Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

On Ingrained Construction of a Newfangled almost GP - Spaces in Simple Extended Topological Spaces

M. Dhanalakshmi* and K. Alli

Department of Mathematics, The M.D.T.Hindu College, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, 627012, Tamil Nadu, India.
Email; dhanagamani@gmail.com, allimdt@gmail.com

ABSTRACT

This paper elucidates about a new fangled form of the almost GP spaces under the ceiling of simple extension topological spaces. In addition it conveys the characteristics of almost P and almost GID spaces in simple extended topological spaces.

KEYWORDS: Meager$^+$ set, Residual$^+$ set, Almost P$^+$ spaces, Almost GP$^+$ spaces, Almost GID $^+$ -spaces.

SUBJECT CLASSIFICATION: 54A05; 54A99; 54C10; 54C20; 54F15

***Corresponding author**

M. Dhanalakshmi

Research Scholar (Reg. No.- 12414), Department of Mathematics, The M.D.T.Hindu College, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, 627012, TamilNadu, India. Email; dhanagamani@gmail.com

1. INTRODUCTION

In 1963, Levine[3] introduced the concept of simple extension of a topology τ by a non open set B as $\tau^+(B)=\{O\cup(O'\cap B) \,/\, O,O'\in \tau, B\notin \tau\}$ in simple extension as well. Let A be a subset of a space X. Then the closure of A is assumed in the extended topology and the interior of A is taken in general topology are denoted by cl^+ (A) and int (A) respectively.

A subset of a topological space is a G_δ-*set* if it is the intersection of countably many open sets; it is an F_σ-*set* if it is the complement of a G_δ-set.

A completely regular space *X* in which every nonempty G_δ-set has nonempty interior is called an almost P-space. Almost P-spaces was first introduced by A. I. Veksler[4] and it was also studied further by R. Levy in [11]. A T_1 topological space *X* is called an almost GP-space (respectively a GID-space) if every dense G_δ-set of *X* has nonempty interior (respectively dense interior).

Throughout this paper, (X, τ^+) (or simply X) represent a non-empty simple extended topological space (or simply extended topological space) on which no separation axioms are assumed, unless and otherwise mentioned.

2. SOME DISPOSITIONS OF DENSE, NOWHEREDENSE, MEAGER AND RESIDUAL SETS IN EXTENDED TOPOLOGICAL SPACES

Definition 2.1: A subset A of a topological space (X, τ^+) is said to be nowhere dense+ in X if $int(cl^+(A)) = \varphi$, i.e the interior of the closure of A is empty. Otherwise put, A is nowhere dense+ iff it is contained in a closed set (τ^+cl)with empty interior.

Proposition 2.2: Let X be an extended topological space. Then:
(a) Any subset of a nowhere dense+ set is nowhere dense+.
(b) The union of finitely many nowhere dense+ sets is nowhere dense+.
(c) The closure of a nowhere dense+ set is nowhere dense+.

Proof: Obvious from the definition and the elementary properties of closure and interior.

Definition 2.3: A subset A ⊆ X is called meager$^+$ (or of first category) in X if it can be written as a countable union of nowhere dense+ sets.

Definition 2.4: Any set that is not meager+ is said to be nonmeager+ (or of second category). The complement of a meager$^+$ set is called residual$^+$ set

Definition 2.5: Let I(X) be the set of all isolated points of X. If I(X) = φ, then the space X is said to be crowded and a spaces is said to be separable if it contains a countable dense+ subset.

Proposition 2.6: Let X be a extended topological space. Then:
(a) Any subset of a meager+ set is meager+.
(b) The union of countably many meager sets is meager+.
(c) If X has no isolated points, then every countable set is meager+.

The following lemma is useful in the sequel.

Lemma 2.7: Let A ⊆ X be a nowhere dense+ set. Then the closure of A is a nowhere dense+ set .

Definition 2.8: A subset A of a topological space (X, τ^+) is said to be dense+ in X if int(cl^+(A)) = X.

3. ALMOST P^+-SPACES, ALMOST GP^+-SPACES AND ALMOST GID^+-SPACES

Definition 3.1: The *almost-P^+-spaces,* consists of those spaces in which $G_\delta$sets have dense$^+$ interiors.

Example 3.2: Let X= {a, b, c} with the topology τ = {X, φ, {c}} and τ^+ = {X, φ, {a}, {c}, {a, c}}.here $G_\delta$sets = {c} and also int(cl^+({c})) = {c}.And hence (X, τ^+) is almost P^+- space.

Recall that completely regular spaces *X* in which every nonempty G_δ-set of *X* has nonempty interior is called an almost P-space.

And hence a completely regular$^+$ space *X* in which every nonempty G_δ-set of *X* has nonempty interior is called an almost P^+-space.

Definition 3.3: A subset of a topological space is a $G_{\delta+}$*-set* if it is the intersection of countably many τ^+-open sets; it is an $F_{\sigma+}$*-set* if it is the complement of a $G_{\delta+}$-set.

In the following definition we give a generalization of almost P^+-spaces as follows:

Definition 3.4: A topological space *X* is called an almost GP^+-space if every dense$^+$ G_δ-set of *X* has nonempty interior in *X*.

Example 3.5: Let X= {a, b, c} with the topology τ = {X, φ, {a}, {a, b}} and τ^+ = {X, φ, {a}, {b}, {a, b}}.here dense$^+$ $G_\delta$sets = {a, b} and also int({a, b})) = {a, b}.And hence (X, τ^+) is almost GP$^+$-space.

The following theorem gives the characterization of Baire space in SETS.

Definition 3.6: A topological space *X* is called an almost GP^{++}-space if int(V) ≠φ for every dense+ sets V and V= $\bigcap_{n\in N} V_n$, V_n‘ s are non-empty τ^+-open sets in (X, τ^+).

In other words, A topological space *X* is called an almost GP^{++}-space if every dense$^+$ $G_{\delta+}$-set of *X* has nonempty interior in *X*.

Theorem 3.7: Suppose (X, τ^+) be SETS. Then X is a Baire space iff every non-empty open set is of II-Category.

Theorem 3.8: If (X, τ^+) be a SETS and T_1 – crowded separable Baire space, then (X, τ^+) is not a almost GP$^+$-space.

Proof: Let D be a countable dense$^+$ subset of X. Then X-D =$\bigcup_{d\in D}\{X-\{d\}\}$. Since X is Baire, X-D is a dense$^+$ set. Thus, X-D is dense$^+$ and X-D = $\bigcup_{d\in D}\{X-\{d\}\}$ where {X-{d}} ‘s are open sets in X. Since D is dense$^+$ in X, int(X-D) = φ. Thus, (X, τ^+) is not an almost GP$^+$-space.

Theorem 3.9: Let Y be an open dense$^+$ subset of (X, τ^+). Then Y is a almost GP$^+$ Space iff X is almost GP$^+$ Space.

Theorem 3.10: Let (X, τ^+) and (Y, σ^+) be two SETS. If f : X → Y is a continuous, feebly open , surjective mapping and X is a almost GP$^+$ space, then Y is a almost GP$^+$ space.

Theorem 3.11: Let (X, τ^+) and (Y, σ^+) be two SETS. If f : X → Y is a homeomorphism and Y is a almost GP$^+$ space, then X is a almost GP$^+$ space .

Theorem 3.12: $\prod_{i\in I} X_i$ is a almost GP$^+$ space iff each X_i is a almost GP$^+$ space.

Theorem3.13: Let ϑ be a collection of open subsets of a SETS X whose union is dense in X. If there is some U$\in\vartheta$ such that U is a almost GP$^+$ space then X is a almost GP$^+$ space.

Theorem3.14: For a SETS X, the following conditions are equivalent.

(a) For every V, if V is a dense set and V =$\bigcap_{n\in N} V_n$ where V_n' are non-empty open sets in X, the cl^+(int(V)) = X.

(b) Every nonempty open subspace of X is a almost GP$^+$-space.

Definition 3.15: A topological space X is called an almost GID$^+$-space if every dense$^+$ G_δ-set of X has dense$^+$ interiors.

Theorem 3.16: If X is a (Baire) almost GID$^+$-space and D is dense$^+$ in X, then D is a (Baire) almost GID$^+$-space.

Theorem 3.17: For a SETS X the following conditions are equivalent:
(1) X is a almost GID$^+$-space.
(2) Every nonempty open subspace of X is a almost GID$^+$-space.
(3) Every nonempty open subspace of X is an almost GP$^+$-space.

Theorem 3.18: Let X be an almost GID$^+$-space. If f: X $\longrightarrow$ Y is continuous, onto and open, then Y is a almost GID$^+$-space.

Proposition 3.19: Let U be a collection of open subsets of the space X whose union is dense in X. Then, every member of U is a GID-space, and then X is a GID-space.

Corollary 3.20: The topological sum of a family of almost GID$^+$-spaces (almost GP$^+$-spaces) is a almost GID$^+$-space (an almost GP$^+$-space).

REFERENCES:

1. Ahmad Al-Omari and MSM Noorani, "Decomposition of continuity via b-open set", Bol. Soc. Paran. Mat. 2008; 26(1-2); 53-64.
2. Cao, J., Ganster, M., and Reilly, I., "Submaximality, extremal disconnectedness and generalized closed sets", Houston J. Math. 1998; 24(4): 681-688.
3. Levine,N., "Simple extension of topology", Amer.Math.Monthly, 1964; 71:22-105.
4. Mohammad Reza Ahmadi Zand, "ALMOST GP-SPACES ", J. Korean Math. Soc. DOI 10.4134/JKMS.2010.47.1.215. 2010; 47(1): 215–222.
5. Nirmala Irudayam, F., and Arokiarani, Sr. I., "A note on the weaker form of bI set and its generalisation in SEITS", International journal of computer applications, 2012; 4(2): 42-54.
6. Palaniappan, N., and Rao, K. C., "Regular Generalized Closed Sets", Kyung-pook Math. J., 1993; 33: 211-219.
7. Park J. H and Park J. K., On πgp-continuous functions in topological spaces,Chaos, Soliton and Fractals, 2004; 20: 467-477.
8. Sinem Cagler Akgun and Gulhan Aslim , "On πgb-closed sets and Related Topics".(2012).
9. Tong, J., "On decomposition of continuity in Topological spaces", Acta .Math. Hungar, 1989; 54(1-2):51-55.

10. UgurSenguil, “On almost b-continuous mappings”, Int. J. Contemp. Math.Sciences, 2008; 3: 1469-1480
11. Vadakasi.S and Renugadevi, “On almost GP spaces in generalised topological spaces”, ANJAC journal of sciences, ISSN:0972-6012, ,August 2016; 15(2).