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Multivalent Harmonic Function Associated With Salagean Operator

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ABSTRACT

In this paper we define , a class $HM(u, v, a)$ of m -valent harmonic functions involving Salagean Operator ${}^{15}D_m^v$ is defined and studied. A subclass $THM(u, v, a)$ of a class $H(u, v, a)$ is also been defined and studied. integral operator, convolution condition, for functions belonging to subclass $THM(u, v, a)$ are obtained.

KEYWORDS : Multivalent , Salagean , convolution , operator.

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1. INTRODUCTION

Definition 1.1

Let f be a harmonic function in a Jordan domain D with boundary C . Suppose f is continuous in \bar{D} and $f(z) \neq 0$ on C . Suppose f has no singular zeros in D , and let m to be sum of the orders of the zeros of f in D . Then $\Delta_c \arg(f(z)) = 2\pi m$, where $\Delta_c \arg(f(z))$ denotes the change in argument of $f(z)$ as z traverses C .

It is also shown that if f is sense-preserving harmonic function near a point z_0 , where $f(z_0) = \omega_0$ and if $f(z) - \omega_0$ has a zero of order m ($m \geq 1$) at z_0 , then to each sufficiently small $\epsilon > 0$ there corresponds a $\delta > 0$ with the property: "for each $\alpha \in N_\delta(\omega_0) = \{\omega : |\omega - \omega_0| < \delta\}$, the function $f(z) - \alpha$ has exactly m zeros, counted according to multiplicity, in $N_\epsilon(z_0)$ ". In particular, f has the open mapping property that is, it carries open sets to open sets.

Let Δ be the open unit disc $\Delta = \{z : |z| < 1\}$ also let $a_k = b_k = 0$ for $0 \leq k < m$ and $a_m = 1$. Ahuja and Jahangiri^{5, 9} introduce and studied certain subclasses of the family $SH(m)$, $m \geq 1$ of all multivalent harmonic and orientation preserving functions in Δ . A function f in $SH(m)$ can be expressed as $f = h + \bar{g}$, where h and g are of the form

$$1.1 \quad h(z) = z^m + \sum_{n=2}^{\infty} a_{n+m-1} z^{n+m-1}$$

$$g(z) = \sum_{n=1}^{\infty} b_{n+m-1} z^{n+m-1}, \quad |b_m| < 1.$$

According to above argument, functions in $SH(m)$ are harmonic and sense-preserving in Δ if $J_f > 0$ in Δ . The class $SH(1)$ of harmonic univalent functions was studied in details by Clunie and Sheil Small¹⁶. It was observed that m -valent mapping need not be orientation-preserving.

Let $TH(m)$ denotes the subclass of $SH(m)$ whose members are of the form

$$h(z) = z^m - \sum_{n=2}^{\infty} |a_{n+m-1}| z^{n+m-1}$$

and

$$g(z) = \sum_{n=1}^{\infty} |b_{n+m-1}| z^{n+m-1}, \quad |b_m| < 1.$$

Definition 1.2

For analytic function $h(z) \in S(m)$ Salagean³³ introduced an operator D_m^v defined as follows:

$$D_m^0 h(z) = h(z), \quad D_m^1 h(z) = D_m(h(z)) = \frac{z}{m} h'(z) \text{ and}$$

$$D_m^v h(z) = D_m(D_m^{v-1} h(z)) = \frac{z(D_m^{v-1} h(z))'}{m}$$

$$= z + \sum_{n=2}^{\infty} \left(\frac{n+m-1}{m} \right)^v a_{n+m-1} z^{n+m-1}, \quad v \in \mathbb{N}.$$

Whereas, Jahangiri et al¹⁷ defined the Salagean operator $D_m^v f(z)$ for multivalent harmonic function as follows:

$$(1.2) \quad D_m^v f(z) = D_m^v h(z) + (-1)^v \overline{D_m^v g(z)}$$

where,

$$D_m^v h(z) = z^m + \sum_{n=2}^{\infty} \left(\frac{n+m-1}{m} \right)^v a_{n+m-1} z^{n+m-1}$$

$$D_m^v g(z) = \sum_{n=1}^{\infty} \left(\frac{n+m-1}{m} \right)^v b_{n+m-1} z^{n+m-1}.$$

Now, a sub class $H_m(\lambda, \nu, \alpha)$ of m -valent harmonic functions involving Salagean operator $D_m^\nu f(z)$ is defined as follows:

Definition 1.3

Let $f(z) = h(z) + \overline{g(z)}$ be the harmonic multivalent function of the form (1.1), then f belongs to $HM(u, \nu, a)$ if and only if

$$(1.3) \quad \operatorname{Re} \left\{ (1 - \lambda) \frac{D_m^\nu f(z)}{z^m} + \lambda \frac{\frac{\partial}{\partial \theta} D_m^\nu f(z)}{\frac{\partial}{\partial \theta} z^m} \right\} > \alpha$$

where $0 \leq \alpha < 1, \lambda \geq 0, z = re^{i\theta} \in \Delta$ and $D_m^\nu f(z)$ is defined by (1.2) and

$$\frac{\partial}{\partial \theta} D_m^\nu f(z) = i \left[z(D_m^\nu h(z))' - (-1)^\nu \overline{z(D_m^\nu g(z))'} \right], \quad \frac{\partial}{\partial \theta} z^m = imz^m.$$

Denote the subclass $THM(u, \nu, a)$ consist of harmonic functions $f_\nu = h + \overline{g_\nu}$ in $HM(u, \nu, a)$ so that h and g_ν are of the form

$$(1.4) \quad h(z) = z^m - \sum_{n=2}^{\infty} |a_{n+m-1}| z^{n+m-1},$$

$$g_\nu(z) = (-1)^\nu \sum_{n=1}^{\infty} |b_{n+m-1}| z^{n+m-1}, \quad |b_m| < 1.$$

Also note that $THM(u, \nu, 0) = THM(u, \nu)$.

The class $HM(u, \nu, 0)$ provides a transition between two classes:

$$\operatorname{Re} \left\{ \frac{D_m^\nu f(z)}{z^m} \right\} > \alpha \quad \text{and} \quad \operatorname{Re} \left\{ \frac{\frac{\partial}{\partial \theta} D_m^\nu f(z)}{\frac{\partial}{\partial \theta} z^m} \right\} > \alpha \quad \text{as } \lambda \text{ moves between } 0 \text{ and } 1.$$

Denote $HM(0, \nu, a)$ by $PM(\nu, a)$ and $HM(1, \nu, a)$ by $QM(\nu, a)$.

Definition 1.4

The generalized Bernardi-Libera-Livingston integral operator $L_c(f(z))$ for m -valent functions is defined by

$$L_c(f(z)) = \frac{c+m}{z^c} \int_0^z t^{c-1} h(t) dt + \overline{\frac{c+m}{z^c} \int_0^z t^{c-1} g(t) dt}, \quad c > -1.$$

2. INTEGRAL OPERATOR

Let f belongs to $THM(u,v,a)$; $\lambda \geq 1$. Thus $L_c(D_m^v f_v(z))$ belongs to the class $THM(u,v,a)$.

Proof

From the representation of $L_c(f(z))$ it follows that

$$\begin{aligned} L_c(D_m^v f_v(z)) &= \frac{c+m}{z^c} \int_0^z t^{c-1} D_m^v h(t) dt + \overline{\frac{c+m}{z^c} \int_0^z t^{c-1} (-1)^v D_m^v g_v(t) dt} \\ &= \frac{c+m}{z^c} \int_0^z t^{c-1} \left(t^m - \sum_{n=2}^{\infty} |a_{n+m-1}| t^{n+m-1} \right) dt \\ &\quad + \overline{\frac{c+m}{z^c} (-1)^v \int_0^z t^{c-1} \left(\sum_{n=2}^{\infty} |b_{n+m-1}| t^{n+m-1} \right) dt} \\ &= z^m - \sum_{n=2}^{\infty} A_{n+m-1} z^{n+m-1} + (-1)^v \sum_{n=1}^{\infty} B_{n+m-1} \bar{z}^{n+m-1} \end{aligned}$$

where, $A_{n+m-1} = \frac{c+m}{n+m-1+c} |a_{n+m-1}|$, $B_{n+m-1} = \frac{c+m}{n+m-1+c} |b_{n+m-1}|$

Therefore,

$$\sum_{n=2}^{\infty} \left(\frac{n+m-1}{m} \right)^v \left[\left\{ \left(\frac{n+m-1}{m} \right) \lambda + (1-\lambda) \right\} \frac{c+m}{n+m-1+c} |a_{n+m-1}| + \right.$$

$$\begin{aligned}
 & + \left\{ \left(\frac{n+m-1}{m} \right)^\lambda - (1-\lambda) \right\} \frac{c+m}{n+m-1+c} |b_{n+m-1}| \Big] \\
 & \leq \sum_{n=2}^{\infty} \left\{ \left(\frac{n+m-1}{m} \right)^\nu \left[\left(\frac{n+m-1}{m} \right)^\lambda + (1-\lambda) \right] |a_{n+m-1}| \right. \\
 & \quad \left. + \left(\frac{n+m-1}{m} \right)^\nu \left[\left(\frac{n+m-1}{m} \right)^\lambda - (1-\lambda) \right] |b_{n+m-1}| \right\} \\
 & \leq (1-\alpha) - (2\lambda - 1) |b_m|
 \end{aligned}$$

and so the proof is complete.

3.CONVOLUTION PROPERTY

Let f_v belongs to $THM(u,v,a)$ and F_v belongs to

$THM(u,v,a)$; $\lambda \geq 1$ then the convolution

$$\begin{aligned}
 (f_v * F_v)(z) &= z^m - \sum_{n=2}^{\infty} |a_{n+m-1} A_{n+m-1}| z^{n+m-1} + \\
 & \quad + (-1)^\nu \sum_{n=1}^{\infty} |b_{n+m-1} B_{n+m-1}| \bar{z}^{n+m-1} \in TH_m(\lambda, v, \alpha).
 \end{aligned}$$

Proof

For F_v belongs to $THM(u,v,a)$ so, $|A_{n+m-1}| \leq 1, |B_{n+m-1}| \leq 1$.

consider,
$$\sum_{n=2}^{\infty} \left[\frac{\left(\frac{n+m-1}{m} \right)^\nu \left[\left(\frac{n+m-1}{m} \right)^\lambda + (1-\lambda) \right] |a_{n+m-1} A_{n+m-1}|}{1-\alpha} \right] +$$

$$\begin{aligned}
 & \left. + \sum_{n=1}^{\infty} \left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu} \left[\left(\frac{n+m-1}{m}\right)^{\lambda - (1-\lambda)} \right] |b_{n+m-1} B_{n+m-1}|}{1-\alpha} \right] \right. \\
 & \leq \sum_{n=2}^{\infty} \left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu} \left[\left(\frac{n+m-1}{m}\right)^{\lambda + (1-\lambda)} \right] |a_{n+m-1}|}{1-\alpha} \right] + \\
 & \left. + \sum_{n=1}^{\infty} \left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu} \left[\left(\frac{n+m-1}{m}\right)^{\lambda - (1-\lambda)} \right] |b_{n+m-1}|}{1-\alpha} \right] \right.
 \end{aligned}$$

≤ 1 using equation coefficient inequality.

Therefore the result follows.

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