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Group Decision Making under Fuzzy Environment by using TOPSIS Method

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ABSTRACT

The objective of this paper is to explain the Group Decision Making under Fuzzy Environment by using TOPSIS method. Owing to the reason that decision data available are vague in nature the crisp value is insufficient to build up a model in real life situations in this article each alternative is rated and each criterion is evaluated by explaining in Linguistic terms which can very well the explain by triangular fuzzy numbers. Subsequently a vertex method is employed to arrive at the distance between two triangular fuzzy numbers. The TOPSIS Method used in this article is for calculating simultaneously the distances between both the fuzzy positive-ideal solution (FPIS) and fuzzy negative – ideal solution (FNIS). An example is used at the end of this article to enlighten the procedure of the proposed method.

KEYWORDS: TOPSIS, Triangular fuzzy numbers, FPIS, FNIS, Linguistic terms, MCDM.

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1. INTRODUCTION

Decision making is a continuous and complex process of selecting the best option from among the possible alternatives. While identifying the best option, multivarious criteria are to be considered and evaluated. The decision maker intends to find the solution for the multiple criteria decision making problem (MCDM). Problem can be clearly and precisely represented by matrix format as D =

$$\begin{array}{c} \\ X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots\ldots. & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \ldots\ldots.. & x_{1n} \\ x_{21} & x_{22} & \ldots\ldots\ldots & x_{2n} \\ & & & \\ & & & \\ x_{m1} & x_{m2} & \ldots\ldots. & x_{mn} \end{bmatrix} \end{array}$$

and

$$W = \left[w_1 \quad w_2 \ldots\ldots.. w_n \right]$$

where $X_1, X_2, \ldots\ldots.., X_m$ are possible alternative decisions that can be arrived at by the decision makers to select $C_1,\ C_2, \ldots\ldots.., C_n$ which are the criteria with the help of which alternative performance of the decision is measured x_{ij} denotes the rating of the alternative X_i with respect to criterion C_j and w_j denotes the weight of the criterion C_j.

In Multiple Criteria decision making, the rating and the weights of the criteria are represented by $[5, 10, 13]$. Hwang and Yoon conducted the survey of these methods. Technique for order performance by similarity to ideal solution (TOPSIS), which is a popular classical MCDM method, was originally propounded by Hwang and Yoon for arriving at a solution to a MCDM problem. It has been founded on the concept that the selected alternative should be at the shortest distance from the positive ideal solution (PIS) and the longest distance from the negative ideal solution (NIS). In the technique for order performance by similarity to ideal solution (TOPSIS) the performance rating of the decisions and the weights of the criteria are presented as crisp values.

In many situations, crisp data are insufficient to develop a model in real life set up. Human evaluations and judgments of the performances of the decisions are very often indefinite and incapable of estimating the performance with a definite numerical value. A more useful and realistic approach is to resort to linguistic assessments instead of numerical values which mean the assessments are the rating and weights of criteria and the problem by using linguistic variables $[1, 3, 4, 6, 9, 15]$. In this article the author extended the concepts of TOPSIS for evolving a new methodology to solve multi- person multi-criteria decision making problems under fuzzy environment. By taking in to consideration of the fuzziness in the data available for decision making and the process of group decision making, Linguistic variables are utilized to evaluate the weights of all criteria and the rating of each alternative decision with regard to each criterion. We can very well transform the decision matrix in to fuzzy decision matrix by pooling the fuzzy ratings made by the decision maker. With respect to the concept of TOPSIS we formulate the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). As a next step the vertex method is used in this paper to find out the distance between two triangular fuzzy rating by utilizing the vertex method we can find out the distance of each alternative from FPIS and FNIS. A closeness coefficient for each of the alternatives is calculated to find out the ranking order of all possible alternatives. The higher value of closeness coefficient denotes that an alternative is close to FPIS and fairly at a distance from FNIS simultaneously.

For the purpose of evolving Linguistic TOPSIS method this article is arranged as shown under. Following this section, the basic definitions and notations of the fuzzy number and Linguistic variable are being introduced. In Section 3 the TOPSIS method group decision making and choice process are elucidated. Subsequent to this, the proposed method is elaborated in appropriate example. In the final section some valid conclusions are arrived at.

2. PRELIMINARIES

Definition: 2.1 A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

Definition: 2.2 A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, Where $\lambda \in [0,1]$.

Definition: 2.3 A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that $\exists x_i \in X$, $\mu_{\tilde{A}}(x_i) = 1$.

Definition: 2.4 A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. A fuzzy number $\tilde{n}$ of the universe of discourse X which is both convex and normal.

Definition: 2.5 The $\alpha -$cut of fuzzy number $\tilde{n}$ is defined $\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i), x_i \in X\}$, Where $\alpha \in [0,1]$.

Definition 2.6 A triangular fuzzy number $\tilde{n}$ can be defined by a triplet. The membership function $\mu_{\tilde{n}}(x)$ is defined as follows

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1 \\ \dfrac{x - n_1}{n_2 - n_1}, & n_1 \le x \le n_2 \\ \dfrac{x - n_3}{n_2 - n_3}, & n_2 \le x \le n_3 \\ 0, & x > n_3 \end{cases}$$

Definition: 2.7 If $\tilde{n}$ is a fuzzy number and $n_l^{\alpha} > 0$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a positive fuzzy number $[2,9]$.

Given any two positive fuzzy numbers $\tilde{m}, \tilde{n}$ and a positive real number r, the $\alpha -$ cut of two fuzzy numbers are $\tilde{m}^{\alpha} = [m_l^{\alpha}, m_u^{\alpha}]$ and $\tilde{n}^{\alpha} = [n_l^{\alpha}, n_u^{\alpha}]$ $(\alpha \in [0,1])$, respectively. According to the interval confidence so main operations of positive fuzzy numbers $\tilde{m}$ and $\tilde{n}$ can be expressed as follows:

$$(\tilde{m}(+)\tilde{n})^{\alpha} = [m_l^{\alpha} + n_l^{\alpha}, m_u^{\alpha} + n_u^{\alpha}]$$

$$(\tilde{m}(-)\tilde{n})^{\alpha} = [m_l^{\alpha} - n_l^{\alpha}, m_u^{\alpha} - n_u^{\alpha}]$$

$$(\tilde{m}(.)\tilde{n})^{\alpha} = [m_l^{\alpha}.n_l^{\alpha}, m_u^{\alpha}.n_u^{\alpha}]$$

$$(\tilde{m}(:)\tilde{n})^{\alpha} = \left[\frac{m_l^{\alpha}}{n_u^{\alpha}}, \frac{m_u^{\alpha}}{n_l^{\alpha}}\right]$$

$$\left(\tilde{m}^{\alpha}\right)^{-1} = \left[\frac{1}{m_u^{\alpha}}, \frac{1}{m_l^{\alpha}}\right]$$

$$(\tilde{m}(.)r)^{\alpha} = \left[m_l^{\alpha}.r, m_u^{\alpha}.r\right]$$

$$(\tilde{m}(:)r)^{\alpha} = \left[\frac{m_l^{\alpha}}{r}, \frac{m_u^{\alpha}}{r}\right].$$

Definition: 2.8 Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. If $\tilde{A} = \tilde{B}$, then $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition 2.9 If $\tilde{n}$ is a triangular fuzzy number and $n_i^{\alpha} > 0, n_u^{\alpha} \le 0$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a normalized positive triangular fuzzy number.

Definition: 2.10 If $\tilde{D}$ is called a fuzzy matrix, if at least an entry in $\tilde{D}$ is a fuzzy number.

Definition: 2.11 A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions. For example,“ weight” is a linguistic variable, its values are very low, low , medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers.

Definition: 2.12 Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the distance between $\tilde{A}$ and $\tilde{B}$ is defined as

$$d\left(\tilde{A}, \tilde{B}\right) = \frac{1}{2}\left\{\max\left(\left|a_1 - b_1\right|, \left|a_2 - b_2\right|\right) + \left|a_3 - b_3\right|\right\}.$$

Definition: 2.13 Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. The fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d\left(\tilde{A}, \tilde{B}\right)$ approaches 0. Many distance measurement functions are proposed. The vertex method is an effective and simple method to calculate the distance between the two triangular fuzzy numbers. Some important properties of the vertex method are described as follows:

Property 1:

If both $\tilde{A}$ and $\tilde{B}$ are real numbers. the distance measurement $d\left(\tilde{A}, \tilde{B}\right)$ is identical to the Euclidean distance .

Proof:

Suppose that both $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two real numbers, then let $a_1 = a_2 = a_3 = a$ and $b_1 = b_2 = b_3 = b$. The distance measurement $d\left(\tilde{A}, \tilde{B}\right)$ can be calculated as

$$d\left(\tilde{A}, \tilde{B}\right) = \frac{1}{2}\left\{\max\left(|a_1 - b_1|, |a_2 - b_2|\right) + |a_3 - b_3|\right\}$$
$$= \frac{1}{2}\left\{\max\left(|a - b|, |a - b|\right) + |a - b|\right\}$$
$$= \frac{1}{2}\left\{|a - b| + |a - b|\right\} = |a - b|$$

Property: 2

Two triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical if and only if, $d\left(\tilde{A}, \tilde{B}\right) = 0$.

Proof:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers.

(I) If $\tilde{A}$ and $\tilde{B}$ are identical, then $a_1 = b_1, a_2 = b_2, \textit{ and } a_3 = b_3$. The distance between $\tilde{A}$ and $\tilde{B}$ is

$$d\left(\tilde{A}, \tilde{B}\right) = \frac{1}{2}\left\{\max\left(|a_1 - b_1|, |a_2 - b_2|\right) + |a_3 - b_3|\right\}$$
$$= \frac{1}{2}\left\{\max(0,0) + 0\right\}$$
$$= 0$$

(II) If $d\left(\tilde{A}, \tilde{B}\right) = 0$, then

$$d\left(\tilde{A}, \tilde{B}\right) = \frac{1}{2}\left\{\max\left(|a_1 - b_1|, |a_2 - b_2|\right) + |a_3 - b_3|\right\} = 0$$

$$\text{that is , } \left\{\max\left(\left|a_1-b_1\right|,\left|a_2-b_2\right|\right)+\left|a_3-b_3\right|\right\}=0$$

$$\max\left(\left|a_1-b_1\right|,\left|a_2-b_2\right|\right)=0 \ and \ \left|a_3-b_3\right|=0$$

$$\left|a_1-b_1\right|=0, \left|a_2-b_2\right|=0 \ and \ \left|a_3-b_3\right|=0$$

Therefore $\tilde{A}$ and $\tilde{B}$ are identical.

Property: 3

Let $\tilde{A}$ $\tilde{B}$ and $\tilde{C}$ be three triangular fuzzy numbers. The fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$ if and only if $d\left(\tilde{A},\tilde{B}\right)<d\left(\tilde{A},\tilde{C}\right)$.

This property is trivial. For example three fuzzy numbers $\tilde{A}=(1,3,5)$, $\tilde{B}=(2,5,8)$ and $\tilde{C}=(4,6,9)$. We can see that the fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ then the other fuzzy number $\tilde{C}$. According to the vertex method, the distance measurement is calculated as

$$d\left(\tilde{A},\tilde{B}\right)=\frac{1}{2}\left\{\max\left(\left|1-2\right|,\left|5-8\right|\right)+\left|3-5\right|\right\}=\frac{1}{2}\left\{\max(1,3)+2\right\}=\frac{1}{2}\left\{5\right\}=\frac{5}{2}$$

$$d\left(\tilde{A},\tilde{C}\right)=\frac{1}{2}\left\{\max\left(\left|1-4\right|,\left|5-9\right|\right)+\left|3-6\right|\right\}=\frac{1}{2}\left\{\max(3,4)+3\right\}=\frac{1}{2}\left\{7\right\}=\frac{7}{2}$$

According to the distance measurement and Definition 2.13, we conclude that the fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$.

Property 4

Let $\tilde{O}=(0,0,0)$ be original. If $d\left(\tilde{A},\tilde{O}\right)<d\left(\tilde{A},\tilde{O}\right)$, then fuzzy number $\tilde{A}$ is closer to original than the other fuzzy number $\tilde{B}$.

According to property 3, for any three fuzzy numbers $\tilde{A}$ $\tilde{B}$ and $\tilde{C}$ if, $d\left(\tilde{A},\tilde{B}\right)<d\left(\tilde{A},\tilde{C}\right)$ then $\tilde{B}$ is closer to $\tilde{A}$ and $\tilde{C}$.

Therefore if $d\left(\tilde{A},\tilde{O}\right)<d\left(\tilde{A},\tilde{O}\right)$, then $\tilde{A}$ is closer to origin than $\tilde{B}$.

3. RANKING ORDER USING TOPSIS METHOD

An attempt is made to extend the TOPSIS to the fuzzy Environment. This method is appropriate for arriving at the solution to the group decision making problems that are usually faced in the fuzzy environment.

In this article the significant weight of various criterion and their rating qualitative criteria are explained in Linguistic forms of variables. These Linguistic forms of variables can be represented by positive triangular numbers. The significance of weight of each criterion can be assessed by direct assignment or indirect using of pairwise comparison. It is recommended that the decision makers can use the Linguistic variables. To assess the significance of the criteria, the assignment of rating of alternative solutions can be made with regard to different criteria.

Let us assume that a decision group has k persons, then the significance of criteria and assigning of rating of alternative decisions with regard to each of the criteria can be computed as

$$\tilde{x}ij = \frac{1}{k}\left[\overset{\sim 1}{xij}(+)\overset{\sim 2}{xij}(+).......(+)\overset{\sim k}{xij}\right]$$

$$\tilde{w}j = \frac{1}{k}\left[\overset{\sim 1}{wj}(+)\overset{\sim 2}{wj}(+).......(+)\overset{\sim k}{wj}\right]$$

where $\overset{\sim k}{xij}$ and $\overset{\sim k}{wj}$ are the rating assign and the importance of weight of the K^{th} decision maker.

As explained above, a fuzzy multi criteria group decision making problem can be precisely explained in the following matrix format

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & . & . & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & . & . & \tilde{x}_{2n} \\ . & . & . & . & . \\ \vdots & \vdots & . & . & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m1} & . & . & \tilde{x}_{mn} \end{bmatrix}$$

$$W = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_2 & . & . & \tilde{w}_n \end{bmatrix}$$

where $\tilde{x}ij, \forall i, j$ and $\tilde{w}j, \ j = 1,2,.......n$ stand for Linguistic variables. These Linguistic variable can be explained with the help of triangular fuzzy numbers $\tilde{x}ij = \left(a_{ij}, b_{ij}, c_{ij}\right)$ and $\tilde{w}j = \left(w_{j1}, w_{j2}, w_{j3}\right)$.

We get over the complication of normalization formula utilized in classical TOPSIS. The linear scale transformation is alternatively utilized here to make various criteria scales comparable. Resultantly we can get the normalized fuzzy decision matrix represented by $R = \left[\tilde{r}ij\right]_{m\times n}$.

Where B and C are the sets of benefit criteria and cost criteria, respectively, and

$$\tilde{r}ij = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), \ j \in B;$$

$$\tilde{r}ij = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \ j \in C;$$

$$c_j^* = \max_i c_{ij} \quad if \ \ j \in B;$$

$$a_j^- = \min_i a_{ij} \quad if \ \ j \in C.$$

The normalization method presented above is to present the feature that ranges normalized triangle fuzzy numbers $[0, 1]$.

By taking into consideration of the difference in the significance of each criteria we can build up the weighted normalized fuzzy decision matrix as follows.

$$\tilde{V} = \left[\tilde{v}ij\right]_{m\times n}, \quad i = 1,2,3........m, \ j = 1,2,......n. \ \text{ where } \tilde{v}ij = \tilde{r}ij(.)\tilde{w}j.$$

with respect the weighted normalized fuzzy decision matrix we come to know that the elements $\tilde{v}ij, \forall i, j$ stand for normalized positive triangular fuzzy numbers and their ranges are with in the closed interval $[0,1]$. We can then explain fuzzy positive ideal solution by $\left(FPIS, A^*\right)$ and fuzzy negative ideal solution by $\left(FNIS, A^-\right)$as

$$P^* = \left(\tilde{v}1^*, \tilde{v}2^*......\tilde{v}n^*\right),$$

$$\bar{N} = \left(\tilde{v}1^-, \tilde{v}2^-......\tilde{v}n^-\right)$$

where $\tilde{v}j^* = (1,1,1)$ and $\tilde{v}j^- = (0,0,0)$, $j = 1,2.....n$

The distance of each alternative from P^* and $\bar{N}$ can be currently calculated as

$$d_i^* = \sum_{j=1}^{n} d\left(\tilde{v}ij, \tilde{v}_j^*\right), i = 1,2,......m$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{v}ij, \tilde{v}_j^-\right), \ j = 1,2,......m$$

where $d(.,.)$ stands for the distance calculated between two fuzzy numbers.

A closeness co-efficient is used to find out the order of ranking of all the alternative decisions after the calculation of d_i^* and d_i^- of each alternatives A_i $(i = 1,2,3......m)$. The closeness co-efficient of each alternative decisions is found out by

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \quad i = 1,2,.....m$$

An alternative decision A_i is found to be closer to FPIS (A^*) and at the distance from FNIS (A^-) when CC_i approaches to 1. The closeness co-efficient can therefore be used to find out the order of ranking of all alternative decisions and identify the best one from out of a set of possible alternatives.

4. ALGORITHMIC APPROACH OF THE METHOD

Step: 1 A committee of decision makers can be constituted and after that the evaluation criteria are to be identified.

Step: 2 Selection of the proper Linguistic variable for finding out the significance of weight of the criteria and the rating of the alternative decisions with regard to criteria.

Step: 3 Adding up weights of criteria to arrive at the aggregate fuzzy weight $\tilde{w}j$ of criterion c_j and collect the decision makers opining to obtain we aggregated fuzzy ratings.

Step: 4 Fuzzy decision matrix to be constructed and normalized.

Step: 5 The weighted normalized fuzzy decision matrix to be constructed.

Step: 6 Determination of FPIS and FNIS

Step: 7 Calculation of the distance of each alternative decision from FPIS and FNIS.

Step: 8 Computation of closeness co-efficient of each alternative decisions.

Step: 9 As a final step ranking order of all the alternative decision is to be determined.

5. NUMERICAL EXAMPLE

The investment decision making criteria in IT companies are evaluated on the basis of five variables. These variables are extended by A_1, A_2 and A_3 the committee of three investors D_1, D_2 and D_3 has been formed to proceed with an evaluation to find out the appropriate.

(1) Equality Capital (C_1)

(2) Earning per share (C_2)

(3) Price to book value (C_3)

(4) Net Profit Margin (C_4)

(5) Dividend payout (C_5)

The three decision makers use the seven point scale linguistic variables whose values are given as triangular fuzzy numbers to express the importance weight /priority to five criteria given by

Very Low (VL)	(0,0,0.1)
Low(L)	(0,0.1,0.3)
Medium Low (ML)	(0.1,0.3,0.5)
Medium(M)	(0.3,0.5,0.7)
Medium High(MH)	(0.5,0.7,0.9)
High(H)	(0.7,0.9,1.0)
Very High (VH)	(0.9,1.0,1.0)

The assessment of the criteria importance by the decision makers are given by

Table 4.1 The importance weight of the criteria

	D_1	D_2	D_3
C_1	H	VH	VH
C_2	H	H	H
C_3	MH	H	MH
C_4	MH	MH	MH
C_5	H	H	H

Based on the above assessment and using the given values of the linguistic variables, the fuzzy weight of each criterion j is found as

$$\tilde{w}j = \frac{1}{3}\left[w_j^{(1)} + w_j^{(2)} + w_j^{(3)}\right]$$

$$\tilde{w}1 = \frac{1}{3}\left[H + VH + VH\right]$$

Therefore $$= \frac{1}{3}\left[(0.7,0.9,1.0) + (0.9,1.0,1.0) + (0.9,1.0,1.0)\right]$$

$$= \frac{1}{3}\left[(2.5,2.9,3.0)\right]$$

$$= (0.83,0.97,1.0)$$

Similarly we can calculate

$$\tilde{w}2 = \frac{1}{3}\left[H + H + H\right] = (0.7,0.9,1.0)$$

$$\tilde{w}3 = \frac{1}{3}\left[MH + H + MH\right]$$

$$= \frac{1}{3}\left[(0.5,0.7,0.9) + (0.7,0.9,1.0) + (0.5,0.7,0.9)\right]$$

$$= (0.57,0.77,0.93)$$

$$\tilde{w}4 = \frac{1}{3}\left[MH + MH + MH\right]$$

$$= \frac{1}{3}\left[(0.5,0.7,0.9) + (0.5,0.7,0.9) + (0.5 + 0.7,0.9)\right]$$

$$= \left[(0.5,0.7,0.9)\right]$$

$$\tilde{w}5 = \frac{1}{3}\left[H + H + H\right]$$

$$= \frac{1}{3}\left[(0.7,0.9,1.0) + (0.7,0.9,1.0),(0.7,0.9,1.0)\right]$$

$$= \left[(0.7,0.9,1.0))\right]$$

Hence the fuzzy weight vector

$\tilde{W} = (\tilde{w}1, \tilde{w}2, \tilde{w}3, \tilde{w}4, \tilde{w}5)$ whose values are give as above .

The three IT companies are assessed by the three decision makers on a seven point linguistic scale whose values are given as

Very poor (VP)	(0,0,1)
Poor (P)	(0,1,3)
Medium Poor (MP)	(1,3,5)
Fair (F)	(3,5,7)
Medium Good(MG)	(5,7,9)
Good (G)	(7,9,10)
Very Good (VG)	(9,10,10)

The ratings or evaluation of the three decision makers under the five criteria are given below.

Table- 4.2

The rating of the three companies by the decision makers under all criteria

Criteria	**Company**	**Decision makers**		
		$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$
$\mathbf{C_1}$	X_1	6	8	7
	X_2	3	4	4
	X_3	4	5	6
$\mathbf{C_2}$	X_1	G	VG	F
	X_2	VG	VG	VG
	X_3	MG	G	VG
$\mathbf{C_3}$	X_1	F	G	G
	X_2	G	G	G
	X_3	G	MG	VG
$\mathbf{C_4}$	X_1	VG	G	G
	X_2	G	G	G
	X_3	G	VG	VG
$\mathbf{C_5}$	X_1	F	F	F
	X_2	G	F	G
	X_3	G	G	G

Combining the opinion of all the decision makers for each criterion, the fuzzy decision matrix of the three alternatives, i.e., three companies are given by:

For the company X_1, under the criterion C_1, the evaluation is

$$\tilde{X}11 = \frac{6+8+7}{3} = 7$$

Under criterion C_2

$$\tilde{X}12 = \frac{1}{3}[G + VG + F]$$
$$= \frac{1}{3}[(7,9,10) + (9,10,10) + (3,5,7)] = (6.3,8,9)$$

Under criterion C_3

$$\tilde{X}13 = \frac{1}{3}[F + G + G]$$
$$= \frac{1}{3}[(3,5,7) + (7,9,10) + (7,9,10)] = (5.7,7.7,9)$$

Under criterion C_4

$$\tilde{X}14 = \frac{1}{3}[VG + G + G]$$
$$= \frac{1}{3}[(9,10,10) + (7,9,10) + (7,9,10)] = (7.7,9.3,10)$$

Under criterion C_5

$$\tilde{X}15 = \frac{1}{3}[F + F + F]$$
$$= \frac{1}{3}[(3,5,7) + (3,5,7) + (3,5,7)] = (3,5,7)$$

Similarly for the brands X_2 and X_3 under the five criteria we can calculate the evaluations $\tilde{X}ij$ where $i = 1,2,3$ and $j = 1,2,3,4,5$

The fuzzy decision matrix $\tilde{F} = \left(\tilde{X}ij\right)$ is given by

$$\tilde{D} = \begin{array}{cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ X_1 & (7.0,7.0,7.0) & (6.3,8,9) & (5.7,7.7,0.9) & (7.7,9.3,10) & (3,5,7) \\ X_2 & (4.0,4.0,4.0) & (9,10,10) & (7,9,10) & (7,9,10) & (5.7,7.7,0.9) \\ X_3 & (5.0,5.0,5.0) & (7,9,10) & (7,9,10) & (8.3,9.7,10) & (7,9,10) \end{array}$$

To find the normalized decision matrix $\tilde{R} = (\tilde{r}ij)$ for the cost criteria C_1,

For the Company X_1 $\quad \tilde{X}11 = (7,7,7)$

X_2 $\quad \tilde{X}21 = (4,4,4)$

X_3 $\tilde{X}31 = (5,5,5)$

$\min \bar{a}1 = \min\{a_{ij}\} = \min(7,4,5) = 4$

$$\tilde{r}11 = \left(\frac{4}{7},\frac{4}{7},\frac{4}{7}\right) = (0.57,0.57,0.57)$$

$$\tilde{r}21 = \left(\frac{4}{4},\frac{4}{4},\frac{4}{4}\right) = (1,1,1)$$

$$\tilde{r}31 = \left(\frac{4}{5},\frac{4}{5},\frac{4}{5}\right) = (0.8,0.8,0.8)$$

For the benefit criterion C_2

$C_2^* = \max_i\{C_{ij}\} = \max\{9,10,10\} = 10$

$$\therefore \quad \tilde{r}_{12} = \left(\frac{6.3}{10},\frac{8}{10},\frac{9}{10}\right) = (0.63,0.8,0.9)$$

$$\tilde{r}_{22} = \left(\frac{9}{10},\frac{10}{10},\frac{10}{10}\right) = (0.9,1,1)$$

$$\tilde{r}_{32} = \left(\frac{7}{10},\frac{9}{10},\frac{10}{10}\right) = (0.7,0.9,1)$$

Similarly for the other benefit criteria C_3, C_4 and C_5 we can calculate the values of $\tilde{r}ij$ for the three alternatives.

Hence the normalized fuzzy decision matrix $\tilde{R} = \tilde{r}ij$ is given by

	C_1	C_2	C_3	C_4	C_5
X_1	(0.57,0.57,0.57)	(0.63,0.8,0.9)	(0.57,0.77,0.9)	(0.77,0.93,1)	(0.3,0.5,0.7)
X_2	(1,1,1)	(0.9,1,1)	(0.7,0.9,1)	(0.7,0.9,1)	(0.57,0.77,0.9)
X_3	(0.8,0.8,0.8)	(0.7,0.9,1)	(0.7,0.9,1)	(0.83,0.97,1)	(0.7,0.9,1)

The fuzzy normalized decision matrix $\tilde{v} = (\tilde{v}_{ij})$

$$\text{Where } \tilde{v}ij = \left(\tilde{r}ij\right)(.)\left(\tilde{w}j\right)$$

For the criterion C_1

$$\tilde{v}_{11} = \left(\tilde{r}_{11}\right)(.)\left(\tilde{w}_1\right)$$

$$= (0.57,0.57,0.57)(.)(0.83,0.97,1) = (0.4731,0.5529,0.57)$$

$$\tilde{v}_{21} = \left(\tilde{r}_{21}\right)(.)\left(\tilde{w}_1\right)$$

$$= (1,1,1)(.)(0.83,0.97,1) = (0.83,0.97,1)$$

$$\tilde{v}_{31} = \left(\tilde{r}_{31}\right)(.)\left(\tilde{w}_1\right)$$
$$= (0.8,0.8,0.8)(.)(0.83,0.97,1) = (0.664,0.776,0.8)$$

For the criterion C_2

$$\tilde{v}12 = \left(\tilde{r}12\right)(.)\left(\tilde{w}2\right) = (0.63,0.8,0.9)(.)(0.7,0.9,1) = (0.441,0.72,0.9)$$
$$\tilde{v}22 = \left(\tilde{r}22\right)(.)\left(\tilde{w}2\right) = (0.9,1,1)(.)(0.7,0.9,1) = (0.63,0.9,1)$$
$$\tilde{v}32 = \left(\tilde{r}32\right)(.)\left(\tilde{w}2\right) = (0.7,0.9,1)(.)(0.7,0.9,1) = (0.49,0.81,1)$$

Similarly for the other criteria C_3, C_4 and C_5 we can calculate the values $\tilde{v}ij$.

$\therefore$ The weighted normalized fuzzy decision matrix is determined to be

$\tilde{v} = (\tilde{v}ij) =$

	C_1	C_2	C_3	C_4	C_5
X_1	(0.4731, .5529, 0.57)	(0.441,0.72,0.9)	(0.3249,0.5929,0.837)	(0.385,0.651,0.9)	(0.21,0.45,0.7)
X_2	(0.83,0.9,1)	(0.63,0.9,1)	(0.399,0.693,0.93)	(035,0.63,0.9)	(0.399,0.693,0.9)
X_3	(0.664, 0.776,0.8)	(0.49,0.81,1)	(0.399,0.639,0.93)	(0.415,0.679,0.9)	(0.49,0.89,1)

Take the fuzzy positive and fuzzy negative ideal solutions to be $P^* = \left(\tilde{V}_1^*, \tilde{V}_2^*, \tilde{V}_3^*, \tilde{V}_4^*, \tilde{V}_5^*\right)$ and

$\tilde{N} = \left(\overline{\tilde{V}}1, \overline{\tilde{V}}2, \overline{\tilde{V}}3, \overline{\tilde{V}}4, \overline{\tilde{V}}5\right)$ respectively such that $\tilde{V}j^* = (1,1,1)$ and $\overline{\tilde{V}}j^* = (0,0,0)$

The distance of each alternative B_i from the positive solution is

$$d_i^+ = \sum_{j=1}^{n} d(V_{ij}, V_j^*)$$

The distance of the 1st alternative from (1, 1, 1) is

$$\begin{aligned} d_1^+ &= d[(0.4731,0.5529,0.57)(1,1,1)] + d[(0.441,0.72,09)(1,1,1)] \\ &\quad + d[(0.3249,0.5929,0.837)(1,1,1)] + d[(0.385,0.651,0.9),(1,1,1)] \\ &\quad + d[(0.21,0.45,0.7)(1,1,1)] \\ &= \frac{1}{2}\{\max(|0.4731-1|,|0.57-1|+|0.5529-1|)\} + \frac{1}{2}\{\max(|0.441-1|,|0.9-1|,|0.72-1|)\} + \\ &\quad \frac{1}{2}\{\max(|0.3249-1|,|0.837-1|,|0.5929-1|)\} + \frac{1}{2}\{\max(|0.385-1|,|0.9-1|+|0.651-1|)\} + \\ &\quad \frac{1}{2}\{\max(|0.21-1|,|0.7-1|,|0.45-1|)\} \\ &= 0.487 + 0.4195 + 0.5411 + 0.482 + 0.67 = 2.5996 \end{aligned}$$

The distance of the 1st alternative from $(0,0,0)$ is

$$\begin{aligned} d_1^- &= d[(0.4731,0.5529,0.57)(0,0,0)] + d[(0.441,0.72,09)(0,0,0)] \\ &\quad + d[(0.3249,0.5929,0.837)(0,0,0)] + d[(0.385,0.651,0.9),(0,0,0)] \\ &\quad + d[(0.21,0.45,0.7)(0,0,0)] \\ &= 0.5615 + 0.81 + 0.71495 + 0.7755 + 0.575 = 3.43695 \end{aligned}$$

Similarly the distance of the 2nd alternative from (1, 1, 1) and from (0,0,0) are

$d_2^+ = 1.753$, $d_2^- = 4.308$ respectively.

The distance of the 3rd alternative from (1, 1, 1) and from (0, 0, 0) are

$d_3^+ = 1.887$, $d_3^- = 4.199$ respectively.

The separation measure from the positive and negative solution are given by

Table 4.3 Separation measure

	d_i^+	d_i^-
X_1	3.0816	3.437
X_2	1.753	4.308
X_3	1.887	4.199

The closeness coefficient $CC_i = \dfrac{d_i^-}{d_i^+ + d_i^-}$

$$CC_1 = \frac{3.437}{3.437 + 2.5996} = \frac{3.437}{6.0366} = 0.5694$$
$$CC_2 = \frac{4.308}{4.308 + 1.753} = \frac{4.308}{6.061} = 0.71077$$
$$CC_3 = \frac{4.199}{4.199 + 1.887} = \frac{4.199}{6.086} = 0.68994$$

According to the closeness coefficient, the ranking order of the three alternatives is $X_2 > X_3 > X_1$. Therefore the best alternative is the company X_2.

6. CONCLUSION

In practical situations, attempts are constantly made to find solutions to varied problems with the help of inadequate, imprecise and uncertain data. Fuzzy analysis is an appropriate approach that can be made to make the imprecise data into a precise one and uncertain data into one with certainty. Linguistic decision process is followed in this article to find solution to the problem selected by giving due importance to multiple criteria.

Decision making is a rigorous exercise undertaken by the assessment of multiple criteria and assignment of weight by using linguistic variables instead of numerical variables. Vertex method is supposed to be a simple and more appropriate method to calculate the distance between two triangular fuzzy numbers. TOPSIS technique in this method is extended to the fuzzy environment. Vertex method is in fact easy method to measure the distance between any two fuzzy numbers whose membership functions is linear. Group Decision making is a process which is the difficult one to use other aggregation functions and pool the fuzzy ratings made by the decision makers in the proposed method. The proposed method explains in this article is of use not only to solve investment decision making problems but also to find solutions to various managerial problems.

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