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# A Note on Largest Eigen Fuzzy Set

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## ABSTRACT

The concept of finite discrete time Fuzzy Possibilistic Markov Chain (FPMC) on the fuzzy possibility space $(X, R, \widetilde{\Pi})$ is introduced. We analyze the classification of its states. Finally, we give the necessary conditions for the occurrence of the ergodicity and find its steady state using Eigen fuzzy sets.

**KEYWORDS:** Possibility measure, fuzzy possibility, ergodicity, steady state

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# 1 INTRODUCTION:

In some circumstances, where the existing information about the system is not enough, the estimation of the probability values is difficult. To overcome these difficulties, the possibility measures have been used since 1965.A Fuzzy Markov Chain [1]was defined with crisp transition probabilities by Bhattacharya in [2] and with fuzzy transition probabilities by Kruse et al[3]. In [4], the authors defined the fuzzyfinite Markov chains based on possibility theory and compared the results of classical Markov chains and FMC.Possibilistic Markov processes and possibilistic Markov chain were defined and analyzed in [6]. Several authors's contributed considerable work in this direction [8, 9, 10, 11, 17]. In[2]Avrachenkov andSanchez, pointed out the difference between the classical and fuzzy Markov chains.

# 2 PRELIMINARIES:

## *DEFINITION 2.1*

*For every possibility measure* $\Pi$ *on* $(X, R)$*, there exists a unique* $R$ *- measurable mapping* $\pi$ :$X$ $L$ *such that for any* $B \in R$, $\Pi(B) = sup_{x\in B}\Pi(x)$ *where* $\Pi(x) = \Pi([x]_R)$, $x \in X$.

## *DEFINITION 2.2*

*Let*$(\Omega,R_\Omega,\Pi_\Omega)$*beanotherpossibilityspace.Amapping*

$f$:$\Omega\rightarrow X$*iscalledpossibilityvariableifffisa*$R_\Omega-R_X$*measurable.Let*$B\in R$*,then*

$$\left.\begin{aligned}\Pi(B) &= \Pi_\Omega\left(f^{-1}(B)\right)\\ \pi(x) &= sup_{f(\omega)\in[x]_R}[\Pi_\Omega(\omega)], \forall x \in X\end{aligned}\right\},(2.1)$$

## *DEFINITION2.3*

*Let* $f:\Omega \rightarrow X$ *be a possibility variable on (X, R). By the transformation of possibility measure,* $\Pi$*is a possibility measure on*$(X,R)$.

$$\left.\begin{aligned}\Pi_f(B) &= (\Pi_\Omega^f R)(B), \forall B \in R = \Pi_\Omega(f^{-1}(B))\\ \Pi_f(x) &= sup_{f(\omega)\in[x]_R}[\Pi_\Omega(\omega)], \forall x \in X\end{aligned}\right\} \qquad (2.2)$$

*Where*$\Pi$ $f$ *is called as the possibility measure of f*

# 3 PROPERTIES AND CLASSIFICATION OF FPMC

## *3.1 Fuzzy Possibilisitic Markovchain ( Fpmc)*

In this subsection, we have defined some new concepts and have introduced FPMC. We have also generalized the properties of classical Markov chain to FPMC using max-min composition.

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## THEOREM 3.1

*Let* $\tilde{A}^{\Pi} = \{x, \mu_{\tilde{A}^{\Pi}}(x) = \tilde{B}_x\}$ *be a normalized type 2 fuzzy set defined on X and* $\tilde{B}_x$. *Then* $\tilde{\Pi}$ *forms a possibility space* $(X,R,\tilde{\Pi})$ *such that there exist a possibility variable f with the possibility distribution* $\mu_{\tilde{A}^{\Pi}}$.

## PROOF:

Let $\tilde{A}^{\Pi}$ be as given in the statement and let $\tilde{\Pi}: R \to F([0,1])$ be the fuzzy Possibility such that $\tilde{\Pi}(B) = sup_{x\in B}[\mu_{\tilde{A}^{\Pi}}(x)] = sup_{x\in B}\tilde{B}_x$ for all $B \in R$. Clearly, $\tilde{\Pi}(\Phi)=0$. Since the given type 2 fuzzy set is normalized, there exists an element with membership grade (1, 1, 1) and it is the supremum among all fuzzy numbers on [0,1]. Hence $\tilde{\Pi}(X) = sup_{x\in X}[\tilde{B}_x] = (1,1,1)$. To prove that $\tilde{\Pi}(\cup_i B_i) = sup_i(\tilde{\Pi}(B_i))$, where $\{B_i\}$ is an arbitrary collection of subsets of X, let $C = \cup_i B_i$. Then $\tilde{\Pi}(C) = sup_{x\in C}[\tilde{B}_x] = \tilde{B}_{x_1}$. Obviously, the corresponding $x_1$ is in at least one of the $B_i$'s. Hence, $sup_i[\tilde{\Pi}(B_i)] = sup_i\{sup_{x\epsilon B_i}[\tilde{B}_x]\} = \tilde{B}_{x_1}$ and p $\tilde{\Pi}(\cup_i B_i) = sup_i(\tilde{\Pi}(B_i))$. This proves that the fuzzy possibility $\tilde{\Pi}$ is a possibility measure on $(X,R)$ with possibility distribution $\tilde{\pi}(x) = \mu_{\tilde{A}^{\Pi}}(x)$ i.e., $(X,R,\tilde{\Pi})$ is possibility space. By (2.2) and (2.3), $f$ is a possibility distribution.

Hence, $\tilde{\Pi}_f = \mu_{\tilde{A}^{\Pi}}$.

**Note:** The possibility space $(X,R,\tilde{\Pi})$ induced by the fuzzy possibility $\tilde{\Pi}$ is named as fuzzy possibility space and the possibility distribution is called fuzzy possibility distribution.

## DEFINITION 3.1

*The set of possibility variables* $\{X(t); t\in T\}$, *defined on the fuzzy possibility space* $(X,R,\tilde{\Pi})$ *is called the fuzzy possibilistic stochastic process (FPSP).*

## DEFINITION 3.2

*The Fuzzy Possibilistic Stochastic Process (FPSP) satisfying the Markovian property is called the fuzzy possibilistic Markov chain (FPMC),*

And $max_j\{\tilde{\pi}_{i,j}\} = (1, 1, 1)$. The transition between the states of the FPMC can be viewed as a fuzzy relation with fuzzy relation matrix $\tilde{H}$. Hence, instead of matrix multiplication, we are using composition of two fuzzy relations (max-min composition).

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### DEFINITION3.3

*AFPMCissaidtobehomogeneous,ifthefuzzytransitionpossibilityfromstate'i' atstepmtostate'j' atstepndoe snotdependonmandn,butonlyonthe difference n-m.*

i.e.,$\tilde{\pi}_{ij}(n,m)=\tilde{\pi}_{ij}(n-m)$ $\tilde{H}(n,m)=\tilde{H}(n-m)$.

### THEOREM 3.2

*Consider a homogeneous FPMC* $\{X(n),\ n = 0,\ 1,\ \ldots\}$ *with m states and with fuzzy Transitionpossibility*$\tilde{\pi}_{ij}$*.Thenforalln*$_1$,$n_2\geq0$,$\tilde{H}(n_1+n_2)=\tilde{H}(n_1)\otimes\tilde{H}(n_2)$.

### PROOF:

*Consider*

$\tilde{\Pi}^{n_1+n_2} = \tilde{\Pi}\{X(n_1 + n_2) = j|\ X(0) = i\}$

$=max_{k=0}^{n}\ min[\tilde{\Pi}_{ik}(n_1), \tilde{\Pi}_{kj}(n_2)]$

Note that $\max_i(\tilde{\Pi}_{ij})$hence proved and itis called the Chapman-Kolmogorov equation of FPMC.Hence

Byinduction$\tilde{H}(\dot{n})=\tilde{H}\cdot$

### DEFINITION 3.4

*Consider a FPMC with m states. Then thefuzzypossibilityofbeinginstate I atnthstepisgivenby,*$\tilde{p}_i(n)=\tilde{\Pi}(X(n)=i)$*.Andtherowvector,* $\tilde{P}(n)=[\ \tilde{p}_0(n),\tilde{p}_1(n),\ldots\tilde{p}_m(n)\ ]$ *iscalledthestatefuzzypossibilitydistributionatthen*$^{th}$*step.Andwe denotethelimitingstatefuzzypossibilitydistributionas*$\lim_{n\to\infty}\tilde{P}(n)=\tilde{\Gamma}$.

### 3.2 Classification Of The States Of Afpmc

In this subsection, we are considering a homogeneous FPMC with m states.

### DEFINITION3.5

Let $\tilde{F}_{ij} = sup_i(\ \tilde{f}_{ij}(n))$ where $\tilde{f}_{ij}(n)$the fuzzy possibility of the first time visit to stage *j*

In the probability space, positive recurrent is defined in terms of mean recurrence time where itisnotpossibleforFPMC.Ifwedefinethemeanrecurrencetimeforarecurrentstate'*i*' as$\tilde{\mu}_i=$ $sup_k[inf(k,\tilde{f}_{ii}(k))]$, since the k-values are positive integers and $\tilde{f}$' $s_i$are triangular fuzzy numbers on[0,1],weget$[inf(k,\ \tilde{f}_{ii}(k))]\ =\tilde{f}_{ii}(k)$provided$k\neq0$.Hence,$\tilde{\mu}_i=[sup_k\tilde{f}_{ii}(k)]=\tilde{f}_{ii}=(1,1,1)$which is always

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finite. Hence the positive recurrence for a state of *FPMC* is defined as follows.

### *DEFINITION 3.6*

*A state 'i'is transient iffthere is a fuzzy possibility that the processwillneverreturntostate'i' and*$\tilde{F}_{ii}$<(1,1,1).

### *DEFINITION3.7*

*Ifthepowersofthe FTPM* $\tilde{H}$ *convergein n steps to a non-periodic solution, then the associated FPMC is said to be a periodic.*

## 4 ERGODICITY OF FPMC

For FPMC, the limiting FTPMwill have stationary solutions depends on the initial state. Hence, in [1] the ergodicity is defined as follows.

### *DEFINITION 4.1*

*A FPMC is said to be ergodic if it is aperiodic and has limiting FTPMwith identical rows.*

### *THEOREM4.1*

*Let*$\tilde{H}_{3\times 3}$*bethefuzzytransitionpossibilitymatrixofaFPMCwiththreestates. Then FPMC is ergodic, if* $\tilde{H}$*possessthefollowingproperties.*

- $\tilde{H}$*hasatleastone column*$j_1$*insuchawaythatall then triesof*$j_1$*areequalto*(1,1,1).
- *If*$\max_i(\tilde{h}_{ij})$for each j*(excludingthe*$j_1^{th}$*columnandrow)arenot thediagonalentries,then*$\tilde{h}_{j_1k} \geq \min\max_i(\tilde{h}_{ij})\ \forall j$ *and for some k.If there is*$\tilde{h}_{k_1k_1} \geq \min\max_i(\tilde{h}_{ij})$ *a diagonalentry, then* $\tilde{h}_{j_1k} \geq \tilde{h}_{k_1k_1}$.
- *Ifadiagonalentry*$\tilde{H}_{ii}(i\neq j_1)$*of*$\tilde{H}$*isthelargestelementin*$i^{th}$*column,then* $\tilde{h}_{j_1i} \geq \tilde{h}_{ij}$.*Ifthereis* $\tilde{h}_{jj} \geq \tilde{h}_{ij}$*then*$\tilde{h}_{j_1i}\geq\tilde{h}_{jj}$.

### *PROOF:*

To get the rows of the limiting *FTPM*to be identical, the *FTPM*should be converge and the entries in each column of the limiting. *FTPM*shouldbeequal. Letusassume that $\tilde{H}$satisfiestheabovethree properties. Now we prove that there exist a liming *FTPM* with identicalrowscorrespondingto$\tilde{H}$.

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***CASE: 1***

SincetherowsoftheFPMCarefuzzypossibilitydistributions,therowmaximumofeachrowof $\tilde{H}$is(1,1,1).Lettheentries $_1$ of$j^{th}$columnbeequalto(1,1,1).Then,thiscolumnentrieswillbe $_1$ retainedinthehigherpowersof$\tilde{H}$.. In the remaining part of this proof, we have considered by$\max_i(\tilde{h}_{ij})$for each jexcluding$j^{th}$columnandrow.

***CASE: 2***

Let none of the column maximums be the diagonal entries of $\widetilde{H}$. Since the given FPMC has three states and the diagonal entries will not be the column maximums, the maximum entries of the $i^{th}$ and $j^{th}$ column will be $\tilde{h}_{ij}$. Let $\tilde{h}_{j_1 i} \geq \min\{\tilde{h}_{ij}, \tilde{h}_{ji}\}$. Since each row's $j_1^{th}$ entry is (1,1,1) and $\tilde{h}_{j_1 i} \geq \min\{\tilde{h}_{ij}, \tilde{h}_{ji}\}$, the entries of $i^{th}$column $\widetilde{H^2}$ are equal to $\tilde{h}_{j_1 i}$. And in the $j^{th}$column $\widetilde{H^2}$, $\tilde{h}^2_{j_1 j}$ ($\therefore \tilde{h}^2_{j_1 j} = \tilde{h}_{ij}\ if\ \tilde{h}_{j_1 i} \geq \tilde{h}_{ij}\ or\ \tilde{h}^2_{j_1 i} = \tilde{h}_{j_1 i}\ if\ \tilde{h}_{j_1 i} \geq \tilde{h}_{ij}$. Even though $\tilde{h}_{j_1 i} \geq \min\{\tilde{h}_{ji}, \tilde{h}_{ij}\}$, if there is a diagonal entry $\tilde{h}_{jj} \geq \min\{\tilde{h}_{ij}, \tilde{h}_{ji}\}$, then$\tilde{h}^n_{jj} = \tilde{h}_{jj}$. If $\tilde{h}_{j_1 i} \geq \tilde{h}_{jj}$then the entries of the $i^{th}$ column of $\widetilde{H^2}$ are equal to $\tilde{h}_{j_1 i}$. Since $\tilde{h}_{j_1 i} \geq \tilde{h}_{jj}$and $\tilde{h}_{jj} \geq \min\{\tilde{h}_{ji}, \tilde{h}_{ij}\}$, $\tilde{h}^2_{j_1 j}[\tilde{h}^2_{j_1 j} = \tilde{h}_{ij}\ if\ \tilde{h}_{j_1 i} \leq \tilde{h}_{ij}\ or\ \tilde{h}^2_{j_1 j} = \tilde{h}_{j_1 i}\ if\ \tilde{h}_{j_1 i} \leq \tilde{h}_{ij}]$ is the$\widetilde{H}^2_{ki} = \tilde{h}_{j_1 i}$. Hence we get the limiting *FTPM* with identical rows

***CASE: 3***

Suppose that $\max_k(\tilde{h}_{kj}) = \tilde{h}_{ij}$and and$\tilde{h}_{j_1 i} \geq \tilde{h}_{ii}$Then ${\tilde{h}_{j_1 j}}^2 = \min\{\tilde{h}_{ii}, \tilde{h}_{ij}\}$

Since $\tilde{h}_{j_1 i} \geq \tilde{h}_{jj}$and $\tilde{h}_{jj} \geq \min\{\tilde{h}_{ii}, \tilde{h}_{ij}\}$, $\tilde{h}^2_{j_1 j}[\tilde{h}^2_{j_1 j} = \tilde{h}_{ij}\ if\ \tilde{h}_{j_1 i} \geq \tilde{h}_{ij}\ or\ \tilde{h}^2_{j_1 j} = \tilde{h}_{j_1 i}\ if\ \tilde{h}_{j_1 i} \leq \tilde{h}_{ij}]$ will be the largest in the $j^{th}$ column. In $\widetilde{H}^3$, all the entries of $j^{th}$ column become equal to ${\tilde{h}_{j_1 j}}^2$. Hence we attain the limiting *FTPM*with identical rows.

***THEOREM4.2***

*Forahomogeneousirreducibleergodic FPMC,let*$\tilde{\Gamma}_j = \lim_{n\to\infty} \tilde{\pi}^n_{ij}$,$j \geq 0$.*Then,* $_{ij}$

$\tilde{\Gamma}=(\tilde{\Gamma}_1,\tilde{\Gamma}_2,\ldots,(\tilde{\Gamma}_m)$*isthegreatesteigenfuzzyset(EFS)of*$\tilde{\Gamma}\otimes\tilde{H}=\tilde{\Gamma}$*suchthat*$\oplus_i\tilde{\Gamma}=(1,1,1)$.*And*$\tilde{\Gamma}$*is called the steady state of FPMC.*

***PROOF:***

The existence of $\tilde{\Gamma}_j = \lim_{n\to\infty}\tilde{\pi}^n_j$

$$\tilde{\pi}(X(n+1)=j) = max_i\{min\,[\tilde{\pi}(X(n+1)=j)|X(n)=i)]\}$$

$= max_i\{min[\tilde{\pi}_i(n), \tilde{\pi}_{ij}]\}$

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$$\lim_{n\to\infty}\left[\tilde{\pi}_j(n+1)\right] = \lim_{n\to\infty} max_i\{\min\left(\tilde{\pi}\tilde{\pi}(n),\tilde{\pi}_{ij}\right\}\}$$

$$= max_i\{min\,[\lim_{n\to\infty}\tilde{p}_i(n),\tilde{\pi}_{ij}]\}$$

$$= max_i\{\min\left[\tilde{\Gamma}_i,\tilde{\pi}_{ij}\right]\}$$

$\tilde{\Gamma}_j=\tilde{\Gamma}_i\otimes\tilde{\pi}_{ij}, j\geq 0$

$\tilde{\Gamma}=\tilde{\Gamma}\otimes\tilde{H}, j\geq 0.$

Since the given FPMC is ergodic, therows of its limiting transition matrix are equal to the greatest EFS of the fuzzyrelationdefinedbyPwhichhasbeenprovedin[1], therowsoft helimiting $FTPM\tilde{H}$ are equal to the greatest EFS of $\tilde{\Gamma} = \tilde{\Gamma} \otimes \tilde{H}$ and also $\tilde{\Gamma}_j = \lim_{n\to\infty}\tilde{\pi}_j^n, \tilde{\Gamma} = \left(\tilde{\Gamma}_1,\tilde{\Gamma}_2,\ldots,\tilde{\Gamma}_m\right)$is the greatest EFS of $\tilde{\Gamma} = \tilde{\Gamma} \otimes \tilde{H}$ and $\bigoplus_i \tilde{\Gamma}_i = (1,1,1)$.

## *4 EXAMPLE*

**The *FTPM*of FPMC is,**

$$\tilde{H}=\begin{pmatrix}(0.1,0.3,0.4) & (1,1,1) & (0.3,0.4,0.6)\\ (0.1,0.2,0.5) & (1,1,1) & (0.3,0.5,0.6)\\ (0.1,0.4,0.5) & (1,1,1) & (0.4,0.6,0.7)\end{pmatrix}$$

Then, $\tilde{H}^2=\tilde{H}^3\ldots= \cdot\begin{pmatrix}(0.1,0.4,0.5) & (1,1,1) & (0.3,0.5,0.6)\\ (0.1,0.4,0.5) & (1,1,1) & (0.3,0.5,0.6)\\ (0.1,0.4,0.5) & (1,1,1) & (0.4,0.6,0.7)\end{pmatrix}$

Since the limiting *FTP M*converges at $T = 2$ with non-identical rows, the given FPMC is not ergodic. And the steady state for this FPMC will not exist

## 5 CONSUMMATION

In this article, we have considered a map, called fuzzy possibility on the ample field of the universe X and have proved that if there exists a normalized type 2 fuzzy set on *X*, then thefuzzy possibility constructs a possibility space on*X*. On this fuzzy possibility space, we have defined the FPMCandtherowsofits$FTPM\tilde{H}$·And we have classified its states. We have considered a FPMC with three states and have found out the necessary conditions for its ergodicity. we have proved that, if a FPMC is ergodic, then its steady state is the greatestEFS of the fuzzy relation defined by $\tilde{H}\cdot$

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