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Some Exponential Product Type Estimators Using Auxiliary Attributes.

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ABSTRACT

In this paper some improved exponential product type estimators of finite population mean have been suggested in presence of auxiliary attributes. For construction of estimators we have used a *priori/a posteriori* knowledge of coefficient variationand auxiliary attributes .The efficiencies of these estimators are compared with the exponential estimator using auxiliary attribute suggested by Singh et.al.¹and among themselves with regard to biases and mean square errors both theoretically and numerically

KEYWORDS: Auxiliary attributes, Population proportion, Simple random sampling, Exponential product type estimators, Bias, Mean square error, Efficiency.

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1. INTRODUCTION

In theory of sampling judicious use of auxiliary information to develop efficient estimators is a long practice. Cochran ² developed a ratio estimator, when the study variable y and auxiliary variable x are positively correlated. However when “ y ” and “ x ” are negatively correlated the ratio estimator does not perform better. Robson³ and Murthy ⁴ have suggested a product estimator to estimate population mean \bar{Y} which perform better than mean per unit estimator \bar{y} , when y and x are highly negatively correlated.

Singh et.al.¹ have suggested some exponential ratio type and product type estimators for positive correlation and negative correlation exist between study variable “ y ” and “ a ” (auxiliary attribute).

In this paper when *a priori or a posteriori* information on population coefficient of variation and auxiliary attributes are available we suggest some improved exponential product type estimators to estimate finite population mean \bar{Y} .

Let there be a finite population U consisting of N unit $U = (U_1, U_2, U_3, \dots, U_i, \dots, U_N)$. The i^{th} unit is indexed by a pair of real value (y_i, a_i) where y_i is the study variable and a_i is the auxiliary attribute. It is assumed that y_i and a_i are negatively correlated and the correlation coefficient between them is denoted by ρ .

2. PROPOSED ESTIMATORS

From the finite population U , a sample of size “ n ” is selected using simple random sampling without replacement (SRSWOR). We denote the sample mean of study variable \bar{y} and sample proportion $\left(\frac{a}{n}\right) = p$ respectively.

Searls⁵ suggested an estimator to estimate population mean \bar{Y} using known population coefficient variation of study variable i.e. $C_y = \frac{S_y}{\bar{Y}}, S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2$. The suggested estimator of Searls is given by $\hat{Y}_s = \frac{\bar{y}}{1 + \theta_1 C_y^2}$, (2.1)

Where, $\theta_1 = \frac{1}{n} - \frac{1}{N}$.

Following Bhal and Tuteja⁶, Singh et.al.¹ have proposed an exponential product type estimator of population mean using population proportion (i.e. in presence of auxiliary attributes), is given by

$$t_{EPP1} = \bar{y} \exp \left[\frac{p - P}{p + P} \right] \tag{2.2}$$

Where, P and p are the population proportion and sample proportion respectively.

Now we proposed an improved exponential product type estimator of population mean when we have a priori knowledge of coefficient of variation of study variable i.e. C_y and presence of auxiliary attributes.

$$t_{EPP2} = \frac{\bar{y}}{1 + \theta_1 C_y^2} \exp \left[\frac{p - P}{p + P} \right] \tag{2.3}$$

Further, if the a priori knowledge of C_y is not known, we can still construct an improved estimator by considering the estimate of population coefficient of variation of study variable y from the sample. The estimator is given by

$$t_{EPP3} = \frac{\bar{y}}{1 + \theta_1 \hat{C}_y^2} \exp \left[\frac{p - P}{p + P} \right] \tag{2.4}$$

Where, $\hat{C}_y^2 = \frac{s_y^2}{\bar{y}^2}$ and $s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2$

Following Upadhyaya and Srivastava, a⁷, and Upadhyaya and Srivastava, b⁸ we suggested another estimator

$$t_{EPP4} = \bar{y}(1 + \theta_1 \hat{C}_y^2) \exp \left[\frac{p - P}{p + P} \right] \tag{2.5}$$

3. BIAS AND MSE OF DIFFERENT ESTIMATORS

Assuming the validity of Taylor’s series expansion of $t_{EPP1}, t_{EPP2}, t_{EPP3}$ and t_{EPP4} , considering the expected value to $O\left(\frac{1}{n}\right)$, the bias of the different estimators are given as.

$$B(t_{EPP1}) = E(t_{EPP1}) - \bar{Y} = \theta_1 \bar{Y} \left[\frac{1}{2} C_{11} - \frac{1}{8} C_{20} \right] \tag{3.1}$$

$$B(t_{EPP2}) = E(t_{EPP2}) - \bar{Y} = \theta_1 \bar{Y} \left[\frac{1}{2} C_{11} - \frac{1}{8} C_{20} - C_{02} \right] \tag{3.2}$$

$$B(t_{EPP3}) = E(t_{EPP3}) - \bar{Y} = \theta_1 \bar{Y} \left[\frac{1}{2} C_{11} - \frac{1}{8} C_{20} - C_{02} \right] \tag{3.3}$$

$$B(t_{EPP4}) = E(t_{EPP4}) - \bar{Y} = \theta_1 \bar{Y} \left[\frac{1}{2} C_{11} - \frac{1}{8} C_{20} + C_{02} \right] \tag{3.4}$$

Where, $C_{rs} = \frac{\mu_{rs}(p, y)}{P^r \bar{Y}^s}$

Where, $\mu_{rs}(p, y)$ in the (r, s)th bivariate moments of p and y.

The Mean Square Error (MSE) of different estimators to $O\left(\frac{1}{n^2}\right)$ are given as

$$\begin{aligned} \text{MSE}(t_{EPP1}) &= E(t_{EPP1} - \bar{Y})^2 \\ &= \bar{Y}^2 \left[\theta_1(C_{02} + \frac{1}{4}C_{20} + C_{11}) + \theta_2(+C_{12} + \frac{1}{4}C_{21} - \frac{1}{8}C_{30} - \frac{5}{8}C_{11}C_{20} + \frac{7}{64}C_{20}^2) \right] \end{aligned} \quad (3.5)$$

Where $\theta_2 = \left(\frac{1}{n^2} - \frac{1}{N^2}\right)$

$$\begin{aligned} \text{MSE}(t_{EPP2}) &= E(t_{EPP2} - \bar{Y})^2 \\ &= \bar{Y}^2 \left[\theta_1(C_{02} + \frac{1}{4}C_{20} + C_{11}) + \theta_2(+C_{12} + \frac{1}{4}C_{21} - \frac{1}{8}C_{30} \right. \\ &\quad \left. - \frac{5}{8}C_{11}C_{20} + \frac{7}{64}C_{20}^2 - 3C_{11}C_{02} - \frac{1}{4}C_{20}C_{02} - C_{02}^2) \right] \end{aligned} \quad (3.6)$$

$$\begin{aligned} \text{MSE}(t_{EPP3}) &= E(t_{EPP3} - \bar{Y})^2 \\ &= \bar{Y}^2 \left[\theta_1(C_{02} + \frac{1}{4}C_{20} + C_{11}) + \theta_2(+C_{12} + \frac{1}{4}C_{21} - \frac{1}{8}C_{30} \right. \\ &\quad \left. - \frac{5}{8}C_{11}C_{20} + \frac{7}{64}C_{20}^2 - C_{11}C_{02} - \frac{1}{4}C_{20}C_{02} + C_{02}^2 - 2C_{02}C_{03} - C_{02}C_{12}) \right] \end{aligned} \quad (3.7)$$

$$\begin{aligned} \text{MSE}(t_{EPP4}) &= E(t_{EPP4} - \bar{Y})^2 \\ &= \bar{Y}^2 \left[\theta_1(C_{02} + \frac{1}{4}C_{20} + C_{11}) + \theta_2(+C_{12} + \frac{1}{4}C_{21} - \frac{1}{8}C_{30} \right. \\ &\quad \left. - \frac{5}{8}C_{11}C_{20} + \frac{7}{64}C_{20}^2 + \frac{1}{4}C_{20}C_{02} + 3C_{11}C_{02} + 3C_{02}^2 + 2C_{02}C_{03} + C_{02}C_{12}) \right] \end{aligned} \quad (3.8)$$

4. COMPARISON OF BIASES AND MEAN SQUARED ERRORS

When the sample is large enough the biases of the estimators $t_{EPP1}, t_{EPP2}, t_{EPP3}$ and t_{EPP4} of $O\left(\frac{1}{n}\right)$ are negligible .

From the above equations (3.2) and (3.3) both the estimators to

$O\left(\frac{1}{n}\right)$ the biases are same. i.e.

$$B(t_{EPP2}) = B(t_{EPP3})$$

However, the estimators t_{EPP2}, t_{EPP3} and t_{EPP4} are more biased than t_{EPP1} .

The mean square errors $t_{EPP1}, t_{EPP2}, t_{EPP3}$ and t_{EPP4} to $O\left(\frac{1}{n}\right)$ are same. Thus for the purpose of comparison of efficiencies, the MSE of the estimators are considered up to $O\left(\frac{1}{n^2}\right)$.

The comparison of efficiencies of different estimators are made under two cases.

Case I: Under general condition

Case II: Under the Bivariate Symmetrical Distribution..

I. t_{EPP2} is more efficient than t_{EPP1} if

$$\text{Case I : } C_{11} > -\frac{1}{12}(C_{20} + 4C_{02}) \quad (4.1)$$

$$\text{Case II : } \rho < -\frac{1}{12W}(W^2 + 4) \quad (4.2)$$

$$\text{Where, } W = \left(\frac{C_{20}}{C_{02}}\right)^{\frac{1}{2}}$$

II. t_{EPP3} is more efficient than t_{EPP1} if

$$\text{Case I : } C_{11} < \frac{1}{C_{02}}\left(-\frac{1}{4}C_{20}C_{02} - C_{02}^2 - 2C_{03} - C_{12}\right) \quad (4.3)$$

$$\text{Case II : } \rho > -\frac{1}{4W}(W^2 - 4) \quad (4.4)$$

III. t_{EPP4} is more efficient than t_{EPP1} if

$$\text{Case I : } C_{11} < \frac{1}{3C_{02}}\left(\frac{1}{4}C_{20}C_{02} + 3C_{02}^2 + 2C_{02}C_{03} + C_{02}C_{12}\right) \quad (4.5)$$

$$\text{Case II : } \rho < \frac{1}{12W}(W^2 + 4) \quad (4.6)$$

IV. t_{EPP3} is more efficient than t_{EPP2} if

$$\text{Case I : } C_{11} < \frac{1}{2C_{02}}(2C_{02}^2 + 2C_{03} + C_{12}) \quad (4.7)$$

$$\text{Case II : } \rho < \frac{1}{W} \quad (4.8)$$

V. t_{EPP4} is more efficient than t_{EPP2} if

$$\text{Case I : } C_{11} < \frac{1}{4C_{02}}\left(\frac{1}{2}C_{02}C_{20} + 2C_{02}^2 - 4C_{03} - 2C_{12}\right) \quad (4.9)$$

$$\text{Case II : } \rho < \frac{1}{8W}(W^2 + 16) \quad (4.10)$$

VI. t_{EPP4} is more efficient than t_{EPP3} if

$$\text{Case I : } C_{11} < \frac{1}{6C_{02}} \left(\frac{1}{2} C_{02} C_{20} + 4C_{02}^2 - 2C_{03} - 2C_{12} \right) \quad (4.11)$$

$$\text{Case II : } \rho < \frac{1}{12W} (W^2 + 8) \quad (4.12)$$

5. NUMERICAL ILLUSTRATION

For comparison of biases and mean square errors, we consider four natural populations. One population is considered from Draper and Smith⁹, one population considered from Swain¹⁰, and two populations are considered from Daniel and Cross¹¹. Biases are calculated considering terms up to first order of approximation and mean square errors are calculated considering second order of approximations. For calculations we consider four datasets showing N, n, \bar{Y}, P and $C_{rs}(p, y)$

$$\text{Where, } C_{rs}(p, y) = \frac{\mu_{rs}(p, y)}{P^r \bar{Y}^s}$$

Data Set-1

The data for the empirical analysis are taken from Natural Population dataset was considered by Draper and Smith [1998, P.40], Appendix 1A, stream plant data.

Y=Response

P= Predictors

$$\bar{Y} = 52.6, P = 0.92, N = 25, n = 6, \rho = -0.3416, C_{20} = 0.0869, C_{02} = 0.1034, C_{11} = -0.0324, \\ C_{30} = -0.0793, C_{03} = -0.0036, C_{12} = -0.0032, C_{21} = 0.0295$$

Data Set-2

The data for the empirical analysis are taken from Natural Population dataset considered by Swain A.K.P.C. [2003, P.274],

Y = No. of Milk Cows , 1956

P = No. of Milk Cows in Rainy Season

$$\bar{Y} = 67.3684, P = 0.4210, N = 19, n = 7, \rho = -0.5899, C_{20} = 1.375, C_{02} = 1.2385, C_{11} = -0.7699, \\ C_{30} = 0.5156, C_{03} = 2.2061, C_{12} = -0.6252, C_{21} = 0.2887$$

Data Set-3

The data for the empirical analysis are taken from Natural Population dataset considered by Daniel and Cross [2015, P.758], Tab No. 14.3.2

Y = Time (Months)

P = Vital Status (Censored and Dead)

$$\bar{Y} = 63.0256, P = 0.5641, N = 39, n = 14, \rho = -0.5362, C_{20} = 0.7727, C_{02} = 1.2629, C_{11} = -0.5297, C_{30} = -0.1756, C_{03} = 2.4308, C_{12} = -0.4395, C_{21} = 0.1204$$

Data Set-4

The data for the empirical analysis are taken from Natural Population dataset considered by Daniel and Cross [2015, P.757], Tab No. 14.3.1

Y= Time (Months)

P= Tumor Grade (Low Grade & High Grade)

$$\bar{Y} = 63.025, P = 0.3589, N = 39, n = 20, \rho = -0.4715, C_{20} = 1.7857, C_{02} = 1.2629, C_{11} = -0.7087, C_{30} = 1.4030, C_{03} = 2.4308, C_{12} = -0.5378, C_{21} = -0.5568$$

TABLE. 1PERCENT OF RELATIVE BIAS OF ESTIMATORS $t_{EPP1}, t_{EPP2}, t_{EPP3}$ and $t_{EPP4} O\left(\frac{1}{n}\right)$

Data set No.	t_{EPP1}	t_{EPP2}	t_{EPP3}	t_{EPP4}
1	0.0312	0.1510	0.1517	0.0871
2	0.1844	0.5674	1.3146	0.1679
3	0.0804	0.3602	0.4631	0.1737
4	0.0898	0.2843	0.3190	0.0955

TABLE. 2MSE OF ESTIMATORS, $\bar{y}, t_{EPP1}, t_{EPP2}, t_{EPP3}$ and $t_{EPP4} O\left(\frac{1}{n^2}\right)$

Data set No.	$t_0 = \bar{y}$	t_{EPP1}	t_{EPP2}	t_{EPP3}	t_{EPP4}
1	36.2490	33.2524	33.1240	32.8448	34.0557
2	507.1931	336.6183	369.9062	68.9115	608.7238
3	229.7066	167.9355	169.3361	102.4355	224.1438
4	122.2039	97.5004	98.7411	78.4485	121.3179

6. CONCLUSIONS

1. It is observed that the suggested estimators t_{EPP2}, t_{EPP3} and t_{EPP4} are more biased than the estimator t_{EPP1} . However the biases are negligible if sample size is large.
2. Comparing the biases of $t_{EPP1}, t_{EPP2}, t_{EPP3}$ and t_{EPP4} we observed

$B(t_{EPP1}) < B(t_{EPP2}) < B(t_{EPP3}) < B(t_{EPP4})$ for all populations.

3. Considering the value of MSE of mean per unit estimator (\bar{y}), t_{EPP1} , t_{EPP2} , t_{EPP3} and t_{EPP4} we observed that the MSE of t_{EPP3} is most efficient for the all populations.

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