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Application of Numerov's Method on Second Order Ordinary Differential Equations

Mahajan Shailly*[1] and Subash Kumar[2]

[1]Mewar University, Chittorgarh (Rajasthan)
E-mail:shaillymahajan@rediffmail.com
[2]Pathankot College of Managemant & Technology Pathankot (Punjab)
E-mail:subhash1950@rediffmail.com

ABSTRACT

This paper explained the differential equation of second order and finds the numerical solutions of second order differential equations with the various Techniques. In this paper discussed the Numeral's method to solve the second order differential equation also some examples are solve for differential equation and also explained the Milne's predictor and corrector formula.

KEYWORDS: second order differential equations, Finite difference method Numeral's method. Milne's predictor and corrector

***Corresponding Author**

Shailly Mahajan

Research Scholar,

Mewar University,

Chittorgarh (Rajasthan)

E-mail:shaillymahajan@rediffmail.com

INTRODUCTION

In physical science, many problems arise and all are expressing in the way of differential equation and therefore study of differential equation is a challenging field. In article we will define the differential equation and its various aspects. This paper contains:

1. Basic idea of second order differential equation, 2. Numeral's method and 3.Finite difference methods Equation involving independent variable *'x'*, dependent variable, *'y'* and their differential coefficients such as $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ …is known as a differential equation

Example (*i*) $\frac{dy}{dx} = Sinx$

(*ii*) $\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + \frac{dx}{dt} = e^t \log t$

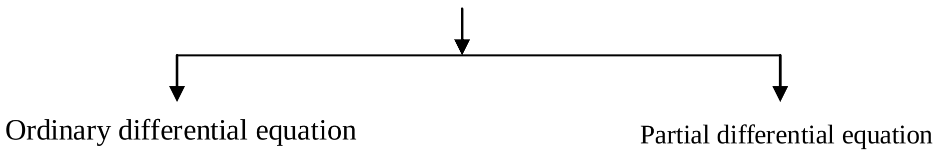
(iii) $\frac{\partial^2 y}{\partial t^2} = \alpha\left(\frac{\partial^3 y}{\partial x^3}\right)^2$, where *y* is a function of *x* and *t* variables.

Differential equation has its own order and degree

Order of differential equation is defined as the order of the highest derivative occurring in the differential equation.

Degree of differential equation is the degree of the highest derivative when differential coefficients are free from radical and fraction. For example, the order of examples(i), (ii) and (iii) is one, four and three respectively and degree is one, one and two respectively.

Classification of differential equation



For example(i) $\frac{dy}{dx} = (Sinx) + x$

(ii) $\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + \frac{dx}{dt} = e^t$

(iii) $K\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$

Partial differential equation is that which involves partial derivatives withrespectto more than one independent variables. For example,

(i) $\Delta^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(ii) $\frac{\partial^2 v}{\partial t^2} = K \frac{\partial^3 v}{\partial x^3}$

Any relation between the dependent and independent variables, when substituted in the differential equation, reduces it to an identity is called a solution of differential equation. Solution does not involve derivative of n^{th} order. This will be desired differential equation of the n^{th} order.

MATERIALS AND METHODS:

Second order differential equation

Consider a second order differential equation as

$$\phi\left(\frac{d^2x}{dt^2}, \frac{dx}{dt}, x, t\right) = 0$$

Under the initial conditions $(x)t_o=x_o$and $(x')\ t_o=x'_o$. This second order equation can be reduced to a system of simultaneous differential equations of first order as follows.

Take $\frac{dx}{dt}$ to be y i.e. $\frac{dx}{dt} = y$ or x′ = y (3.1.1)

And $\frac{dx}{dt} = \phi[x, y, t]$ (3.1.2)

Initial conditions being $(x)t_o=x_o$ and $(y')\ t_o=x'_o$it is worth noticing that an nth order differential equation yields n simultaneous differential equation of first order.

Numeral's method to solve second order differential equation

This method is used only when there is a second order equation without $*y^{(1)}$ i.e. the differential equation is of the type $*y^{(2)} = \phi(x, y)$, we shall develop the formula by using the method of undetermined coefficients. Let x_1, x_2, x_3..., x_k, be the equidistant values of x at the equispacedpoints 1, 2, 3, … k, Let

y_{k-1} and $y_k$be known so that $\phi_{k-1}=\phi\ (x_{k-1}, y_{k-1})$ and$\phi_k=\phi(x_k,y_k)$ also become known.

To integrate the differential equation, we assume a corrector type formula of the form

$$y_{k+1} = Ay_k + By_{k-1} + h^2\ [C\phi_{k+1} + D\phi_k + E_{k-1}] + R \qquad (3.2.1)$$

Where R denotes the error term and A, B, C, D, E are unknown coefficients to be determined.

Let $y_k^{(P)}$denotes the p^{th} derivative of y at $x = x_k$ then by Taylor′s series expansion, we get

$$y_{k+1} = y_k + hy_k^{(1)} + \frac{h^2}{2!}y_k^{(2)} - \frac{h^3}{3!}y_k^{(3)} + \frac{h^4}{4!}y_k^{(4)} + \frac{h^5}{5!}y_k^{(5)} + \frac{h^6}{6!}y_k^{(6)}$$

$$y_{k-1} = y_k - hy_k^{(1)} + \frac{h^2}{2!}y_k^{(2)} - \frac{h^3}{3!}y_k^{(3)} + \frac{h^4}{4!}y_k^{(4)} - \frac{h^5}{5!}y_k^{(5)} + \frac{h^6}{6!}y_k^{(6)}$$

and $$\phi_{k+1} = \phi_k + h\phi_k^{(1)} + \frac{h^2}{2!}\phi_k^{(2)} + \frac{h^3}{3!}\phi_k^{(3)} + \frac{h^4}{4!}\phi_k^{(4)} + - - -$$

Again, we have $\phi = y^{(2)}$

$$\therefore \phi^{(1)} = y^{(3)}, \phi^{(2)} = y^{(4)}, \phi^{(3)} = y^{(5)}, \phi^{(4)} = y^{(6)} \text{etc.}$$

$$h^2\phi_{k-1} = h^2 y_k^{(2)} + h^3 y_k^{(3)} + \frac{h^4}{2!}y_k - \frac{h^5}{3!}y_k^{(5)} + \frac{h^6}{4!}y_k^{(6)}$$

and $$h^2\phi_{k-1} = h^2 y_k^{(2)} - h^3 y_k^{(3)} + \frac{h^4}{2!}y_k^{(4)} - \frac{h^5}{3!}y_k^{(5)} + \frac{h^6}{4!}y_k^{(6)}$$

The unknown coefficients are so determined that the corrector type formula agrees with the Taylor's expansion of y_{K+1} up to fourth order. Now substituting the values of y_{k+1}, y_k, y_{k-1}, ϕ_{k+1}, ϕ_k, ϕ_{k-1}, *in* (3.2.1) and then comparing the coefficients of various powers of h (up to fourth power of h) on both sides, we get

$A + B = 1, B = -1, (B/2) + C + D + E = (1/2)$

$- (B/6) + C - E = (1/6), (B/24) + (E/2) + (C/2) = (1/24)$

These give $A = 2, B = -1, C = (1/12), D = (5/6), E = (1/12)$

It is worth noticing that the coefficient of fifth powers of h are also equal, of course we matched the terms only up to fourth power. Making these substitutions in the corrector type formula, we at once obtain

$$y_{k+1} = 2y_k - y_{k-1} + (h^2/12)\,[\phi_{k+1}, +10\phi_k + \phi_{k-1}] + R \qquad (B)$$

It is known as **Numerov's formula.**

This formula involves an error of order six and if we assume all the terms of $y^{(6)}$ to be equal, then the estimate of the error is $-[h^6 y_k^{(6)}/240]$.

Again in (B), y_{K+1} appears on both sides hence this formula becomes a corrector formula. Thus to obtain y_{k+1}, some previous approximation for y is necessary.

For this reason, ignoring the term ϕ_{k+1}, in (B), we readily obtain

$$y_{k+1} = 2y_k - y_{k-1} + (h^2/12)\,[10\phi_k + \phi_{k-1}] \qquad (C)$$

The formula of (C) can now be used as predictor formula.

Finite difference method

Most of require the solution in a region R subject to various conditions on the boundary of R. We shall discuss two point linear boundary value problems as given below:

(i) $$\frac{d^2y}{dx^2} + \lambda(x)\frac{dy}{dx} + \mu(x)y = y(x); y(x_0) = a \qquad y(x_n) = b$$

(ii) $$\frac{d^4y}{dx^4} + \lambda(x)y = \mu(x); y(x_0) = y'(x_0) = a; \qquad y(x_n) = y'(x_n) = b$$

As a matter of fact, there exist many numerical methods for solving such boundary value problems but the method of finite differences is most common.

The finite difference approximations to the various derivatives are derived below.

Let *y(x)* and its derivatives be single valued continuous functions of *x* by Taylor′s expansion, we get

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + --- \qquad (3.3.1)$$

$$y(x-h) = y(x) - hy'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + --- \qquad (3.3.2)$$

$$\Rightarrow y'(x) = 1/h\,[y(x+h) - y(x)] - h/2\, y''(x) -$$

$$= 1/h\,[y(x+h) - y(x)] + O(h)$$

This is the forward difference approximation of *y′(x)* with an error of order h.Similarly (3.3.2) gives $y'(x) = 1/h\,[y(x) - y(x-h)] + O(h)$

This is the backward difference approximation of *y′(x)* with an error of order h. Now subtracting (3.3.2) from (3.2.1) we get

$$y'(x) = 1/2h\,[y(x+h) - y(x-h)] + O(h^2)$$

This is the central difference approximation of *y′(x)* with an error of the order h^2.

Clearly central difference approximation to *y′(x)* is better than the forward or backward difference approximations and hence it should be preferred.

Then adding (3.3.1) and (3.3.2), we obtain

$$y''(x) = 1/h^2\,[y(x+h) - 2y(x) + y(x+h)] + O(h^2)$$

This is the central difference approximation of *y″(x)*. Similarly, we can obtain central difference approximation of higher derivatives.

Now the working expressions for the central difference approximation of the first four derivatives of y_i are as given below:

$$y_i^{I} = \frac{1}{2h}\left(y_{i+1} - y_{i-1}\right) \qquad (3.3.3)$$

$$y_i^{II} = \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right) \qquad (3.3.4)$$

$$y_i^{III} = \frac{1}{2h^3}\left(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right) \qquad (3.3.5)$$

$$y_i^{iV} = \frac{1}{h^4}\left(y_{i+2} - 4y_{i+1} + 6y_1 - 4y_{i\text{-}1} + y_{i-2}\right) \qquad (3.3.6)$$

As a matter of fact, the accuracy of the method depends on the size of the sub-interval h and also on the order of approximation. As we reduce h, though the accuracy improves put the number of equations to be solved also increases.

DISCUSSIONS AND RESULTS:

Numerical Examples

Example 1: Find the differential equation of the family of curves $y=e^x(A\ cosx + B\ sinx)$, where A and B are arbitrary constants.

Solution: Given equation

$Y = e^x\ (A\ Cos\ x + B\ Sin\ x\ )$ *(i)*

Differentiating (i) with respect to x

$y' = e^x\ (A\ Cos\ x + B\ Sin\ x) + e^x\ (-A\ Sin\ x + B\ Cos\ x)$

$= (-A\ Sin\ x + B\ Cos\ x)\ e^x + y$ *(ii)*

Differentiating (ii) with respect to x

$y'' = (-A\ Cos\ x + B\ Sin\ x)\ e^x + e^x\ (-A\ Sin\ x + B\ Cos\ x) + y'$ *(iii)*

$y'' = -\ (A\ Cos\ x - B\ Sin\ x)\ e^x + (-A\ sin\ x + B\ cos\ x)\ e^x + y$

$= (-\ A\ sin\ x + B\ cos\ x)\ e^x + y' - y$ *(iv)*

$e^x\ (-A\ sin\ x + B\ cosx)$

Put the values from (ii) in the equation (iv)

$y'' = y' = y + y' - y$

$= 2\ y' - 2y$ *or*

$$y'' - 2y' + 2y = 0$$

Example 2: Show that $Ax^2 + By^2 = 1$ is the solution of

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Solution: Given that $Ax^2 + By^2 = 1$ *(i)*

Differentiating (i) $2Ax + 2By\frac{dy}{dx} = 0$

$$Ax + By\frac{dy}{dx} = 0 \qquad (ii)$$

Differentiating (ii) $A + \left[By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2\right] = 0$ *(iii)*

Multiplying (iii) by x $Ax + Bx\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = 0$ *(iv)*

Subtracting (ii) from (iv)

$Bx\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - By\frac{dy}{dx} = 0$ which implies $x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$

of dependent variable with respect to independent variable. Numerical solution of differential equations in second order

Example 3: Use Taylor's series method to obtain the power series in t for x and y satisfying the differential equations.

$$(dx / dt) = x + y + t, (d^2y / dt^2) = x - t$$

under the initial conditions

$$x_{t=o} = 0, y_{t=o} = 1, (dy / dt)_{t=o} = -1$$

Solution: Consider $dy / dt = z$, so that the given equation are $dx / dt = x + y + t$,

$dz / dt = x - 't$ *and* $dy /dt = z$.

The initial conditions become $x_{t=0} = 0, y_{t=0} = 1$ *and* $(z)_{t=0} = -1$

Expanding x, y, z in power series of t, we get

$$x = x_0 + x_o't + (t^2 / 2!)\, x_0'' + (t^3 / 3!)\, x_0''' + \ldots.$$

$$y = y_0 + y_o't + (t^2 / 2!)\, y_0'' + (t^3 / 3!)\, y_0''' + \ldots. \qquad (A)$$

And $z = z_0 + z_o't + (t^2 / 2!)\, z_0' + (t^3 / 3!)\, z_0''' + \ldots.$

Now, we have $x' = x + y + t,\ y' = z_1\ z' = x - t$

$x'' = x' + y' + 1 \quad = \quad x + y + z + t + 1$

$y'' = z' \quad = \quad x - t$

$z'' = x' - 1 \quad = \quad x + 1 + t - 1$

$x''' = x'' + y'' \quad = \quad 2x + y + z + 1$

$y''' = z'' \quad = \quad x + y + t - 1$

$z''' = x''' \quad = \quad x + y + z + t + 1$ etc

when substituting $x_o = 0, y_o = 1, z_o = -1$ we get

$x_0' = 1, y_0' = -1, z_0' = 0$

$x_0'' = 1, y_0'' = -0, z_0'' = 0$

$x_0''' = 1, y_0''' = 0, z_0''' = 1$ etc.

Making these substitutions in (A), we get

$x = x_0 + x_o't + x_0'' (t^2 / 2!) + x_0''' (t^3 / 3!) + \ldots.$

$= t + (t^2/2!) + (t^3/3!) + (t^4/4!) + (2t^5/5!) + \ldots$

$y = 1 - t + (t^4/4!) + (t^5/5!) + \ldots$

$z = -1 + (t^3/3!) + (t^4/4!) + (t^5/5!) + \ldots$

Example 4: Using Runge- Kutta method, solve $y'' = xy'^2 - y^2$ *for* $x = 0.2$ correct to 4 decimal places. Initial conditions are $x = 0, y = 1, y' = 0.$

Solution: Let $(dy/dx) = z = f(x, y, z)$ then $\frac{dz}{dx} = xz^2 - y^2 = \phi[x, y, z]$

But $x_o = 0, y_o = 1, z_o = 0, h = 0.2$

Then Runge-Kutta formulae become

$K_1 = hf(x_o, y_o, z_o) \quad = 0.2\,(0) = 0$

$K_2 = hf(x_o + ½ h, y_o + ½ k_1, z_o + ½ l_1) \quad = 0.2\,(-0.1) = -0.02$

$K_3 = hf(x_o + ½ h, y_o + ½ k_2, z_o + ½ l_2) \quad = 0.2\,(-0.0999) = -0.02$

$K_4 = hf(x_o + h, y_o + k_3, z_o + l_3) \quad = 0.2\,(-0.1958) = -0.0392$

$\therefore K = 1/6\,(k_1 + 2k_2 + 2k_3 + k_4) \quad = -0.0199$

$l_1 = h\phi(x_o, y_o, z_o) \quad = 0.2\,(-1) = -0.2$

$l_2 = h\phi(x_o + ½ h, y_o + ½ k_1, z_o + ½ l_1) \quad = 0.2\,(-0.999) = -0.1998$

$l_3 = h\phi\,(x_o + ½\, h, y_o + ½\, k_2, z_o + ½\, l_2) \quad = 0.2\,(-0.9791) = -0.1958$

$l_4 = h\phi\,(x_o + h, y_o + k_3, z_o + l_3) \quad = 0.2\,(0.9527) = -0.1905$

$\therefore\ l = 1/6\,(l_1 + 2l_2 + 2l_3 + l_4) \quad = -0.1970$

Thus at $x = 0.2$, we have $\quad y = y_o + k = 1 - 0.0199 = 0.9801$

and $\quad y' = z = z_o + l = 0\text{-}0.1970 = -0.1970$

Example5: Using Numerov's formula, solve $y''=y$ given that $y_o = 1, y'(0)=-1$

Given $y'' + xy' + y = 0, y(0) = 1, y'(0) = 0$, obtain y *for* $x = 0, (0.1), 0.3$ by any method. Further, continue the solution by Milne's method to calculate $y\,(0.4)$.

Solution: Putting $y' = z$, the given equation reduces to:

$$z' + xz + y = 0,\ y' = Z \qquad (1)$$

Now we use Taylor's series method to find y. Differentiating the given equation n times, we have

$Y_{n+2} + xy_{n+1} + ny_n + y_n = 0 \Rightarrow (y_{n+2})_0 = -\,(n+1)\,(y_n)_0$

$\therefore y(0) = 1$, *gives* $y_2\,(0) = -1, y_4\,(0) = 3, y_6(0) = -5 \times 3, \ldots$

And $y_1(0) = 0$ *yields* $y_3(0) = y_5(0) = \ldots = 0$

The expanding $y(x)$ by Taylor's series, we get

$$y(x) = y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + ---$$

$$\Rightarrow y(x) = 1 - \frac{x^2}{2!} + \frac{3}{4!}x^4 - \frac{5 \times 3}{6!}x^6 + --- \qquad (2)$$

and $$z(x) = y'(x) = -x + \frac{1}{2}x^3 - \frac{1}{8}x^5 + --- \quad = -xy \qquad (3)$$

Then from (2), we have

Solution: Taking the interval of differentiating to be 0.5 and equal throughout, we may take $x_0=0$, $x_1=0.5$, $x_2=1$. The corresponding values by y shall be denoted by y_0, $y_1$and y_2 respectively. We are interested in finding out the value of y_2. By putting K = 1 in (C) we get

$$y_2 = 2y_1 - y_0 + (h^2/12)\,[10\phi_1 + \phi_0] \qquad (1)$$

Now $y_o = 1, y'_o = -1, y''_o = 1, y'''_o = -1, y^{iv}_o = 1$ *etc.*

$$\therefore\ y_1 = y_0 + hy'_o + \frac{h^2}{2!}y''_o + \frac{h^3}{3!}y'''_0 + ---$$

$= 1 - 0.5 + 0.125 - 0.0208 + 0.0026 - 0.0003 = .6065$

And from (1)

$y_2 = 2\,(.6065) - 1 + \{(0.5)^2 / 12\}\{10 \text{ x } .6065 + 1\} = 0.3602$

$\Rightarrow \phi_2 = \phi\,(x_2, y_2) = y_2 = 0.3602$

Hence using (B), the first corrected value by y_2 is given by

$y_2 = 2y_1 - y_0 + (h^2/12)\,[\phi 2 + 10\phi_1 + \phi_0] = 2 \text{ x } (.6065) - 1 + \{(0.5)^2 / 12\}\,[.3602 + 6.065 + 1] = 0.3677$

Again using corrector formula, we get $y_2 = .2130 + \{(0.5)^2 / 12\}\,[.3677 + 6.065 + 1\} = 0.3678$

$\Rightarrow (y)_{x=1} = 0.3678$

$$y(0.1) = 1 - \frac{(0.1)^2}{2} + \frac{1}{8}(0.1)^4 - .... = 0.995$$

$$y(0.2) = 1 - \frac{(0.2)^2}{2} + \frac{(0.2)^4}{8} - .... = 0.9802$$

$$y(0.3) = 1 - \frac{(0.3)^2}{2} + \frac{(0.3)^4}{8} - \frac{(0.3)^6}{48} + \;... = 0.956$$

From (3), we also have

$z\,(0.1) = -0.0995,\; z\,(0.2) = -\,0.196,\; z\,(0.3) = -0.2863$

Further from $(1),\; z'(x) = -(xz+y)$

$z'\,(0.1) = 0.985,\; z'\,(0.2) = -0.941,\; z'\,(0.3) = -0.87$

Now applying Milne′s predictor formula, first to z and then to y

We readily obtain

$z\,(0.4) = z\,(0) + 4/3\,(0.1)\,\{\,2\,z'\,(0.1) - z'\,(0.2) + 2z'\,(0.3)\} = 0 + (0.4/3)\,\{\,-1.79 + 0.941 - 1.74\} = -0.3692$ and $y\,(0.4) = y(0) + 4/3\,(0.1)\,\{\,2y'(0.1) - y(0.2) + 2y'\,(0.3)\}\; 0 + (0.4/3)\,\{-0.199 + 0.196 - 0.5736\}\,[\;\; y'=z] = 0.9231$. Also $z'\,(0.4) = -\{x(0.4)\,z\,(0.4) + y\,(0.4)\} = -\,\{0.4\,(-0.3692) + 0.9231\} = -0.7754$

Further applying Milne′s corrector formula, we readily get

$z\,(0.4) = z(0.2) = h/3\,\{z'\,(0.2) + 4\,z'\,(0.3) + z'\,(0.4)\} = -0.196 + (0.1/3)\,\{-0.941 - 3.48 - 0.7754\} = -0.3692$ and $y\,(0.4) = y(0.2) + h/3\,\{y'\,(0.2) + 4y'\,(0.3) + y'\,(0.4)\} = 0.9802 + (0.1/3)\,\{-0.196 - 1.1452 - 0.3692\} = 0.9232$

$\therefore y\,(0.4) = 0.9232$ and $z\,(0.4) = -0.3692$

Example 6: Solve the equation $y'' = x+y$ with the boundary conditions $y(0) = y(1) = 0$.

Solution: Let us divide the interval (0, 1) into four sub intervals, so that $h = ¼$. The pivot points are $x_0=0, x_1=1/4, x_2=1/2, x_3=3/4$ and $x_4=1$.

Now the differential equation is approximated as

$$1/h^2 [y_{i+1} - 2y_i + y_{i-1}] = x_i + y_i$$

$\Rightarrow$ $16y_{i+1} - 33y_i + 16 y_{i-1} = x_i$, $i = 1, 2, 3$ $(h=1/4)$ then using $y_o = y_4=0$, we get the system of equations.

$16y_2 - 33y_1 = ¼$, $16 y_3 – 33y_2 + 16 y_1 = ½$ and $–33y_3 + 16y_2 = ¾$

Solving $y_1 = - 0.03488$, $y_2 = -0.05632$, $y_3 = -0.05003$

The exact solution is $y(x) = \frac{Sinhx}{Sinh\ 1} - x$. Now we frame the following table

X	Computed value y (x)	Exact value y (x)	Error
0.25	-0.03488	-0.03505	0.00017
0.5	-0.05632	-0.05659	0.00027
0.75	-0.05003	-0.05028	0.00025

Example 7: Determine values of y at the pivotal points of the interval (0, 1) if y satisfies the boundary value problem $y^{iv} + 81y = 81 x^2$

$$y (0) = y (1) = y'' (1) = 0. \text{ (Take } n=3)$$

Solution: $h = 1/3$ and the pivotal points *are* $x_o=0$, $x_1 = 1/3$, $x_2 = 2/3$, $x_3 = 1$. The corresponding y values are $y_o(=0)$, y_1, y_2, y_3 $(=0)$

Then replacing $y^{iv}$by its central difference approximation, the differential equation yields

$$1/h^4 (y_{i+2} - 4y_{i+1} + 6 y_i – 4y_{i-1} + y_{i-2}) + 81y_I = 81x_i^2$$

$$\Rightarrow y_{i+2} - 4y_{i+1} + 7y_i – 4y_{i-1} + y_{i-2} =x_i^2, \ i = 1, 2$$

Putting $i = 1$, $y_3 – 4y_2 + 7y_1 – 4y_o + y_{-1}= 1/9$

Putting $i = 2$, $y_4 – 4y_3 + 7y_2 – 4y_1 + y_0 = 4/9$

Now, using $y_o = y_3 = 0$, we obtain

$-4 y_2 + 7y_1 + y_{-1} = 1/9$ (1)

$y_4 + 7y_2 – 4y_1 = 4/9$ (2)

Further, regarding the conditions $y_0'' = y_3'' = 0$

We have $y_i'_I = 1/h^2 (y_{i+1} – 2y_i + y_{i-1})$

Putting $i = 0$, $y_o'' = 9 (y_1 – 2y_0 + y_{-1})$

$\Rightarrow y_{-1} = -y_1 \quad [\quad y_o = y_o'' = 0]$ (3)

Putting $i = 3, y_3'' = 9\ (y_4 - 2y_3 + y_3)$

$\Rightarrow y_4 = -y_2 \quad [\quad y_3 = y_3'' = 0]$ (4)

Using (3), the equation $(1) \Rightarrow -4y_2 + 6y_1 = 1/9$ (5)

Using (4), equation $(2) \Rightarrow 6y_2 - 4y_1 = 4/9$ (6)

Solving (5) and (6), we get $y_1 = 11/90$ and $y_2 = 7/45$

$\therefore y\ (1/3) = 0.1222$ and $y\ (2/3) = 0.1556$

CONCLUSIONS:

This Article explained the differential equation of second order and how to find the numerical solutions of second order differential equations with the various Technique. In this paper discussed the Numerov's method to solve the second order differential equation to obtain predictor formula by using the method of undetermined coefficients and for better understanding given some examples and also explained the Milne's predictor and corrector formula. In Finite Difference Method as we reduce h, the accuracy improves. Some application and numerical results are given to demonstrate the high efficiency of the approach

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