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On (1,2)*- G‴-Regular Space In Bitopological Spaces

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ABSTRACT

In this paper, we introduce (1,2)*- g‴-regular space in bitopological spaces. We obtain several characterizations of (1,2)*- g‴-regular space in some preservation theorems for (1,2)*-g‴-regular.

KEYWORDS:(1,2)*- g‴-regular spaces, (1,2)*-g‴-regular, (1,2)*- g‴-neighborhood, (1,2)*- g‴-open set, (1,2)*-gT g‴-space.

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1. INTRODUCTION

Using g-closed sets, Munshi[1]introduced g-regular in topological spaces. In a similar way, Sheik John[2]introduced ω-regular using ω-closed sets in topological spaces.

2. SOME DEFINITIONS AND THEOREMS

Definition 2.1

A function f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called a weakly (1,2)*- continuous if for each point x ∈X and each $\sigma_{1,2}$-open set V in (Y, σ_1, σ_2) containing f(x), there exists an $\tau_{1,2}$-open set U containing x such that f(U) ⊆ $\sigma_{1,2}$-cl(V).

Definition 2.2

A space (X, τ_1, τ_2) is called a (1,2)*-gT g‴-space if every (1,2)*-g- closed set in it is (1,2)*- g‴-closed.

Definition 2.3

A bitopological space (X, τ_1, τ_2) will be termed symmetric if and only if for x and y in (X, τ_1, τ_2), x ∈ $\tau_{1,2}$-cl(y) implies that y ∈ $\tau_{1,2}$-cl(x).

Definition 2.4

For a subset A of a bitopological space (X, τ_1, τ_2), $\tau_{1,2}$-cl_θ(A) = {x ∈X:$\tau_{1,2}$-cl(U) ∩ A ≠ φ, U is $\tau_{1,2}$ –open set containing x}.

Theorem 2.5

A set A is (1,2)*- g‴-open if and only if F ⊆ $\tau_{1,2}$-int(A) whenever F is(1,2)*-gs-closed and F ⊆A.

Theorem 2.6

The space (X, τ_1, τ_2) is symmetric if and only if {x} is (1,2)*-g-closed in (X, τ_1, τ_2) for each point x of (X, τ_1, τ_2).

Definition 2.7

Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. Then f is called

(i) (1,2)*- g‴-continuous if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)*- g‴-closed in X.

(ii) (1,2)*- g‴-irresolute if the inverse image of every (1,2)*- g‴-closed set of Y is (1,2)*- g‴-closed in X.

(iii) pre-(1,2)*-gs-open if the image of every (1,2)*-gs-open set of X is(1,2)*-gs-open set in Y.

(iv) (1,2)*-open if the image of every $\tau_{1,2}$-open set of X is $\sigma_{1,2}$-open in Y.

(v) (1,2)*-gs-irresolute if the inverse image of every (1,2)*-gs-closed set ofY is (1,2)*-gs-closed in X.

(vi) (1,2)*- g‴-closed if the image of every $\tau_{1,2}$-closed set of X is (1,2)*- g‴- closed in Y.

(vii) (1, 2)*-continuous [3] if for each $\sigma_{1,2}$-open set V of Y, $f^{-1}(V)$ is $\tau_{1,2}$- open in X.

Theorem 2.8

If f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is bijective, pre-(1,2)*-gs-open and (1,2)*-g‴-continuous, then f is (1,2)*- g‴-irresolute.

Definition 2.9

Let (X, τ_1, τ_2) be a bitopological space. Let x be a point of X and G be a subset of X. Then G is called an (1,2)*- g‴-neighborhood of x (briefly, (1,2)*- g‴-neighborhood of x) in X if there exists an (1,2)*-g‴-open set U of X such that $x \in U \subseteq G$.

3. (1,2)*- α g‴-REGULAR SPACE

We introduce the following definition.

Definition 3.1

A space (X, τ_1, τ_2) is said to be (1,2)*- g‴-regular if for every (1,2)*- g‴-closed set F and each point $x \notin F$, there exist disjoint $\tau_{1,2}$-open sets U and V such that $F \subseteq U$ and $x \in V$.

Theorem 3.2

Let (X, τ_1, τ_2) be a bitopological space.

Then the following statements are equivalent:

(i) (X, τ_1, τ_2) is a (1,2)*- g‴-regular space.

(ii) For each $x \in X$ and (1,2)*- g‴-neighborhood W of x there exists an $\tau_{1,2}$- open neighborhood V of x such that $\tau_{1,2}$-cl(V) $\subseteq W$.

Proof

(i) ⇒(ii). Let W be any (1,2)*- g‴-neighborhood of x. Then there exist a (1,2)*- g‴-open set G such that $x \in G \subseteq W$. Since G^c is (1,2)*- g‴-closed and $x \notin G^c$, by hypothesis there exist $\tau_{1,2}$-open sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \varphi$ and so $V \subseteq U^c$. Now, $\tau_{1,2}$-cl(V) $\subseteq$ $\tau_{1,2}$-cl(U^c) = U^c and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq W$. Therefore $\tau_{1,2}$-cl(V) $\subseteq W$.

(ii) ⇒(i). Let F be any (1,2)*- g‴-closed set and $x \notin F$. Then $x \in F^c$ and F^c is (1,2)*- g‴-open and so F^c is a (1,2)*- g‴-neighborhood of x. By hypothesis, there exists an τ1,2-open neighborhood V of x such that $x \in V$ and τ1,2-cl(V) $\subseteq F^c$, which implies $F \subseteq$(τ1,2-cl(V))^c. Then (τ1,2-cl(V))^c is an $\tau_{1,2}$-open set containing F and $V \cap (\tau_{1,2}\text{-cl}(V))^c = \varphi$. Therefore, X is (1,2)*- g‴-regular[4,5,6,7].

Theorem 3.3

For a space (X, τ_1, τ_2) the following are equivalent:

(i) (X, τ_1, τ_2) is (1,2)*-normal.

(ii) For every pair of disjoint $\tau_{1,2}$-closed sets A and B, there exist (1,2)*- g‴- open sets U and V such that A ⊆U, B ⊆V and U ∩ V = φ.

Proof

(i) ⇒(ii). Let A and B be disjoint $\tau_{1,2}$-closed subsets of (X, τ_1, τ_2). By hypothesis, there exist disjoint $\tau_{1,2}$-open sets (and hence (1,2)*- g‴-open sets) U and V such that A ⊆U and B ⊆V.

(ii) ⇒(i). Let A and B be $\tau_{1,2}$-closed subsets of (X, τ_1, τ_2). Then by assumption, A ⊆G, B ⊆H and G ∩ H = φ, where G and H are disjoint (1,2)*-g‴-open sets. Since A and B are (1,2)*-gs-closed, by Theorem 2.5, A ⊆ $\tau_{1,2}$- int(G) and B ⊆ $\tau_{1,2}$-int(H). Further, $\tau_{1,2}$-int(G) ∩ $\tau_{1,2}$-int(H) = $\tau_{1,2}$-int(G ∩ H)= φ[8].

Theorem 3.4

A (1,2)*-gT g‴-space (X, τ_1, τ_2) is symmetric if and only if {x} is(1,2)*- g‴-closed in (X, τ_1, τ_2) for each point x of (X, τ_1, τ_2).

Proof

Follows from Definitions 2.2, 2.3 and Theorem 2.6.

Theorem 3.5

A bitopological space (X, τ_1, τ_2) is (1,2)*- g‴-regular if and only if for each (1,2)*- g‴-closed set F of (X, τ_1, τ_2) and each x ∈Fc there exist $\tau_{1,2}$-open sets U and V of (X, τ_1, τ_2) such that x ∈U, F ⊆V and $\tau_{1,2}$-cl(U) ∩ $\tau_{1,2}$-cl(V) = φ.

Proof

Let F be a (1,2)*- g‴-closed set of (X, τ_1, τ_2) and x ∉F. Then there exist $\tau_{1,2}$-open sets U_0 and V of (X, τ_1, τ_2) such that x ∈U=, F ⊆V and U_0∩ V = φ, which implies U_0∩ $\tau_{1,2}$-cl(V) = φ. Since $\tau_{1,2}$-cl(V) is $\tau_{1,2}$-closed, it is (1,2)*-g‴-closed and x ∉ $\tau_{1,2}$-cl(V). Since (X, τ_1, τ_2) is (1,2)*- g‴-regular, there exist $\tau_{1,2}$-open sets G and H of (X, τ_1, τ_2) such that x ∈G, $\tau_{1,2}$-cl(V) ⊆H and G ∩ H = φ, which implies $\tau_{1,2}$-cl(G) ∩ H = φ. Let U = U_0∩ G, then U and V are $\tau_{1,2}$-open sets of (X, τ_1, τ_2) such that x ∈U, F ⊆V and $\tau_{1,2}$-cl(U) ∩ $\tau_{1,2}$- cl(V) = φ[9].

Converse part is trivial.

Corollary 3.6

If a space (X, τ_1, τ_2) is (1,2)*- g‴-regular, symmetric and (1,2)*-gT g‴- space, then it is (1,2)*-Urysohn.

Proof

Let x and y be any two distinct points of (X, τ_1, τ_2). Since (X, τ_1, τ_2) issymmetric and (1,2)*-gTg‴-space, {x} is (1,2)*- g‴-closed by Theorem 3.4. Therefore, by Theorem 3.5, there exist $\tau_{1,2}$-open sets U and V such that x ∈U, y ∈V and $\tau_{1,2}$-cl(U) ∩ $\tau_{1,2}$-cl(V) = φ[10].

Theorem 3.7

Let (X, τ_1, τ_2) be a bitopological space. Then the following statements are equivalent:

(i) (X, τ_1, τ_2) is (1,2)*-g‴-regular.

(ii) For each point $x \in X$ and for each (1,2)*- g‴-neighborhood W of x, there exists an $\tau_{1,2}$-open neighborhood V of x such that $\tau_{1,2}$-cl(V) $\subseteq$W.

(iii) For each point $x \in X$ and for each (1,2)*- g‴-closed set F not containing x, there exists an $\tau_{1,2}$-open neighborhood V of x such that $\tau_{1,2}$-cl(V) $\cap$ F = φ.

Proof

(i) $\Leftrightarrow$(ii). It is obvious from Theorem 3.2.

(ii) $\Rightarrow$(iii). Let $x \in X$ and F be a (1,2)*- g‴-closed set such that $x \notin F$. Then Fc is a (1,2)*- g‴-neighborhood of x and by hypothesis, there exists an $\tau_{1,2}$-open neighborhood V of x such that $\tau_{1,2}$-cl(V) $\subseteq$Fc and hence $\tau_{1,2}$-cl(V) $\cap$ F = φ.

(iii) $\Rightarrow$(ii). Let $x \in X$ and W be a (1,2)*- g‴-neighborhood of x. Then there exists a (1,2)*- g‴-open set G such that $x \in G \subseteq W$. Since G^c is (1,2)*- g‴- closed and $x \notin G^c$, by hypothesis there exists an $\tau_{1,2}$-open neighborhood V of x such that $\tau_{1,2}$-cl(V) $\cap$ G^c = φ. Therefore, $\tau_{1,2}$-cl(V) $\subseteq G \subseteq$W[11].

Theorem 3.8

The following are equivalent for a space (X, τ_1, τ_2).

(i) (X, τ_1, τ_2) is (1,2)*- g‴-regular.

(i) $\tau_{1,2}$-cl_θ(A) = (1,2)*- g‴-cl(A) for each subset A of (X, τ_1, τ_2).

(ii) $\tau_{1,2}$-cl_θ(A) = A for each (1,2)*- g‴-closed set A.

Proof

(i) $\Rightarrow$(ii). For any subset A of (X, τ_1, τ_2), we have always A $\subseteq$(1,2)*- g‴-cl(A)$\subseteq$ $\tau_{1,2}$-cl_θ(A). Let x$\in$((1,2)*- g‴-cl(A))^c. Then there exists a (1,2)*- g‴-closed set F such that $x \in F^c$ and A $\subseteq$F. By assumption, there exist disjoint $\tau_{1,2}$-open sets U and V such that x $\in$U and F $\subseteq$V. Now, x$\in$U$\subseteq$ $\tau_{1,2}$-cl(U) $\subseteq V^c \subseteq$Fc$\subseteq$Ac and therefore $\tau_{1,2}$-cl(U) $\cap$ A = φ. Thus, x $\in(\tau_{1,2}$-cl_θ(A))^c and hence $\tau_{1,2}$- cl_θ(A) = (1,2)*- g‴-cl(A).

(ii) $\Rightarrow$(iii). It is trivial.

(iii) $\Rightarrow$(i). Let F be any (1,2)*- g‴-closed set and $x \in F^c$. Since F is (1,2)*- g‴- closed, by assumption x $\in(\tau_{1,2}$-cl_θ(F))^c and so there exists an $\tau_{1,2}$-open set U such that x $\in$U and $\tau_{1,2}$-cl(U) $\cap$ F = φ. Then F $\subseteq(\tau_{1,2}$-cl(U))^c. Let

$V = (\tau_{1,2}\text{-cl}(U))^c$. Then V is an $\tau_{1,2}$-open such that $F \subseteq V$. Also, the sets U and V are disjoint and hence (X, τ_1, τ_2) are (1,2)*- g‴-regular[12, 13].

Theorem 3.9

If (X, τ_1, τ_2) is a (1,2)*- g‴-regular space and $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is bijective, pre-(1,2)*-gs-open, (1,2)*- g‴-continuous and (1,2)*-open, then (Y, σ_1, σ_2) is (1,2)*- g‴-regular.

Proof

Let F be any (1,2)*- g‴-closed subset of (Y, σ_1, σ_2) and $y \notin F$. Since the function f is (1,2)*- g‴-irresolute by Theorem 2.8, we have $f^{-1}(F)$ is (1,2)*-g‴-closed in (X, τ_1, τ_2). Since f is bijective, let $f(x) = y$, then $x \notin f^{-1}(F)$. By hypothesis, there exist disjoint $\tau_{1,2}$-open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$. Since f is (1,2)*-open and bijective, we have $y \in f(U)$, $F \subseteq f(V)$ and $f(U) \cap f(V) = \varphi$. This shows that the space (Y, σ_1, σ_2) is also (1,2)*- g‴-regular[14].

Proposition 3.10

If f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-gs-irresolute (1,2)*- g‴-closed and A is an (1,2)*- g‴-closed subset of X then f(A) is (1,2)*- g‴-closed in Y.

Proof

Let U be a (1,2)*-gs-open set in Y such that $f(A) \subseteq U$. Since f is (1,2)*- gs-irresolute, $f^{-1}(U)$ is a (1,2)*-gs-open set containing A. Hence $\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(U)$ as A is (1,2)*- g‴-closed in X. Since f is (1,2)*- g‴-closed, $f(\tau_{1,2}\text{-cl}(A))$ is an (1,2)*- g‴-closed set containedin the
(1,2)*-gs-open set U, which implies that $\sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) \subseteq U$ and $\sigma_{1,2}\text{-cl}(f(A)) \subseteq U$. Therefore f(A) is an (1,2)*- g‴-closed set in Y[15].

Theorem 3.11

If $f : (X, \tau_1, \tau^2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-gs-irresolute (1,2)*- g‴-closed (1,2)*-continuous injection and (Y, σ_1, σ_2) is (1,2)*- g‴-regular, then (X, τ_1, τ_2) is (1,2)*- g‴-regular.

Proof

Let F be any (1,2)*- g‴-closed set of (X, τ_1, τ_2) and $x \notin F$. Since f is (1,2)*-gs-irresolute (1,2)*- g‴-closed, by Proposition 3.3.10, f(F) is (1,2)*- g‴- closed in (Y, σ_1, σ_2) and $f(x) \notin f(F)$. Since (Y, σ_1, σ_2) is (1,2)*- g‴-regular and so there exist disjoint $\sigma_{1,2}$-open sets U and V in (Y, σ_1, σ_2) such that $f(x) \in U$ and $f(F) \subseteq V$. i.e., $x \in f^{-1}(U)$, $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varphi$. Therefore, (X, τ_1, τ_2) is (1,2)*- g‴-regular[16].

Theorem 3.12

If f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is weakly (1,2)*-continuous (1,2)*- g‴- closed injection and (Y, σ_1, σ_2) is (1,2)*-g‴-regular, then (X, τ_1, τ_2) is (1,2)*-regular.

Proof

Let F be any $\tau_{1,2}$-closed set of (X, τ_1, τ_2) and $x \notin F$. Since f is (1,2)*- g‴- closed, f(F) is (1,2)*- g‴-closed in (Y, σ_1, σ_2) and $f(x) \notin f(F)$. Since (Y, σ_1, σ_2) is (1,2)*- g‴-regular
by Theorem 3.5 there exist $\sigma_{1,2}$-open sets U and V such that $f(x) \in U$, $f(F) \subseteq V$ and $\sigma_{1,2}\text{-cl}(U) \cap \sigma_{1,2}\text{-cl}(V) = \varphi$. Since f is weakly (1,2)*-continuous it follows that $x \in f^{-1}(U) \subseteq \tau_{1,2}\text{-int}(f^{-1}(\sigma_{1,2}\text{-cl}(U)))$, $F \subseteq f^{-1}(V) \subseteq \tau_{1,2}\text{-int}(f^{-1}(\sigma_{1,2}\text{-cl}(V)))$ and $\tau_{1,2}\text{-int}(f^{-1}(\sigma_{1,2}\text{-cl}(U))) \cap \tau_{1,2}\text{-int}(f^{-1}(\sigma_{1,2}\text{- cl}(V))) = \varphi$. Therefore, (X, τ_1, τ_2) is (1,2)*-regular[17].

REFERENCES:

1. Munshi BM et al.ActaCienciaIndica, 1986;12: 140-144
2. Tong J et al. On decomposition of continuity in topological spaces, Acta Math. Hungar., 1989; 54:51-55.
3. Ravi O,LellisThivagarMet al. On stronger forms of (1,2)*-quotient mappings in bitopological spaces, Internat. J. Math. Game Theory and Algebra. 2004; 14(6) : 481-492.
4. Reilly IL, Ganster, M et al. Locally closed sets and LC-continuous functions, Internat. J. Math. Sci. 1989;12(3) :417-424.
5. Hatir E, Noiri T, Yuksel S et al. A decomposition of continuity, Acta Math. Hungar., 70, No. 1996;2:145-150.
6. LellisThivagar M, NirmalaMariappan, Jafari S et al. On (1,2)*-αĝ- closed sets, J. Adv. Math. Studies, 2009;2(2):25-34.
7. Ravi O, Thivagar ML, Joseph Israel, M.Kayathri K et al. Decompositions of (1,2)*-rg-continuous maps in bitopological spaces, Antarctica J. Math., 2009;6(1): 13-23.
8. Kelly J C et a, Bitopological spaces, Proc. London Math. Soc., 1963; 13:71-89.
9. Kopperman R, Meyer P, Kong T et al. A topological approach to digital topology, Amer. Math. Monthly.,1991; 98:901-917.
10. Ravi O, Abd El-MonsefME,LellisThivagar M et al. Remarks on bitopological (1,2)*-quotient mappings, J. Egypt Math. Soc.2008; 16(1) :17-25.
11. Ravi O, Thivagar ML et al. Remarks on λ-irresolute functions via (1,2)*-sets, Advances in App. Math. Analysis. 2010; 5(1) :1-15.
12. Ravi O, Ekici E, LellisThivagar M et al.On (1,2)*-sets and decompositions of bitopological (1,2)*-continuous mappings, Kochi J. Math. 2008;3:181-189.

13. Ravi O, KayathriK,Thivagar ML, Joseph Israel M et al.Mildly (1,2)*-normal spaces and some bitopological functions, MathematicaBohemica. 2010; 135(1):1-15.
14. Ravi O, Pious MissierS, SalaiParkunan T et al. On bitopological (1,2)*-generalized homeomorphisms, Int J. Contemp. Math. Sciences.2010;5(11):543-557.
15. Ravi O, Thivagar ML, Nagarajan A et al. (1,2)*- α g-closed sets and (1,2)*-g α -closed sets (submitted).
16. Ravi O, Thivagar ML, Ekici E et al.Decomposition of (1,2)*- continuity and complete (1,2)*-continuity in bitopological spaces, AnaleleUniversitatii Din Oradea. Fasc. Matematica Tom XV. 2008; 2:29-37.
17. Ravi O, Pandi A, Pious MissierS, SalaiParkunan T et al. Remarks on bitopological (1,2)*-rω-Homeomorphisms, Int. J. of Math. Arc. 2011;2(4) :465-475.