y' and $t'\le 0$ (5a)

$u'=Ut',\ w'=0,\ T'=T'_w,\ C'=C'_w$ as $y'=0$ for $t'>0$ (5b)

$u'\to 0,\ w'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty$ as $y'\to\infty$ for $t'>0$ (5c)

Non-dimensional quantities are,

$$u = \frac{u'}{U_0}, w = \frac{w'}{U_0}, y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^{\ 2}}{\nu}, t = \frac{t' U_0^{\ 2}}{\nu}, Sr = \frac{D_M K_T (T'_w - T'_\infty)}{\nu T_M (C'_w - C'_\infty)}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$\phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, K_1 = \frac{K'_1 U_0^{\ 2}}{\nu^2}, Du = \frac{D_M K_T (C'_w - C'_\infty)}{\nu C_s C_p (T'_w - T'_\infty)}, Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0^{\ 3}}, Gm = \frac{g\beta'\nu(C'_w - C'_\infty)}{\nu_0^{\ 3}},$$

$$Q = \frac{\nu Q_0}{\rho C_p U_0^{\ 2}}, Kr = \frac{\nu Kr'}{U_0^{\ 2}}, M^2 = \frac{\sigma B_0^{\ 2}\nu}{\rho U_0^{\ 2}}, \Pr = \frac{\mu C_p}{K}, K^2 = \frac{\nu\Omega}{U_0^{\ 2}}, Sc = \frac{\nu}{D_M}, U = \frac{U_0^{\ 3}}{\nu}$$

The non- dimensional form of the equations (1)-(4) using the above non- dimensional quantities are given below,

$$\frac{\partial u}{\partial t} + 2K^2 w = \frac{\partial^2 u}{\partial y^2} + \frac{M^2}{1+m^2}(u + mw) + Gr\theta + Gm\phi - \frac{u}{K_1} \tag{6}$$

$$\frac{\partial w}{\partial t} - 2K^2 u = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1+m^2}(mu - w) - \frac{w}{K_1} \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - Q\theta + Du\frac{\partial^2 \phi}{\partial y^2} \tag{8}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Kr\phi + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

The non-dimensional initial and boundary conditions are given below,

$u=w=0,\ \theta=0,\ \varphi=0$ for all y and $t\le 0$ (10a)

$u=t,\ w=0,\ \theta=1,\ \varphi=1$ at $y=0$ for $t>0$ (10b)

$u\to 0,\ w\to 0,\ \theta\to 0,\ \varphi\to 0$ as $y\to\infty$ and $t>0$ (10c)

The set of above partial differential equations along with the boundary conditions is the formulation of our problem.

SOLUTION OF THE PROBLEM

We have used the finite difference scheme to solve the above (1)-(4) partial differential equations along with the initial and boundary conditions (5). We followed the technique given below to convert the set of PDE to set of algebraic equations,

$$\frac{\partial Z}{\partial t} = \frac{Z(i+1, j) - Z(i, j)}{\Delta t}$$

$$\frac{\partial Z}{\partial y} = \frac{Z(i, j+1) - Z(i, j)}{\Delta y}$$

$$\frac{\partial^2 Z}{\partial y^2} = \frac{Z(i, j+1) - 2Z(i, j) + Z(i, j-1)}{\Delta y^2},$$

Where Z stands for any variable such as velocity, non-dimensional temperature or concentration. i and j are node locations for t and y directions respectively.

The obtained algebraic equations are,

$$u(i, j) = \frac{1}{\Delta t \Delta y^2 (2 + \frac{M^2}{1+m^2} + \frac{1}{K^2})}[u(i+1, j)\Delta y^2 + \Delta t \Delta y^2 \{-2K^2 w(i, j) - \frac{M^2}{1+m^2} mw(i, j) + Gr\theta(i, j) + Gm\phi(i, j)\} + \Delta t\{u(i, j+1) + u(i, j-1)\}] \quad (11)$$

$$w(i, j) = \frac{1}{-\Delta y^2 + 2\Delta t + \frac{M^2}{1+m^2}\Delta t \Delta y^2 + \frac{\Delta t \Delta y^2}{K_1}}[\{2K^2 u(i, j) + \frac{M^2}{1+m^2} mu(i, j)\}\Delta t \Delta y^2 + \{w(i, j+1) + w(i, j-1)\}\Delta t - w(i+1)\Delta y^2] \quad (12)$$

$$\theta(i, j) = \frac{1}{-\Delta y^2 + 2\frac{\Delta t}{\Pr} + Q\Delta t \Delta y^2}[\frac{\Delta t}{\Pr}\{\theta(i, j+1) + \theta(i, j-1)\} - \theta(i+1, j)\Delta y^2 + Du\{\phi(i, j+1) - 2\phi(i, j) + \phi(i, j-1)\}\Delta t] \quad (13)$$

$$\phi(i, j) = \frac{1}{\Delta y^2 + 2\frac{\Delta t}{Sc} + Kr\Delta t \Delta y^2}[\frac{1}{Sc}\{\phi(i, j+1) + \phi(i, j-1)\}\Delta t + Sr\{\theta(i, j+1) - 2\theta(i, j) + \theta(i, j-1)\}\Delta t - \phi(i+1, j)\Delta y^2] \quad (14)$$

The above algebraic equations from (11)-(14) were solved in MATLAB (R2017b, Licence number: 40497131).

RESULT AND DISCUSSION

To observe the physical significance of this study, the changes of the velocity profile, temperature profile and concentration profile with respect to the changes in the parameters Du, Sr, Pr, Q and K is depicted in graphs. The variations in the skin friction, the Nusselt number and the Sherwood number are displayed in tables.

The value of Schmidt number is considered as 0.22 which represent H_2 in one atmospheric pressure. The *Pr* of air is in between 0 and 1, therefore *Pr* is chosen as 0.1, 0.4, 0.7 and 1. The other physical parameters are chosen arbitrarily.

Our observation stated that *Sr* reverses the effect of *Du* in all the profiles. That is in the figures 2-5, u, w, θ, φ decrease with respect to increasing *Sr* on the other hand in figures 6-9, u, w, θ, φ increases with increasing *Du*. *Sr* is the ratio of temperature difference to the concentration. At $t>0$, near to the plate $T'=T'_w$. *Sr* is $(T'-T'_\infty)/C'$, e. i. increase in *Sr* increases temperature difference which implies an increase in θ. As the fluid flows away from the plate then $T'\to T'_\infty$ e.i. $T'-T'_\infty\to 0$, i.e. increase in *Sr* leads to decrease in θ. In far away from the plate, θ itself tends to zero e.i. increase in *Sr* leads to decrease in concentration which implies a decrease in θ. In the case of φ, an increase in *Sr* implies to decrease in φ. But far away from the plate, φ increases with increasing *Sr*. Table 1 and table 2 displayed the change in skin friction, Nusselt number and Sherwood number with respect to *Sr* and *Du*. *Sr* has the tendency to thicken the concentration boundary layer, therefore increasing *Sr* leads to a decrease in the rate of mass transfer at the plate. Accelerated *Sr* causes an increase in chemical thermal diffusivity at the surface of the plate which leads to a fall in secondary skin friction. Dufour effect tends to thicken the thermal boundary layer which should lead to a decrease in the rate of heat transfer but maybe the rotation of the system resist this fall of Nusselt number.

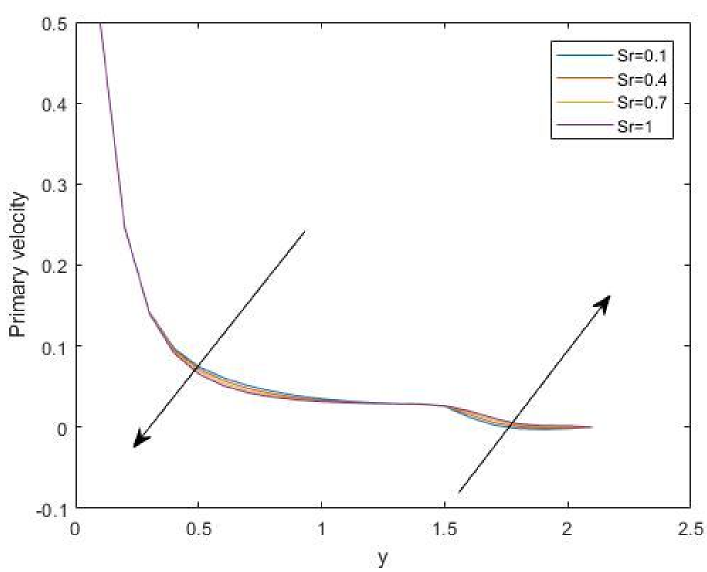

Figure 2: Change in Primary Velocity with *Sr*

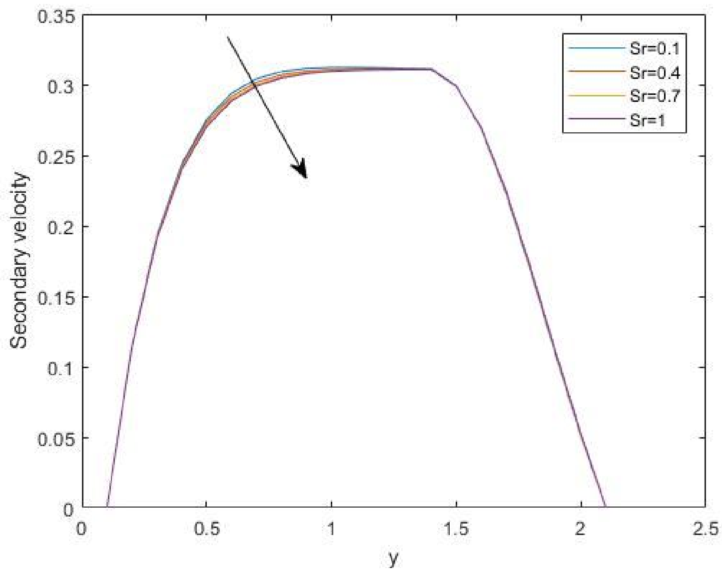

Figure 3: Change in Secondary Velocity with *Sr*

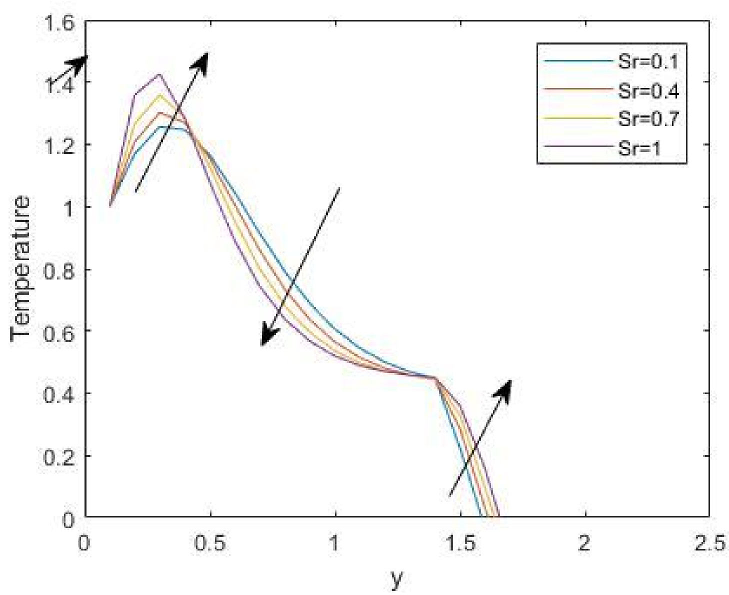

Figure 4: Change in Temperature with *Sr*

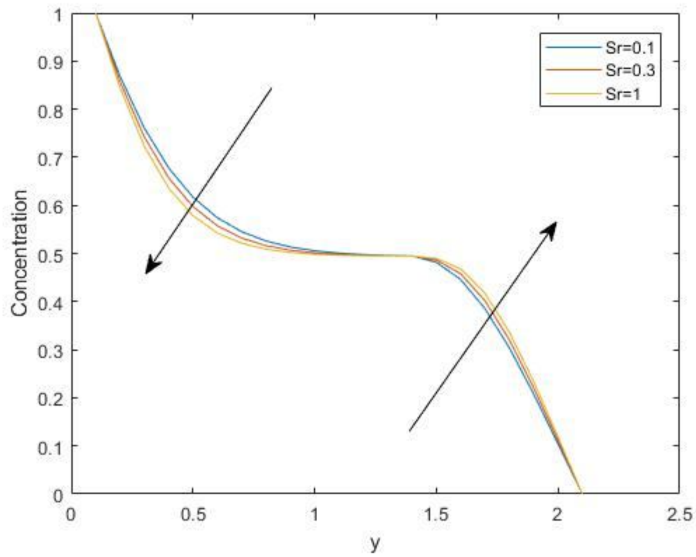

Figure 5: Change in Concentration with *Sr*

Table 1: Change of Skin Friction, Nusselt Number and Sherwood Number w.r.t. *Sr*

Sr	*Cfu*	*Cfw*	*Nu*	*Sh*
0.1	-2.479595	-13.725376	-0.367800	-1.298457
0.4	-2.509272	-13.726404	-0.250220	-1.614274
0.7	-2.539926	-13.727384	-0.060432	-2.052093
1.0	-2.574084	-13.728362	0.276829	-2.753349

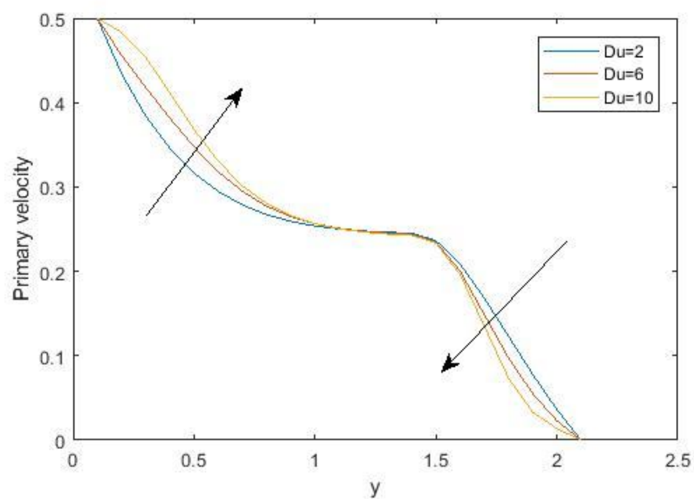

Figure 6: Change in Primary Velocity with *Du*

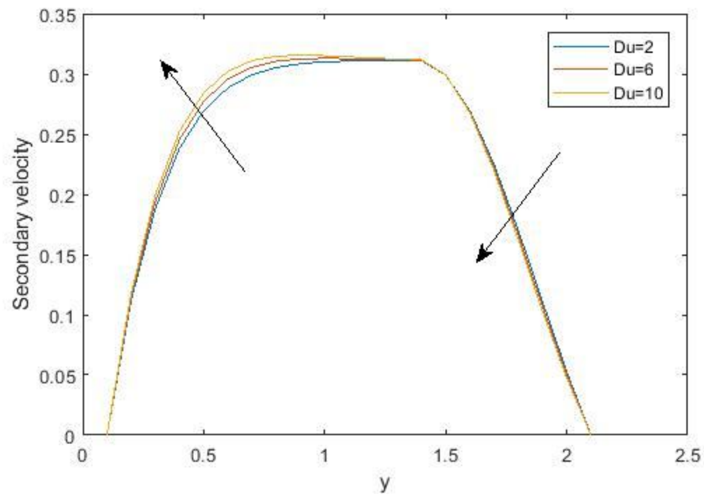

Figure 7: Change in Secondary Velocity with *Du*

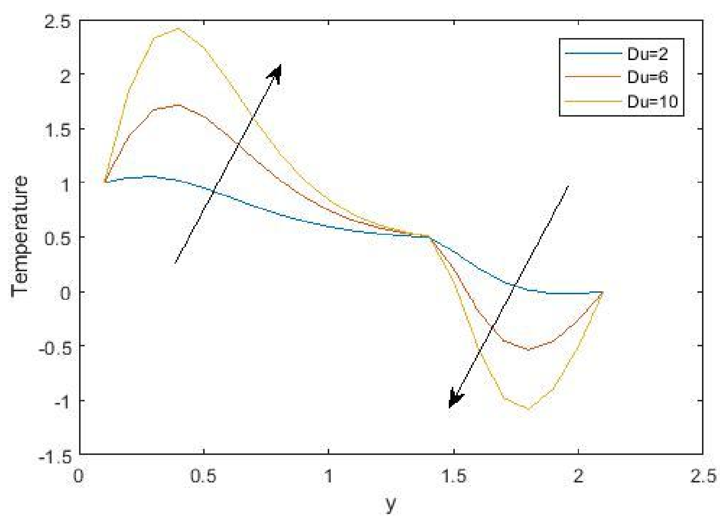

Figure 8: Change in Temperature with *Du*

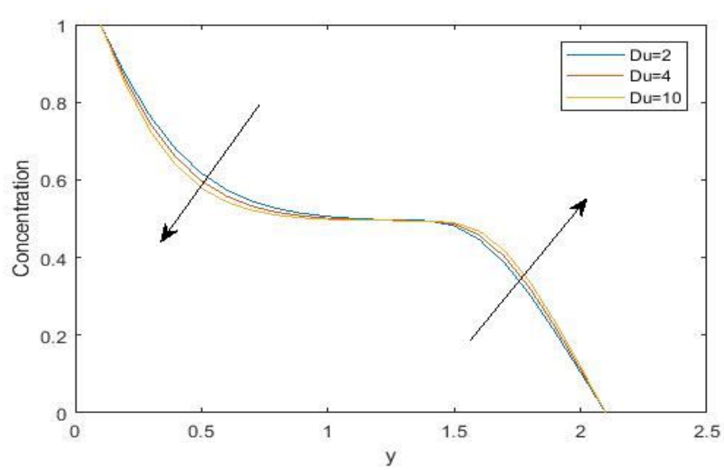

Figure 9: Change in concentration with *Du*

Table 2: Change of Skin Friction, Nusselt Number, Sherwood Number w.r.t. *Du*

Du	*Cfu*	*Cfw*	*Nu*	*Sh*
2	-4.824324	-14.017858	0.573428	-1.297866
6	-4.606383	-14.005898	4.489006	-1.416148
10	-4.382688	-13.994048	8.812370	-1.546675

Figures 10-11 show that increasing porosity parameter accelerates both the secondary and primary velocities. Increase in porosity in the medium helps the fluid to flow rapidly.

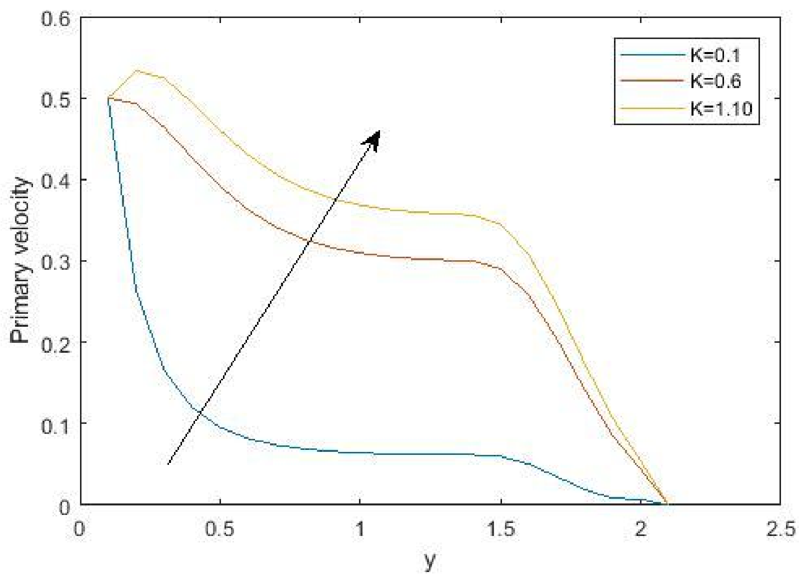

Figure 10: Change in Primary Velocity with *K*

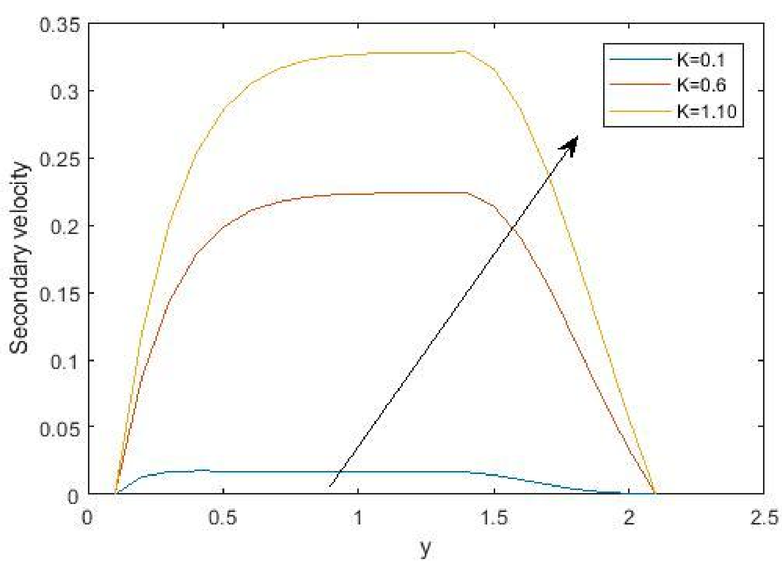

Figure 11: Change in Secondary Velocity with *K*

Pr is the ratio of momentum diffusivity to heat diffusivity. Figure 12 and figure 13 shows that both the velocities increases near to the plate with increasing Pr. Acceleration of the vertical plate may influence the velocities near to the plate. Far away from the plate deceleration of velocities is observed. As Pr gradually increases from 0.1 to 1, the boundary layer thickness increases which lead to a decrease in velocities. Figure 14 describes that temperature increases near the plate and decreases far away from the plate with increasing Pr. It can be again explained with ignoring the acceleration factor of the plate far away, increasing Pr leads to a decrease in thermal diffusivity which leads to decrease in temperature. Figure 15 describes that the concentration profile decreases near to the plate and increases far away from the plate with increasing Pr.

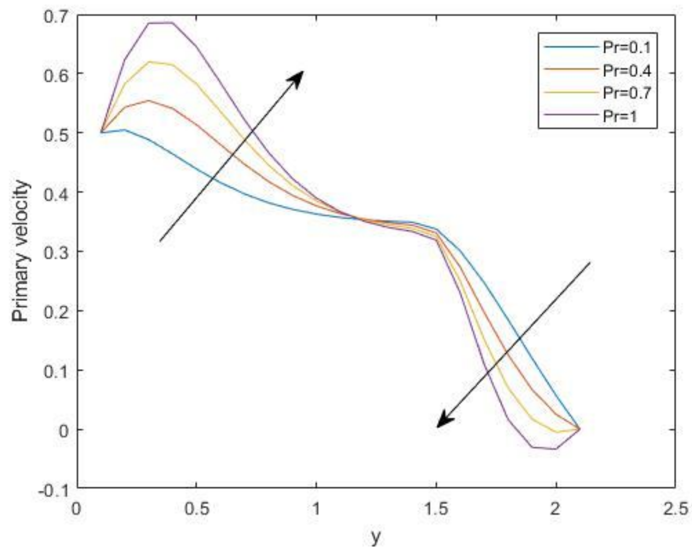

Figure 12: Change in Primary Velocity with *Pr*

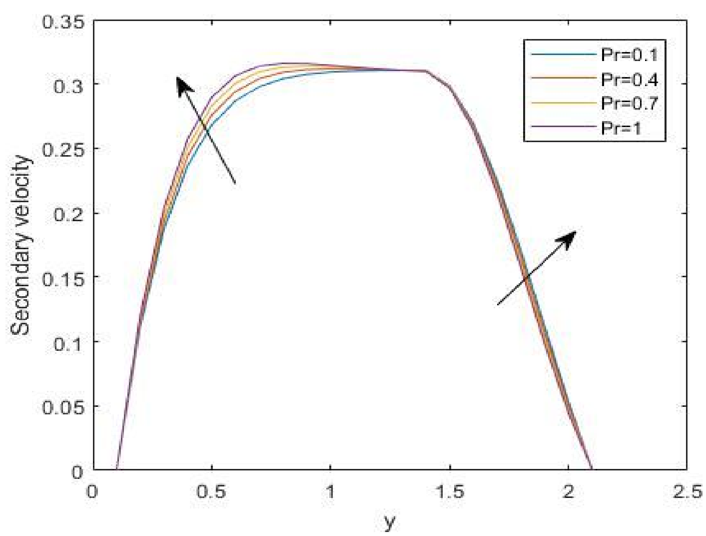

Figure 13: Change in Secondary Velocity with *Pr*

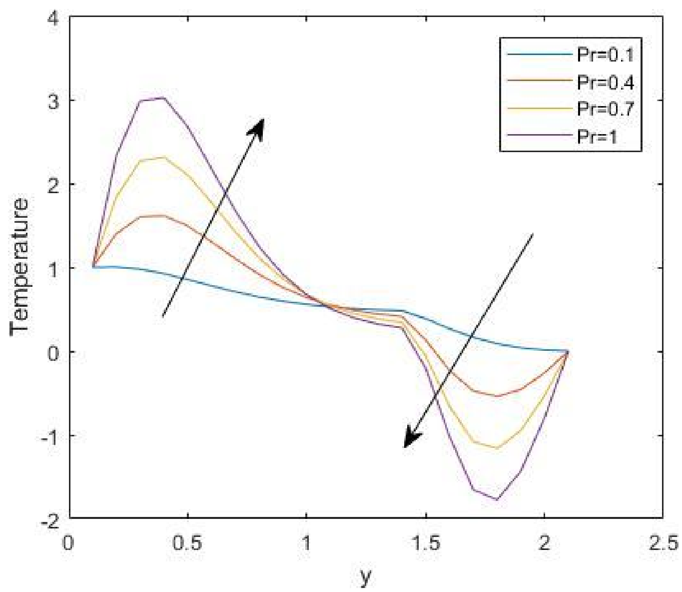

Figure 14: Change in Temperature with Pr

Table 3: Change in Skin Friction, Nusselt Number, Sherwood Number w.r.t. *Pr*

Pr	*Cfu*	*Cfw*	*Nu*	*Sh*
0.1	-3.647651	-13.523502	0.028191	-1.431498
0.4	-3.264771	-13.493364	4.087007	-1.550814
0.7	-2.869068	-13.463377	8.654691	-1.685325
1.0	-2.455713	-13.433319	13.874293	-1.839293

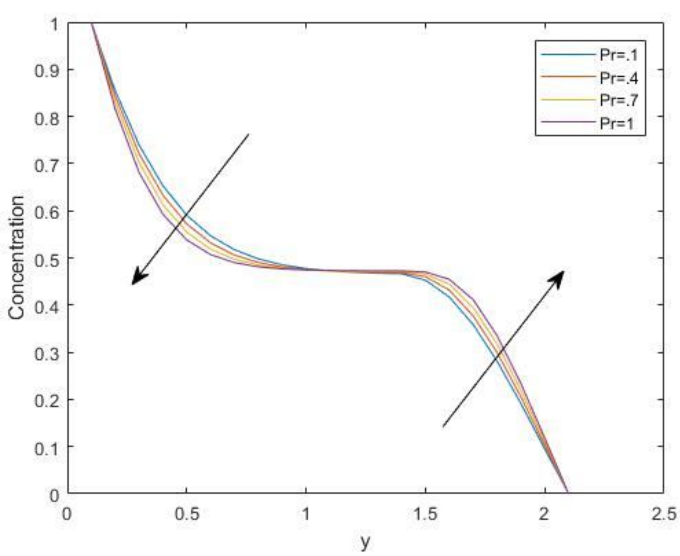

Figure 15: Change in Concentration with Pr

Figures 16-19 show the changes in the velocities, temperature and concentration along with the increasing value of the heat source parameter. From figure 10 and figure 11 we observed a deceleration near to the plate and acceleration far away from the plate in both the velocities with increasing Q. Again, increase in Q leads to increase in volumetric rate of heat generation which implies increase in molecular mobility which leads to increase in temperature and concentration near to the plate and a decrease in both profiles away from the plate.

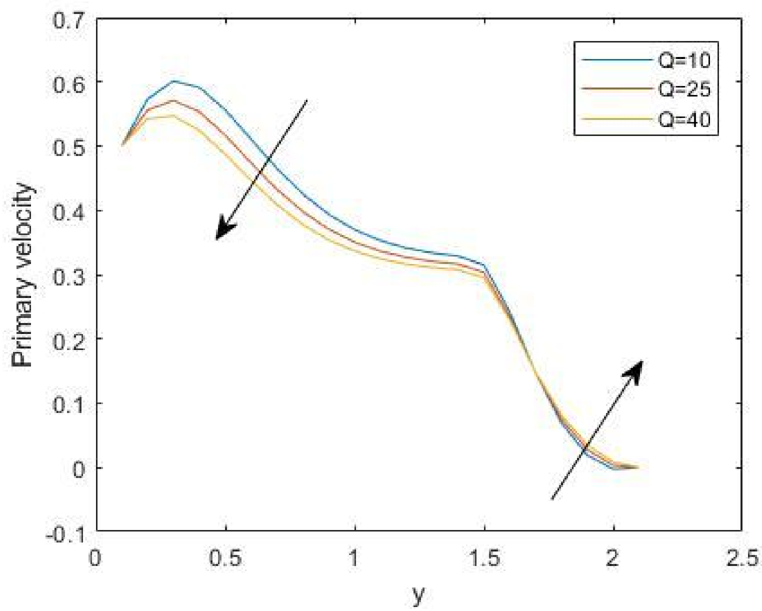

Figure 16: Change in Primary Velocity with *Q*

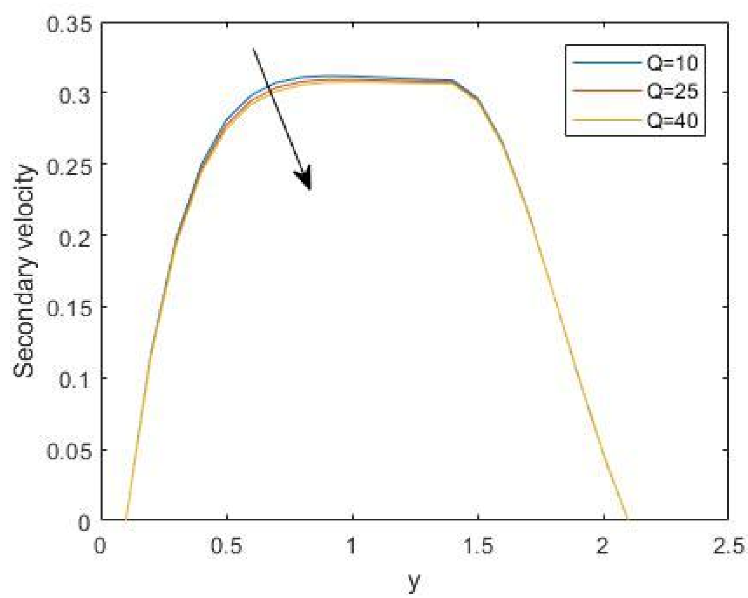

Figure 17: Change in Secondary Velocity with *Q*

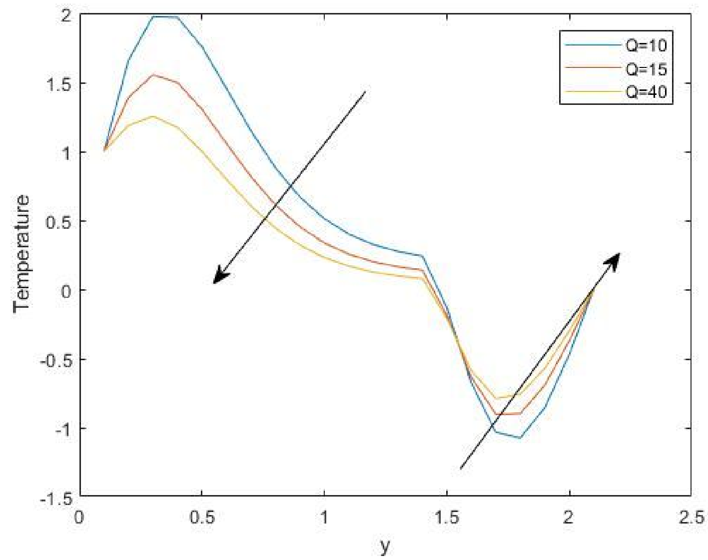

Figure 18: Change in temperature with Q

Table 3: Change in Skin friction, Nusselt Number and Sherwood Number w.r.t. *Q*

Q	*Cfu*	*Cfw*	*Nu*	*Sh*
10	-2.965667	-13.469422	6.834081	-1.631459
25	-3.136013	-13.480718	4.058538	-1.549427
40	-3.272228	-13.489949	1.996167	-1.488511

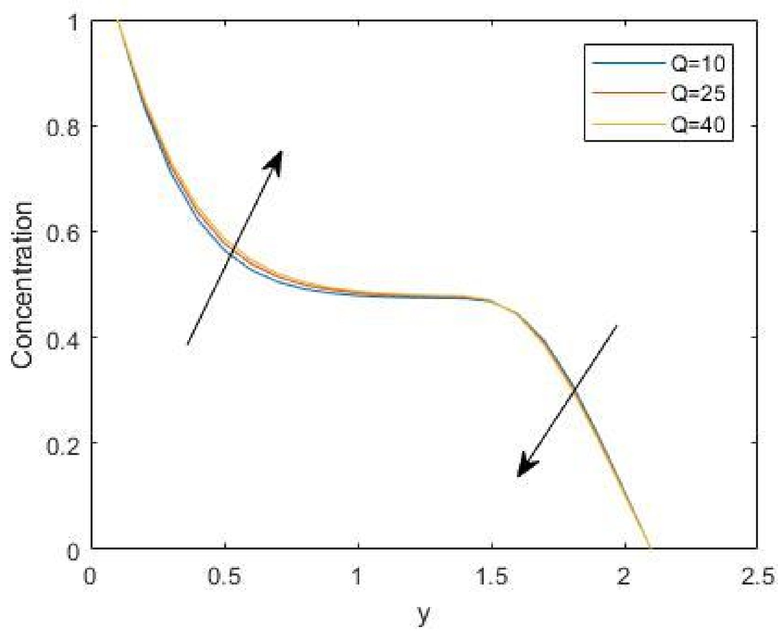

Figure 19: Change in Concentration with *Q*

CONCLUSION

The Dufour and Soret effect on a free convection MHD, unsteady heat and mass transfer flow was observed in a rotating system in the presence of a magnetic field. The fluid was electrically conducting, viscous and incompressible. A significant effect of Dufour number, Soret number, Prandtl number and heat source parameter in the profiles is observed. The effect of Dufour number in the rate of heat transfer in the surface of the plate is different.

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NOMENCLATURE

C'	species concentration	β'	volumetric coefficient of expansion for concentration, K^{-1}
C_p	specific heat at constant pressure, J. kg^{-1}. K^{-1}	β	thermal expansion coefficient, K^{-1}
D_M	coefficient of mass diffusivity, m2.s^{-1}	ϕ	dimensionless concentration
Du	dimensionless Dufour number	Θ	dimensionless temperature
g	acceleration due to gravity, m.s^{-2}	ν	kinematic viscosity, m^2s^{-1}
Gr	dimensionless solutal Grashof number	μ	dynamic viscosity, kg. m^{-1}.s^{-1}
Gm	dimensionless thermal Grashof number	ρ	fluid density, kg.m^{-3}
K^2	dimensionless rotational parameter	σ	electrical conductivity, Sm^{-1}
K_1	dimensionless permeability of the porous medium	τ_e	electron collision time
Kr	dimensionless chemical reaction parameter	τ_u	skin friction due to primary velocity
K_T	thermal diffusion ratio, m.s^{-2}	τ_w	skin friction due to secondary velocity
k	thermal conductivity, W.m^{-1}. K^{-1}		
Nu	Nusselt number		
M^2	dimensionless magnetic parameter		
m	Hall current parameter,		
Pr	dimensionless Prandtl number		
Q	dimensionless heat source parameter		
Sc	dimensionless Schmidt number		

Sh	Sherwood number
T'	temperature of the fluid, K
T_M	mean fluid temperature
u'	fluid velocity in *x'* direction, m.s^{-1}
w'	fluid velocity in z' direction, m.s^{-1}
w_e	cyclotron frequency, e.m^{-1}.B^{-1}