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# Odd Prime Labeling of Various Snake Graphs

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## ABSTRACT

For a graph $G$ $(V, E)$, a function $f$ is called an odd prime labeling, given that $f$ is a bijection from $V$ to $\{1, 3, 5, \ldots, 2|V|-1\}$, satisfying $gcd\ (f(u), f(v)) = 1$, for each $uv \in E$. A graph admitting this labeling is called an odd prime graph. Various snake graphs like $n$-polygonal snake, double $n$-polygonal snake, alternate $n$-polygonal snake, double alternate triangular snake, irregular triangular snake, irregular quadrilateral snake are proved to be odd prime graphs.

**KEYWORDS:** Odd prime graph, $n$-polygonal snake, double $n$-polygonal snake, alternate $n$-polygonal snake, double alternate triangular snake, irregular triangular, irregular quadrilateral snake

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## INTRODUCTION

Finite, simple, undirected, non-trivial and connected graphs have been considered in this paper. $V$ or $V(G)$ and $E$ or $E(G)$ of a graph $G(V, E)$ are the vertex and edge set respectively, where $|V|$ and $|E|$ are the number of elements in $V$ and $E$ respectively. Gross and Yellen[1] is referred for graph theoretical notations and terminologies. Burton[2] is referred for number theoretical notations. Assignment of integers to vertices and/or edges of a graph subject to certain conditions is known as graph labeling.[3] Gallian[3] is referred for the latest survey of graph labeling. Different labeling has been proved on various snake graphs including prime labeling. Odd prime labeling is a variation of prime labeling. This paper attempts to prove various snake graphs to have odd prime labeling. We start with few notations and definitions required in the paper.

**Notation: 1** For each natural number $n$, $[n]$ and $O_n$ respectively are the set of first $n$ natural numbers and the set of first $n$ odd natural numbers. i.e $[n]=\{1, 2, 3, \dots, n\}$ and $O_n=\{1, 3, 5, \dots, 2n-1\}$.[4, 5]

**Definition: 2** Let $P_k$ $(k \geq 2)$ be a path with consecutive vertices $v_1, v_2, \dots, v_k$. An ***n*-polygonal snake** $(n \geq 3)$ is obtained from the path $P_k$ whose vertex set $V=V(P_k)\cup\{u_{i,j} \mid i\in[k-1], j\in[n-2]\}$ and the edge set is $E=E(P_k)\cup\{v_i u_{i,1}, u_{i,j-1}u_{i,j}, u_{i,n-2}v_{i+1} \mid i\in[k-1], j\in[n-2]-\{1\}\}$. We denote it as $S_k(C_n)$. i.e. a snake of path $P_k$ where each edge of $P_k$ is replaced by a cycle $C_n$.[6]

**Definition: 3** An **alternate triangular snake** is obtained from the path $P_n$ by replacing each alternate edge of the path by a triangle $C_3$. It is denoted as $A(T_n)$.[7]

Similar to alternate triangular snake, we define the following:

**Definition: 4** An **alternate *n*-polygonal snake** $(n \geq 3)$ is obtained from the path $P_k$ $(k \geq 2)$ by replacing each alternate edge of the path by $C_n$. We denote it as $AS_k(C_n)$.

**Definition: 5** A **double triangular snake** consists of two triangular snakes that have a common path $P_n$. It is denoted by $D(T_n)$.[8]

Similar to double triangular snake, we define the following:

**Definition: 6** A **double *n*-polygonal snake** $(n \geq 3)$ consists of two $n$-polygonal snakes that have a common path $P_k$ $(k \geq 2)$. We denote it as $DS_k(C_n)$.

**Definition: 7** A **double alternate triangular snake** consists of two similar alternate triangular snakes that have a common path $P_n$. It is denoted as $DA(T_n)$.[8]

**Definition: 8** Let $P_n$ $(n \geq 3)$ be a path with consecutive vertices $v_1, v_2, \ldots, v_n$. An **irregular triangular snake** $(n \geq 3)$ is obtained from the path $P_n$ whose vertex set $V = V(P_n) \cup \{u_i \mid i \in [n-2]\}$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i v_{i+2} \mid i \in [n-2]\}$. It is denoted as $IT_n$.[9]

**Definition: 9** Let $P_n$ $(n \geq 3)$ be a path with consecutive vertices $v_1, v_2, \ldots, v_n$. An **irregular quadrilateral snake** $(n \geq 3)$ is obtained from the path $P_n$ whose vertex set $V = V(P_n) \cup \{u_i, w_i \mid i \in [n-2]\}$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i w_i, w_i v_{i+2} \mid i \in [n-2]\}$. It is denoted as $IQ_n$.[9]

Prime labeling was originated by Entringer and first introduced by Tout et al[10] in their paper. This labeling is defined as follows:

**Definition: 10** For a graph $G(V,E)$ with $n$ vertices, a bijection $f : V \rightarrow [n]$ is called **prime labeling** if $\gcd(f(u), f(v)) = 1$, for each $uv \in E$. The graph admitting prime labeling is called a **prime graph**.[10]

Carlson[11] proved that generalized books are prime graphs. Seoud and Youssef[12] proved that $C_m$-snakes are prime graphs. Ganesan et al[13] proved that cycle-cactus $C_k^{(n)}$ and triangular book $B_{3,n}$ are prime graphs. Vaidya and Prajapati[14] proved that tadpole, also called kite or dragon is a prime graph.

A variation of prime labeling, called odd prime labeling was introduced by Prajapati and Shah.[5]

**Definition: 11** For a graph $G(V,E)$ with $n$ vertices, a bijective function $f : V \rightarrow O_n$ is called **odd prime labeling** if for each $uv \in E$, $\gcd(f(u), f(v)) = 1$. The graph admitting this labeling is called an **odd prime graph**.[5]

Prajapati and Shah[5] proved that graphs like path, ladder graph, complete graph $K_n$ iff $n \leq 4$, complete bipartite graphs under certain conditions, wheel graph, helm graph, fan graph, friendship graph, Petersen graph $P(n,2)$ and many more are odd prime graphs. Graphs obtained by duplicating each vertex by an edge and each edge by a vertex in a path, star graph, cycle and wheel graph are all odd prime graphs is also proved by them.[15]

## MAIN RESULTS

**Theorem: 1** $S_k(C_n)$, $n \geq 3$, $k \geq 2$ is an odd prime graph.

**Proof:** Consider an *n*-polygonal snake $S_k(C_n)$ on a path with *k* consecutive vertices $v_1, v_2, \ldots, v_k$. Let the vertex set of $S_k(C_n)$ be $V = V(P_k) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence, $|V| = nk - (n+k) + 2$ and its edge set is $E = E(P_k) \cup \{v_i u_{i,1}, u_{i,j-1}u_{i,j}, u_{i,n-2}v_{i+1} \mid i \in [k-1], j \in [n-2]-\{1\}\}$. Let $f : V \to O_{|V|}$ be defined as

$$f(x) = \begin{cases} (2n-2)i - 2n + 3, & \text{if } x = v_i, i \in [k]; \\ (2n-2)i - 2n + 2j + 3, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$$

For each $e \in E$, if

1. $e = v_i v_{i+1}$, $i \in [k-1]$, $\gcd(f(v_i), f(v_{i+1})) = \gcd((2n-2)i - 2n + 3, (2n-2)i + 1) = 1$;

2. $e = v_i u_{i,1}$, $i \in [k-1]$, $\gcd(f(v_i), f(u_{i,1})) = \gcd((2n-2)i - 2n + 3, (2n-2)i - 2n + 5) = 1$;

3. $e = u_{i,n-2}v_{i+1}$, $i \in [k-1]$, $\gcd(f(u_{i,n-2}), f(v_{i+1})) = \gcd((2n-2)i - 1, (2n-2)i + 1) = 1$;

4. $e = u_{i,j-1}u_{i,j}$, $i \in [k-1]$, $j \in [n-2]-\{1\}$, $\gcd(f(u_{i,j-1}), f(u_{i,j}))$

$$= \gcd((2n-2)i - 2n + 2j + 1, (2n-2)i - 2n + 2j + 3) = 1.$$

This shows that *f* admits odd prime labeling on $S_k(C_n)$ and hence it is an odd prime graph.

**Theorem: 2** $DS_k(C_n)$, $n \geq 3$, $k \geq 2$ is an odd prime graph.

**Proof:** Consider a double *n*-polygonal snake on path of *k* consecutive vertices $v_1, v_2, \ldots, v_k$. Let the vertex set of $DS_k(C_n)$ be $V = V(P_k) \cup \{u_{i,j}, w_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence, $|V| = 2nk - 2n - 3k + 4$ and its edge set is $E = E(P_k) \cup \{v_i u_{i,1}, u_{i,j-1}u_{i,j}, u_{i,n-2}v_{i+1}, v_i w_{i,1}, w_{i,j-1}w_{i,j}, w_{i,n-2}v_{i+1} \mid i \in [k-1], j \in [n-2]-\{1\}\}$.

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} (4n-6)i - 4n + 7, & \text{if } x = v_i, i \in [k]; \\ (4n-6)i - 4(n-j) + 5, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]; \\ (4n-6)i - 4(n-j) + 7, & \text{if } x = w_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

For each $e \in E$, if

1. $e = v_i v_{i+1}$, $i \in [k-1]$, $\gcd\left(f(v_i), f(v_{i+1})\right) = \gcd\left((4n-6)i-4n+7, (4n-6)i+1\right) = 1$;

2. $e = v_i u_{i,1}$, $i \in [k-1]$, $\gcd\left(f(v_i), f(u_{i,1})\right) = \gcd\left((4n-6)i-4n+7, (4n-6)i-4n+9\right) = 1$;

3. $e = u_{i,n-2} v_{i+1}$, $i \in [k-1]$, $\gcd\left(f(u_{i,n-2}), f(v_{i+1})\right) = \gcd\left((4n-6)i-3, (4n-6)i+1\right) = 1$;

4. $e = u_{i,j-1} u_{i,j}$, $i \in [k-1]$, $j \in [n-2]-\{1\}$, $\gcd\left(f(u_{i,j-1}), f(u_{i,j})\right)$

   $= \gcd\left((4n-6)i-4(n-j)+1, (4n-6)i-4(n-j)+5\right) = 1$;

5. $e = v_i w_{i,1}$, $i \in [k-1]$, $\gcd\left(f(v_i), f(w_{i,1})\right) = \gcd\left((4n-6)i-4n+7, (4n-6)i-4n+11\right) = 1$;

6. $e = w_{i,n-2} v_{i+1}$, $i \in [k-1]$, $\gcd\left(f(w_{i,n-2}), f(v_{i+1})\right) = \gcd\left((4n-6)i-1, (4n-6)i+1\right) = 1$;

7. $e = w_{i,j-1} w_{i,j}$, $i \in [k-1]$, $j \in [n-2]-\{1\}$, $\gcd\left(f(w_{i,j-1}), f(w_{i,j})\right)$

   $= \gcd\left((4n-6)i-4(n-j)+3, (4n-6)i-4(n-j)+7\right) = 1$.

This shows that *f* admits odd prime labeling on $DS_k(C_n)$ and hence it is an odd prime graph.

**Theorem: 3** $AS_m(C_n)$, $m, n \geq 3$ is an odd prime graph.

**Proof:** Consider an alternate *n*-polygonal snake $AS_m(C_n)$ on a path with *m* consecutive vertices $v_1, v_2, \ldots, v_m$.

1. Let *m* be odd. In this case, $AS_m(C_n)$ is obtained from a path of $m = 2k-1, (k \geq 2)$ consecutive vertices $v_1, v_2, \ldots, v_{2k-1}$. Consider the vertex set $V = V(P_{2k-1}) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence $|V| = n(k-1)+1$ and the edge set $E = E(P_{2k-1}) \cup \{v_{2i-1}u_{i,1}, u_{i,n-2}v_{2i}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2]-\{1\}\}$.

   Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2ni-1, & \text{if } x = v_{2i}, i \in [k-1]; \\ 2n(i-1)+2j-1, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

2. Let *m* be even. In this case, $AS_m(C_n)$ is obtained from a path of $m = 2k, (k \geq 2)$ consecutive vertices $v_1, v_2, \ldots, v_{2k}$. Here two non-isomorphic graphs are obtained:

   (a) When the polygon starts with the first edge, the vertex set $V = V(P_{2k}) \cup \{u_{i,j} \mid i \in [k], j \in [n-2]\}$ with $|V| = nk$ and

the edge set $E = E(P_{2k}) \cup \{v_{2i-1}u_{i,1}, u_{i,n-2}v_{2i}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2]-\{1\}\}$

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2ni-1, & \text{if } x = v_{2i}, i \in [k]; \\ 2n(i-1)+2j-1, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

(b) When the polygon starts with the second edge,

the vertex set $V = V(P_{2k}) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$ with $|V| = n(k-1)+2$ and

the edge set $E = E(P_{2k}) \cup \{v_{2i}u_{i,1}, u_{i,n-2}v_{2i+1}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2]-\{1\}\}$

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2n(i-1)+3, & \text{if } x = v_{2i}, i \in [k-1]; \\ 2n(i-1)+2j+3, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

In each of the cases, it is easy to check that if $n-1=2^t$, then the functions defined in the respective cases will admit odd prime labeling on $AS_m(C_n)$.

If $n-1 \neq 2^t$, assume that *d* is an odd divisor of *n* – 1.

For each $v_l \in V(P_{2k-1})$, whenever $f(v_l) = qd$ for some $q \in \mathbb{N}$, $\gcd(f(v_l), f(v_{l+1})) \neq 1$. In this case, either $\gcd(f(v_{l-1}), f(v_{l+1})) = 1$ or $\gcd\left(f\left(u_{\left\lceil \frac{l}{2} \right\rceil, 1}\right), f(v_{l+1})\right) = 1$. Interchange $f(v_l)$ by $f(v_{l-1})$ or $f\left(u_{\left\lceil \frac{l}{2} \right\rceil, 1}\right)$ accordingly and the function thus obtained will admit odd prime labeling on $AS_m(C_n)$ and it is an odd prime graph.

**Theorem: 4** $DA(T_n)$, $n \geq 3$ is an odd prime graph.

**Proof:** Consider a double alternate triangular snake graph $DA(T_n)$ on a path with *m* consecutive vertices $v_1, v_2, \ldots, v_m$.

1. Let $n$ be odd. In this case, $DA(T_n)$ is obtained from the path of $n=2k-1,(k\geq 2)$ consecutive vertices $v_1,v_2,\ldots,v_{2k-1}$. Consider the vertex set $V=V(P_{2k-1})\cup\{u_i,w_i \mid i\in[k-1]\}$. Hence $|V|=4k-1$ and the edge set $E=E(P_{2k-1})\cup\{v_{2i}u_i,u_iv_{2i+1},v_{2i}w_i,w_iv_{2i+1} \mid i\in[k-1]\}$.

Let $f:V\rightarrow O_{|V|}$ be defined as
$$f(x)=\begin{cases} 24i-23, & \text{if } x=v_{6i-5}, i\in\left[\left\lfloor\frac{k+2}{3}\right\rfloor\right];\\ 24i-19, & \text{if } x=v_{6i-4}, i\in\left[\left\lfloor\frac{2k+3}{6}\right\rfloor\right];\\ 24i-17, & \text{if } x=v_{6i-3}, i\in\left[\left\lfloor\frac{k+1}{3}\right\rfloor\right];\\ 24i-13, & \text{if } x=v_{6i-2}, i\in\left[\left\lfloor\frac{2k+1}{6}\right\rfloor\right];\\ 24i-7, & \text{if } x=v_{6i-1}, i\in\left[\left\lfloor\frac{k}{3}\right\rfloor\right];\\ 24i-5, & \text{if } x=v_{6i}, i\in\left[\left\lfloor\frac{2k-1}{6}\right\rfloor\right];\\ f(v_{2i+1})-4, & \text{if } x=u_i, i\in[k-1];\\ f(v_{2i})+4, & \text{if } x=w_i, i\in[k-1]. \end{cases}$$

2. Let $n$ be even. In this case, $DA(T_n)$ is obtained from a path of $n=2k,(k\geq 2)$ consecutive vertices $v_1,v_2,\ldots,v_{2k}$. Here two non-isomorphic graphs are obtained:

   (a) When the triangle starts from the first edge, the vertex set $V=V(P_{2k})\cup\{u_i,w_i \mid i\in[k]\}$ with $|V|=4k$ and the edge set $E=E(P_{2k})\cup\{v_{2i-1}u_i,u_iv_{2i},v_{2i-1}w_i,w_iv_{2i} \mid i\in[k]\}$

   Let $f:V\rightarrow O_{|V|}$ be defined as
$$f(x)=\begin{cases} 16i-13, & \text{if } x=v_{4i-3}, i\in\left[\left\lfloor\frac{2k+3}{4}\right\rfloor\right];\\ 16i-11, & \text{if } x=v_{4i-2}, i\in\left[\left\lfloor\frac{k+1}{2}\right\rfloor\right];\\ 16i-3, & \text{if } x=v_{4i-1}, i\in\left[\left\lfloor\frac{2k+1}{4}\right\rfloor\right];\\ 16i-5, & \text{if } x=v_{4i}, i\in\left[\left\lfloor\frac{k}{2}\right\rfloor\right];\\ 8i-7, & \text{if } x=u_i, i\in[k];\\ 8i-1, & \text{if } x=w_i, i\in[k]. \end{cases}$$

   (b) When the triangle starts from the second edge, the vertex set $V=V(P_{2k})\cup\{u_i,w_i \mid i\in[k-1]\}$ with $|V|=4k-2$ and the edge set $E=E(P_{2k})\cup\{v_{2i}u_i,u_iv_{2i+1},v_{2i}w_i,w_iv_{2i+1} \mid i\in[k-1]\}$.

Let $f:V\to O_{|V|}$ be defined as $f(x)=\begin{cases} 24i-23, & \text{if } x=v_{6i-5}, i\in\left[\left\lfloor\frac{k+2}{3}\right\rfloor\right]; \\ 24i-19, & \text{if } x=v_{6i-4}, i\in\left[\left\lfloor\frac{2k+3}{6}\right\rfloor\right]; \\ 24i-17, & \text{if } x=v_{6i-3}, i\in\left[\left\lfloor\frac{k+1}{3}\right\rfloor\right]; \\ 24i-13, & \text{if } x=v_{6i-2}, i\in\left[\left\lfloor\frac{2k+1}{6}\right\rfloor\right]; \\ 24i-7, & \text{if } x=v_{6i-1}, i\in\left[\left\lfloor\frac{k}{3}\right\rfloor\right]; \\ 24i-5, & \text{if } x=v_{6i}, i\in\left[\left\lfloor\frac{2k-1}{6}\right\rfloor\right]; \\ 8k-5, & \text{if } x=v_{2k}; \\ f(v_{2i+1})-4, & \text{if } x=u_i, i\in[k-1]; \\ f(v_{2i})+4, & \text{if } x=w_i, i\in[k-1]. \end{cases}$

In each of the cases, it is easy to check that the function defined in the respective cases admit odd prime labeling and so $DA(T_n)$ is an odd prime graph.

**Theorem: 5** $IT_n$, $n\geq 3$ is an odd prime graph.

**Proof:** Consider an irregular triangular snake graph $IT_n$ on a path with $n$ consecutive vertices $v_1,v_2,\ldots,v_n$. Let the vertex set of $IT_n$ be $V=V(P_n)\cup\{u_i \mid i\in[n-2]\}$. Hence $|V|=2n-2$ and the edge set is $E=E(P_n)\cup\{v_iu_i,u_iv_{i+2} \mid i\in[n-2]\}$.

Let $f:V\to O_{|V|}$ be defined as $f(x)=\begin{cases} 12i-11, & \text{if } x=v_{3i-2}, i\in\left[\left\lfloor\frac{n+1}{3}\right\rfloor\right]; \\ 12i-7, & \text{if } x=v_{3i-1}, i\in\left[\left\lfloor\frac{n}{3}\right\rfloor\right]; \\ 12i-5, & \text{if } x=v_{3i}, i\in\left[\left\lfloor\frac{n-1}{3}\right\rfloor\right]; \\ 4n-3, & \text{if } x=v_n; \\ 12i-9, & \text{if } x=u_{3i-2}, i\in\left[\left\lfloor\frac{n}{3}\right\rfloor\right]; \\ 12i-3, & \text{if } x=u_{3i-1}, i\in\left[\left\lfloor\frac{n-1}{3}\right\rfloor\right]; \\ 12i-1, & \text{if } x=u_{3i}, i\in\left[\left\lfloor\frac{n-2}{3}\right\rfloor\right]. \end{cases}$

It is easy to check that the function defined here admits odd prime labeling and hence $IT_n$ is an odd prime graph.

**Theorem: 6** $IQ_n$, $n \geq 3$ is an odd prime graph.

**Proof:** Consider an irregular quadrilateral snake graph $IQ_n$ on a path with *n* consecutive vertices $v_1, v_2, \ldots, v_n$. Let the vertex set of $IQ_n$ be $V = V(P_n) \cup \{u_i, w_i \mid i \in [n-2]\}$. Hence $|V| = 3n-4$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i w_i, w_i v_{i+2} \mid i \in [n-2]\}$.

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 6i-5, & \text{if } x = v_i, i \in [n-1]; \\ 6n-9, & \text{if } x = v_n; \\ 6i-3, & \text{if } x = u_i, i \in [n-2]; \\ 6i-1, & \text{if } x = w_i, i \in [n-2]. \end{cases}$

It is easy to check that the function defined here admits odd prime labeling and hence $IQ_n$ is an odd prime graph.

## CONCLUSION

Various snake graphs like *n*-polygonal snake, double *n*-polygonal snake, alternate *n*-polygonal snake, double alternate triangular snake, irregular triangular snake and irregular quadrilateral snake have been shown to be odd prime graph in this paper.

## OPEN PROBLEMS

Investigating double alternate *n*-polygonal snake and irregular *n*-polygonal snakes for odd prime labeling can be considered as open problems for research work in future.

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