Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Some Functions Concerning Neutrosophic Feebly Open & Closed Sets

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ABSTRACT

Inthis paper, we introduce and investigate a new class functions in neutrosophic topological space called neutrosophic feebly irresolute functions.Furthermore, the concepts of strongly neutrosophic feebly continuous functions and perfectly neutrosophic feebly continuous functions in terms of neutrosophic feebly open sets and neutrosophic feebly closed sets are introduced and several properties of them are investigated. Also we introduce neutrosophic feebly closed maps and neutrosophic feebly open maps in neutrosophic topological spaces and obtain certain characterizations of these classes of maps.

AMS Subject Classification: 03E72

KEYWORDS AND PHRASES: Neutrosophic feebly closed set, Neutrosophic feebly open set, Neutrosophic feebly irresolute function, strongly Neutrosophic feebly continuous function,perfectly Neutrosophic feebly continuous function, Neutrosophic feebly closed maps and Neutrosophic feebly open maps.

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I.INTRODUCTION

After the advent of the notion of fuzzy set by Zadeh[16], C. L. Chang[4] introduced the notion of fuzzy topological space and many researchers converted, among others, general topological notions in the context of fuzzy topology. The notion of Intuitionistic fuzzy set was introduced by Atanassov[3] in 1983 is one of the generalization of the notion of fuzzy set.Later, Coker[5] by using the notion of the intuitionistic fuzzy set, offered the useful notion of intuitionistic fuzzy topological space. The neutrosophic set was introduced by Smarandache[9] and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set. After the introduction of the concepts of neutrosophy and neutrosophic set, In 2012, Salama, Alblowi[15], introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element.

The purpose of this article is to introduce neutrosophic feebly irresolute functions in neutrosophic topological space. Furthermore, the concepts of strongly neutrosophic feebly continuous functions and perfectly neutrosophic feebly continuous functions in terms of neutrosophic feebly open sets and neutrosophic feebly closed sets are introduced and several properties of them are investigated.

Section II briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic mapping. The Section III deals with the concept of neutrosophic feebly irresolute functions.Section IV explains about strongly neutrosophic feebly continuous functions and perfectly neutrosophic feebly continuous functions in terms of neutrosophic feebly open sets and neutrosophic feebly closed sets. In the fourth section, we introduce neutrosophic feebly closed maps and neutrosophic feebly open maps in neutrosophic topological spaces and obtain certain characterizations of these classes of maps.

II. PRELIMINARIES

Definition 2.1[14]Let X be a non-empty fixed set. A neutrosophic set (NF for short) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

For basic notations and definitions of Neutrosophic Theory is not given here, the reader can refer [5-14].

Definition 2.2.[10] A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly open if there is a neutrosophic open set U in X such that $U \leq A \leq NScl(U)$.

Definition 2.3.[10] A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly closed if there is a neutrosophic closed set U in X such that $NSint(U) \leq A \leq U$.

Lemma 2.4. [10](i)A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly closed if and only if $Ncl\left(Nint\left(Ncl(A)\right)\right) \leq A$.

(ii) A neutrosophic subset A is a neutrosophic feebly closed set if and only if A^c is neutrosophic feebly open.

Definition 2.5.[10]Let (X, τ) be neutrosophic topological space and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a neutrosophic set in X. Then neutrosophic feebly interior of A is defined by $NFint(A) = \vee\{G: G$ is a neutrosophic feebly open set in X and $G \leq A\}$.

Lemma 2.6. [10]Let (X, τ) be neutrosophic topological space. Then for any neutrosophic feebly subsets A and B of a neutrosophic topological space X, we have

(i) $NFint(A) \leq A$

(ii) A is neutrosophic feebly open set in $X \Leftrightarrow NFint(A) = A$

(iii) If $A \leq B$, $NFint(A) \leq NFint(B)$.

Definition 2.7. [10]Let (X, τ) be neutrosophic topological space and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a neutrosophic set in X. Then the neutrosophic feebly closure is defined by $NFcl(A) = \wedge\{K: K$ is a neutrosophic feebly closed set in X and $A \leq K\}$.

Lemma 2.8. [10]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subset A of X,

(i) $(NFint(A))^c = NFcl(A^c)$

(ii) $(NFcl(A))^c = NFint(A^c)$.

Lemma 2.9. [10]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subsets A and B of a neutrosophic topological space X,

(i) $A \leq NFcl(A)$

(ii) A is a neutrosophic feebly closed set in $X \Leftrightarrow NFcl(A) = A$

(iii) $NFcl(NFcl(A)) = NFcl(A)$

(iv) If $A \leq B$ then $NFcl(A) \leq NFcl(B)$.

Definition 2.10.[14]Let X and Y be two neutrosophic sets and $f: X \to Y$ be a function. (i) If $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle$ is a neutrosophic sets in Y, then the pre image of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(ii) If $A = \{\langle x, \alpha_A(x), \delta_A(x), \lambda_A(x)\rangle : x \in X\}$ is a neutrosophic sets in X, then the image of A under f is denoted and defined by

$$f(A) = \{\langle y, f(\alpha_A)(y), f(\delta_A)(y), f\_(\lambda_A)(y)\rangle : y \in Y\} \text{where} f\_(\lambda_A) = (f(\lambda_A{}^C))^C.$$

In (i), (ii), since μ_B, σ_B, γ_B, α_A, δ_A, $\lambda_A$are neutrosophic sets, we explain that $f^{-1}(\mu_B)(x) = \mu_B(f(x))$, and $f(\alpha_A)(y) = \begin{cases} \sup \alpha_A(x) \text{ if } y \in f^{-1}(x) \\ 0 \quad \text{Otherwise} \end{cases}$.

Lemma 2.11. [11] Let $f: X \to Y$ be a function. Then the following statements hold.

(i) If A and B are neutrosophic subsets of X such that $A \leq B$, then $f(A) \leq f(B)$,

(ii) If A and B are neutrosophic subsets of Y such that $A \leq B$, then $f^{-1}(A) \leq f^{-1}(B)$.

Lemma 2.12.[11]Let $f: X \to Y$ be a function. If A is a neutrosophic subset of X and μ is a neutrosophic subset of Y. Then

(i) $f(f^{-1}(A)) \leq A$

(ii) $f(f^{-1}(A)) = A \Leftrightarrow f$ is surjective.

$(iii) f^{-1}(f(A)) \geq A$

(iv)$f^{-1}(f(A)) = A$ whenever f is injective.

Definition 2.13[11]Let $A, A_i (i \in J)$ be neutrosophic subsets in X and B, B_j $(j \in K)$ be neutrosophic subsets in Y and $f: X \to Y$ be the neutrosophic function. Then

(i) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(ii)$f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(iii) $f^{-1}(1_N) = 1_N$, $f^{-1}(0_N) = 0_N$

(iv) $f^{-1}(B^C) = (f^{-1}(B))^c$

(v) $f(\cup A_i) = \cup f(A_i)$.

Definition 2.14.[11] Let (X, τ) and (Y, σ) be neutrosophic topological spaces. Then a map $f: (X, \tau) \to (Y, \sigma)$ is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in (Y, σ) is neutrosophic open set in (X, τ).

Definition 2.15 [10]Let α, β, γ∈ [0, 1] and α+β+γ≤ 1. A neutrosophic point with support$x_{(\alpha,\beta,\gamma)} \in X$ is a neutrosophic set of X is defined by $x_{(\alpha,\beta,\gamma)} = \begin{cases} (\alpha, \beta, \gamma), y = x \\ (0,0,1), \ y \neq x \end{cases}$

In this case, x is called the support of $x_{(\alpha, \beta, \gamma)}$ and α, β and γ are called the value, intermediate value and the non – value of $x_{(\alpha, \beta, \gamma)}$ respectively. A neutrosophic point $x_{(\alpha, \beta, \gamma)}$ is said to belong to a neutrosophic set A = { ⟨ x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ ⟩ : x∈X } is denoted by two ways

(i) $x_{(\alpha, \beta, \gamma)} \in A$ if $\alpha \leq \mu_A(x)$, $\beta \leq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$.

(ii) $x_{(\alpha, \beta, \gamma)} \in A$ if $\alpha \leq \mu_A(x)$, $\beta \geq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$.

Clearly a neutrosophic point can be represented by an ordered triple of neutrosophic set as follows : $x_{(\alpha, \beta, \gamma)} = (x_\alpha, x_\beta, C(x_{c(\gamma)}))$. A class of all neutrosophic points in X is denoted as NP(X).

Definition 2.16[10] For any two neutrosophic subsets A and B, we shall write AqB to mean that A is quasi-coincident (q- coincident, for short) with B if there exists $x \in X$ such that $A(x) + B(x) > 1$. That is $\{\langle x, \mu_A(x) + \mu_B(x), \sigma_A(x) + \sigma_B(x), \gamma_A(x) + \gamma_B(x) \rangle : x \in X\} > 1$.

Definition 2.17 [10] Let λ and μ be any two neutrosophic subsets of a neutrosophic topological space. Then A is q-neighbourhood with B (q-nbd, for short) if there exists a neutrosophic open set O with $AqO \leq B$.

III. NEUTROSOPHIC FEEBLY IRRESOLUTE FUNCTIONS

In this section, we introduce the concept of neutrosophic feebly irresolute functions in neutrosophic topological spaces. Also, we discuss the relation with neutrosophic feebly continuous functions.

Definition 3.1. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \to Y$ is called neutrosophic feebly irresolute if the inverse image of every neutrosophic feebly open set in Y is neutrosophic feebly open in X.

Theorem 3.2. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \to Y$ is neutrosophic feebly irresolute if and only if the inverse image of every neutrosophic feebly closed set in Y is neutrosophic feebly closed in X.

Proof. Let A be any neutrosophic feebly closed set in Y. Then A^c is neutrosophic feebly open set in Y. Since f is neutrosophic feebly irresolute, $f^{-1}(A^c)$ is neutrosophic feebly open set in X and $f^{-1}(A^c) = [f^{-1}(A)]^c$ which implies that is $f^{-1}(A)$ is neutrosophic feebly closed set in X.

Conversely, let B be any neutrosophic feebly open set in Y. Then B^c is neutrosophic feebly closed set in Y. Thus $f^{-1}(B^c)$ is neutrosophic feebly closed set in X and

$f^{-1}(B^c) = [f^{-1}(B)]^c$ which implies that is $f^{-1}(B)$ is neutrosophic feebly open set in X. Hence $f: X \to Y$ is neutrosophic feebly irresolute.

Theorem 3.3. Every neutrosophic feebly irresolute function is neutrosophic feebly continuous.

Proof. Let V be a neutrosophic open set in Y. Since every neutrosophic open set is neutrosophic feebly open, V is neutrosophic feebly open. Since f is neutrosophic feebly irresolute, $f^{-1}(V)$ is neutrosophic feebly open in X. Therefore f is neutrosophic feebly continuous.

Remark 3.4. The converse of above is not true as shown in the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$. Define the neutrosophic sets as follows:

$A = \langle x, (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) \rangle$

$B = \langle x, (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) \rangle$

$C = \langle x, (0.5, 0.4, 0.2), (0.2, 0.3, 0.1), (0.6, 0.9, 0.8) \rangle$ and

$D = \langle x, (0.4, 0.2, 0.5), (0.1, 0.1, 0.2), (0.5, 0.6, 0.8) \rangle$. Now $\tau = \{0_N, A, B, 1_N\}$ and $\sigma = \{0_N, C, D, 1_N\}$ are neutrosophic topologies on X. Thus (X, τ) and (Y, σ) are neutrosophic topological spaces. Also, we define $f: (X, \tau) \to (Y, \sigma)$ as follows: $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is neutrosophic feebly continuous function. But f is not neutrosophic feebly irresolute function. Since $E = \langle x, (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) \rangle$ is a neutrosophic feebly open in (Y, σ), but $f^{-1}(E)$ is not neutrosophic feebly open set in (X, τ).

Theorem 3.6. Let $f: X \to Y$ be a function. Then the following are equivalent:

(i) f isneutrosophic feebly irresolute.

(ii) $NFcl(f^{-1}(B)) \leq f^{-1}(NFcl(B))$ for every neutrosophic set B of Y.

(iii) $f(NFcl(A)) \leq NFcl(f(A))$ for every neutrosophic set A of X.

(iv) $f^{-1}(NFint(B)) \leq NFint(f^{-1}(B))$ for every neutrosophic set B of Y.

Proof. (i) ⇒ (ii): Let B be any neutrosophic set in Y. Then by Lemma 2.9, $NFcl(B)$ is neutrosophic feebly closed in Y. Since f is neutrosophic feebly irresolute,

$f^{-1}(NFcl(B))$ is neutrosophic feebly closed in X. Then $NFcl(f^{-1}(NFcl(B))) = f^{-1}(NFcl(B))$. By Lemma 2.9 (i) and (iv), $NFcl(f^{-1}(B)) \leq NFcl(f^{-1}(NFcl(B))) = f^{-1}(NFcl(B))$. This proves (ii).

(ii) ⇒ (iii): Let A be any neutrosophic set in X. Then $f(A) \leq Y$. By (ii), $NFcl\left(f^{-1}(f(A))\right) \leq f^{-1}\left(NFcl(f(A))\right)$. But $Fcl(A)) \leq NFcl(f^{-1}(f(A))), NFcl(A) \leq f^{-1}(NFcl(f(A)))$. That implies, $f(NFcl(A)) \leq NFcl(f(A))$.

(iii) ⇒ (i): Let F be any neutrosophic feebly closed set in Y. Then $f^{-1}(F) = f^{-1}(NFcl(F))$. By (iii), $f(NFcl(f^{-1}(F))) \leq NFcl(f(f^{-1}(F))) \leq NFcl(F) = F$. That implies, $(NFcl(f^{-1}(F))) \leq f^{-1}(F)$. But $f^{-1}(F) \leq NFcl(f^{-1}(F))$, $NFcl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is neutrosophic feebly closed set X. Therefore f is neutrosophic feebly irresolute.

(i) ⇒ (iv): Let B any neutrosophic set in Y.By Lemma 2.6, $NFint(B)$ is neutrosophic feebly open in Y. Since f is neutrosophic feebly irresolute, $f^{-1}(NFint(B))$ is neutrosophic feebly open in X. Then $f^{-1}(NFint(B))$=$NFint(f^{-1}(NFint(B))) \leq NFint(f^{-1}(B))$.

(iv) ⇒ (i): Let V be any neutrosophic feebly in Y. Then by (iv), $f^{-1}(V)$=$f^{-1}(NFint(V)) \leq NFint(f^{-1}(V))$. But, $NFint(f^{-1}(V)) \leq f^{-1}(V)$, $NFint(f^{-1}(V))$=$f^{-1}(V)$ and by Lemma 2.6(ii), $f^{-1}(V)$ is neutrosophic feebly open. Thus f is neutrosophic feebly irresolute.

Theorem 3.7. If $f: X \to Y$ and $g: Y \to Z$ are neutrosophic feebly irresolute, then their composition $g \circ f: X \to Z$ is also neutrosophic feebly irresolute.

Proof. Let V be a neutrosophic feebly open set in Z. Since $g$is a neutrosophic feebly irresolute function, $g^{-1}(V)$ is neutrosophic feebly open in Y. Since $f$is a neutrosophic feebly irresolute function, $f^{-1}(g^{-1}(V))$= $(g \circ f)^{-1}(V)$ is neutrosophic feebly open in X. Therefore $g \circ f$ is neutrosophic feebly irresolute.

Theorem 3.8. If $f: X \to Y$ is neutrosophic feebly irresolute and $g: Y \to Z$ are neutrosophic feebly continuous then their composition $g \circ f: X \to Z$ is also neutrosophic feebly continuous.

Proof. Let V be a neutrosophic open set in Z. Since $g$is a neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic feebly open in Y. Since f is a neutrosophic feebly irresolute function, $f^{-1}(g^{-1}(V))$ =$(g \circ f)^{-1}(V)$ is neutrosophic feebly open in X. Therefore $g \circ f$ is neutrosophic feebly continuous.

IV.STRONGLY NEUTROSOPHIC FEEBLY CONTINUOUS AND PERFECTLY NEUTROSOPHIC FEEBLY CONTINUOUS

In this section, we introduce the concept of strongly neutrosophic feebly continuous and perfectly neutrosophic feebly continuous functions in neutrosophic topological spaces and we discuss the relation with the above-mentioned functions.

Definition 4.1. Let $(X,\tau)$and (Y,σ) be two neutrosophic topological spaces. A function $f:(X,\tau) \to (Y,\sigma)$ is called strongly neutrosophic feebly continuous if the inverse image of every neutrosophic feebly open set in Y is neutrosophic open in X.

Definition 4.2. Let $(X,\tau)$and (Y,σ) be two neutrosophic topological spaces. A function $f:(X,\tau) \to (Y,\sigma)$ is called a perfectly neutrosophic feebly continuous if the inverse image of every neutrosophic feebly open set in Y is neutrosophic clopen in X.

Theorem 4.3. Let $(X,\tau)$and (Y,σ) be two neutrosophic topological spaces and $f:(X,\tau) \to (Y,\sigma)$ be a function.

(i) If $f$is perfectly neutrosophic feebly continuous, then $f$is perfectly neutrosophic continuous.

(ii) If f is strongly neutrosophic feebly continuous, then f is neutrosophic continuous.

Proof. (i)Let $f: X \to Y$ be perfectly neutrosophic feebly continuous. Let V be a neutrosophic open set in Y. Then by Lemma 2.6, V is neutrosophic feebly open in Y. Since f is perfectly neutrosophic feebly continuous, $f^{-1}(V)$ is neutrosophic clopen in X. Therefore f is perfectly neutrosophic continuous.

(ii) Let $f: X \to Y$ be strongly neutrosophic feebly continuous. Let G be a neutrosophic open set in Y. Then by Lemma 2.6, G is neutrosophic feebly open in Y. Since f is strongly neutrosophic feebly continuous, $f^{-1}(G)$ is neutrosophic open in X. Therefore f is neutrosophic continuous.

Theorem 4.4. Let $f : X \to Y$ be strongly neutrosophic feebly continuous and A be neutrosophic open in X. Then the restriction, $f_A: A \to Y$ is strongly neutrosophic feebly continuous.

Proof. Let V be any neutrosophic feebly open set in Y. Since f is strongly neutrosophic feebly continuous, $f^{-1}(V)$ is neutrosophic open in X. But $f_A^{-1}(V)$=A$\wedge f^{-1}(V)$. Since A and $f^{-1}(V)$ are neutrosophic open, $f_A^{-1}(V)$ is neutrosophic open in X. Hence f_A is strongly neutrosophic feebly continuous.

Theorem 4.5. Every perfectly neutrosophic feebly continuous is strongly neutrosophic feebly continuous.

Proof. Let $f: X \to Y$ be perfectly neutrosophic feebly continuous and V be neutrosophic feebly open in Y. Since f is perfectly neutrosophic feebly continuous, $f^{-1}(V)$ is neutrosophic clopen in X. That is, $f^{-1}(V)$ is both neutrosophic open and neutrosophic closed in X. Hence f is strongly neutrosophic feebly continuous.

Theorem 4.6. If $f: X \to Y$ and $g: Y \to Z$ are strongly neutrosophic feebly continuous, then their composition $g \circ f: X \to Z$ is also strongly neutrosophic feebly continuous.

Proof. Let V be a neutrosophic feebly open set in Z. Since g is a strongly neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic open in Y. Then by Lemma 2.6, $g^{-1}(V)$ is neutrosophic feebly open in X. Since f is a strongly neutrosophic feebly continuous function, $f^{-1}(g^{-1}(V))$=$(g \circ f)^{-1}(V)$ is neutrosophic open in X. Therefore $g \circ f$ is strongly neutrosophic feebly continuous.

Theorem 4.7. If $f: X \to Y$ and $g: Y \to Z$ are perfectly neutrosophic feebly continuous, then their composition $g \circ f: X \to Z$ is also perfectly neutrosophic feebly continuous.

Proof. Let V be a neutrosophic feebly open set in Z. Since g is a perfectly neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic clopen in Y. That is $g^{-1}(V)$ is both neutrosophic open and neutrosophic closed. Then by Lemma 2.6, $g^{-1}(V)$ is neutrosophic feebly open in X. Since f is a perfectly neutrosophic feebly continuous function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is neutrosophic clopen in X. Therefore $g \circ f$ is perfectly neutrosophic feebly continuous.

Theorem 4.8. Let $f: X \to Y$ and $g: Y \to Z$ be functions. Then,

(i) If g is strongly neutrosophic feebly continuous and f is neutrosophic feebly continuous, then $g \circ f$ is neutrosophic feebly irresolute.

(ii) If g is perfectly neutrosophic feebly continuous and f is neutrosophic continuous, then $g \circ f$ is strongly neutrosophic feebly continuous.

(iii) If g is strongly neutrosophic feebly continuous and f is perfectly neutrosophic feebly continuous, then $g \circ f$ is perfectly neutrosophic feebly continuous.

(iv) If g is neutrosophic feebly continuous and f is strongly neutrosophic feebly continuous, then $g \circ f$ is neutrosophic continuous.

Proof. (i)Let V be a neutrosophic feebly open set in Z. Since $g$is a strongly neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic open in Y. Since $f$is a neutrosophic feebly continuous function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is neutrosophic feebly open in X. Hence $g \circ f$ is neutrosophic feebly irresolute.

(ii) Let V be a neutrosophic feebly open set in Z.Since$g$is a perfectly neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic clopen in Y. That is, $g^{-1}(V)$ is both neutrosophic open and neutrosophic closed. Since $f$is a neutrosophic continuous function, $f^{-1}(g^{-1}(V)) =(g \circ f)^{-1}(V)$ is neutrosophic open in X. Therefore $g \circ f$ is strongly neutrosophic feebly continuous.

(iii) Let V be a neutrosophic feebly open set in Z. Since $g$is a strongly neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic open in Y. By Lemma 2.6, $g^{-1}(V)$ is neutrosophic feebly open in X. Since $f$is a perfectly neutrosophic feebly continuous function, $f^{-1}(g^{-1}(V))$ $=(g \circ f)^{-1}(V)$ is neutrosophic clopen in X. Hence $g \circ f$ is perfectly neutrosophic feebly continuous.

(iv) Let V be a neutrosophic open set in Z. Since $g$is a neutrosophic feebly continuous function, $g^{-1}(V)$ is neutrosophic feebly open in Y. Since $f$is a strongly neutrosophic feebly continuous function, $f^{-1}(g^{-1}(V))$ $=(g \circ f)^{-1}(V)$ is neutrosophic open in X. Therefore $g \circ f$ is neutrosophic continuous.

V. NEUTROSOPHIC FEEBLY CLOSED MAP AND NEUTROSOPHIC FEEBLY OPEN MAP

In this section, we introduce neutrosophic feebly closed maps and neutrosophic feebly open maps in neutrosophic topological spaces and obtain certain characterizations of these classes of maps.

Definition 5.1. Let $(X, \tau)$and (Y, σ) be two neutrosophic topological spaces. A function $f: X \rightarrow Y$ is said to be neutrosophic feebly closed if the image of each neutrosophic closed set in X is neutrosophic feebly closed in Y.

Definition 5.2. Let $(X,\tau)$and (Y,σ) be two neutrosophic topological spaces. A function $f: X \to Y$ is said to be neutrosophic feebly open if the image of each neutrosophic open set in X is neutrosophic feebly open in Y.

Theorem 5.3. A function $f: X \to Y$ is neutrosophic feebly closed if and only if $NFcl(f(A)) \leq f(Ncl(A))$ for every neutrosophic set $A$of X.

Proof. Suppose $f: X \to Y$ is a neutrosophic feebly closed function and A is any neutrosophic set in X. Then $Ncl(A)$ is a neutrosophic closed set in X. Since f is neutrosophic feebly closed, $f(Ncl(A))$ is a neutrosophic feebly closed set in Y. Then by Lemma 2.9 (ii), $NFcl(f(Ncl(A))) = f(Ncl(A))$. Therefore $NFcl(f(A)) \leq NFcl\left(f(Ncl(A))\right) = f(Ncl(A))$. Hence $NFcl(f(A)) \leq f(Ncl(A))$.

Conversely, Let A be a neutrosophic closed set in X. Then $Ncl(A) = A$ and so $f(A) = f(Ncl(A))$. By our assumption $NFcl(f(A)) \leq f(A)$. But $f(A) \leq NFcl(f(A))$. Hence $NFcl(f(A)) = f(A)$ and therefore by Lemma 2.9(ii), $f(A)$ is neutrosophic feebly closed in Y. Thus f is a neutrosophic feebly closed map.

Theorem 5.4. A map $f: X \to Y$ is neutrosophic feebly closed if and only if for each neutrosophic set S of Y and for each neutrosophic open set U of X containing $f^{-1}(S)$ there exists a neutrosophic feebly open set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Proof. Suppose f is a neutrosophic feebly closed map. Let S be any neutrosophic set in Y and U be a neutrosophic feebly open set of X such that $f^{-1}(S) \leq U$. Then $V = (f(U^c))^c$ is neutrosophic feebly open set containing S such that $f^{-1}(V) \leq U$. Conversely, Let S be a neutrosophic closed set of X. Then $f^{-1}((f(S))^c) \leq S^c$ and S^c is neutrosophic open in X. By assumption, there exists a neutrosophic feebly open set V of Y such that $(f(S))^c \leq V$ and $f^{-1}(V) \leq S^c$ and so $S \leq (f^{-1}(V))^c$. Hence $V^c \leq f(S) \leq f((f^{-1}(V))^c) \leq V^c$, which implies $f(S) = V^c$. Since V^c is neutrosophic feebly closed, $f(S)$ is neutrosophic feebly closed and f is neutrosophic feebly closed map.

Remark 5.5. The composition of two neutrosophic feebly closed maps need not be a neutrosophic feebly closed map, which is shown in the following example.

Example 5.6. Let $X = \{p, q, r\}$ and $\tau = \{0_N, A, B, C, D, 1_N\}$ be a neutrosophic topology on X, where

$$A = < x, (0.4, 0.3, 0.2), (0.3, 0.4, 0.5), (0.2, 0.3, 0.2) >$$

$$B = < x, (0.2,0.4, 0.6), (0.3, 0.2, 0.1), (0.5., 0.4,0.3) >$$

$$C = < x, (0.4, 0.4, 0.6), (0.3, 0.2, 0.1), (0.2.,0.3,0.2) >$$

$$D = < x, (0.2, 0.3, 0.2), (0.3, 0.4, 0.5), (0.5., 0.4, 0.3) >$$

Let $Y = \{p, q, r\}$ and $\sigma = \{O, E, F, G, H, I_N\}$ be a neutrosophic topology on Y, where

$$E = < y, (0.1, 0.2, 0.3), (0.3, 0.2, 0.3), (0.5, 0.6, 0.4) >$$

$$F = < y, (0.4,0.3,0.2), (0.3,0.4,0.5), (0.2., 0.3,0.3) >$$

$$G = < y, (0.4, 0.3, 0.3), (0.3,0.2,0.3), (0.2.,0.3,0.2) >$$

$$H = < y, (0.1,0.2,0.2), (0.3, 0.5, 0.5), (0.5.,0.6,0.4) >$$

Let $f: X \to Y$ be defined by $f\ (p) = p_1 f(q) = q_1, f(r) = r$

Assume F is neutrosophic open set in Y, then $f^{-1}(F) = A$ is neutrosophic feebly open set in X. Hence f is neutrosophic feebly continuous. Suppose G is neutrosophic feebly closed set in Y. Then $f^{-1}(G)$ is not neutrosophic feebly closed set in X.

Theorem 5.7. Let $f: X \to Y$ be a neutrosophic closed map and $g: Y \to Z$ be a neutrosophic feebly closed map. Then their composition $g \circ f: X \to Z$ is neutrosophic feebly closed.

Proof. Let F be a neutrosophic closed set in X.Since$f$is neutrosophic closed, $f(F)$ is neutrosophic closed in Y. Since g is neutrosophic feebly closed, $g(f(F))=(g \circ f)(F)$ is neutrosophic feebly closed in Z. Hence $g \circ f$ is a neutrosophic feebly closed map.

Theorem 5.8. Let $f: X \to Y$ and $g: Y \to Z$ be two mappings such that their composition $g \circ f: X \to Z$ is neutrosophic feebly closed. Then the followings are true.

(i) If f is neutrosophiccontinuous and surjective, then g is neutrosophic feebly closed.

(ii) If g is neutrosophic feebly irresolute and injective, then f is neutrosophic feebly closed.

Proof. (i)Let A be a neutrosophic closed set of Y. Since f is neutrosophic continuous, $f^{-1}(A)$ is neutrosophic closed in X. Since $g \circ f$ is neutrosophic feebly closed, $(g \circ f)(f^{-1}(A))$ is neutrosophic feebly closed in Z. Since f is surjective, $g(A)$ is neutrosophic feebly closed in Z. Hence g is neutrosophic feebly closed.

(ii) Let B be any neutrosophic closed set of X. Since $g \circ f$ is neutrosophic feebly closed, $(g \circ f)(B)$ is neutrosophic feebly closed in Z. Since $g$is neutrosophic feebly irresolute, $g^{-1}(g \circ f(B))$ is neutrosophic feebly closed in Y. Since $g$is injective, $f(B)$ is neutrosophic feebly closed in Y. Hence f is neutrosophic feebly closed.

Theorem 5.9. Let $f: X \to Y$ be neutrosophic feebly closed.

(i) If A is neutrosophic closed set of X, then the restriction $f_A: A \to Y$ is neutrosophic feebly closed.

(ii) If $A = f^{-1}(B)$ for some neutrosophicclosed set B of Y, then the restriction $f_A : A \to Y$ is neutrosophic feebly closed.

Proof. (i) Let B be any neutrosophic closed set of A. Then $B = A \wedge F$ for some neutrosophic closed set F of X and so B is neutrosophic closed in X. By hypothesis, $f(B)$ is neutrosophic feebly closed in Y. But $f(B)$=$f_A(B)$, therefore f_A is a neutrosophic feebly closed map.

(ii) Let D be a neutrosophic closed set of A. Then $D = A \wedge H$, for some neutrosophic closed set H in X. Now, $f_A(D)$=$f(D)$=$f(A \wedge H)$= $f(f^{-1}(B) \wedge H)$=$B \wedge f(H)$. Since f is neutrosophic feebly closed, $f(H)$ is neutrosophic feebly closed in Y. Hence f_A is a neutrosophic feebly closed map.

Theorem 5.10. A function $f: X \to Y$ is neutrosophic feebly open if and only if $f(Nint(A)) \leq NFint(f(A))$, for every neutrosophic set A of X.

Proof. Suppose $f: X \to Y$ is a neutrosophic feebly open function and A is any neutrosophic set in X.Then$Nint(A)$ is a neutrosophic open set in X. Since f is neutrosophic feebly open, $f(Nint(A))$ is a neutrosophic feebly open set. Since $NFint\left(f\left(Nint(A)\right)\right) \leq NFint\left(f(A)\right)$, $f(Nint(A)) \leq NFint(f(A))$.

Conversely, $f(Nint(A)) \leq NFint(f(A))$ for every neutrosophic set A in X.Let U be a neutrosophic open set in X. Then $Nint(U) = U$ and by hypothesis, $f(U) \leq NFint(f(U))$. But $NFint(f(U)) \leq f(U)$. Therefore, $f(U) = NFint\left(f(U)\right)$. Then by Lemma 2.6(ii), $f(U)$ is neutrosophic feebly open. Hence f is a neutrosophic feebly open map.

Theorem 5.11. Let $f: X \to Y$ be a mapping. Then the following statements are equivalent.

(i) $f$is a neutrosophic feebly open mapping.

(ii) For a subset $A$of X, $f(Nint(A)) \leq NFint(f(A))$.

(iii) For each $x_{(\alpha,\beta,\gamma)} \in X$ and for each neutrosophic neighbourhood$U$ of $x_{(\alpha,\beta,\gamma)}$in X, there exists Aneutrosophic feeblyneighbourhood$W$ of $f(x_{(\alpha,\beta,\gamma)})$ in Y such that $W \leq f(U)$.

Proof. (i) ⇒ (ii): Suppose $f: X \to Y$ is a neutrosophic feebly open function and $A \leq X$. Then $Nint(A)$ is a neutrosophic open set in X. Since f is neutrosophic feebly open, $f(Nint(A))$ is a neutrosophic feebly open set. Since $NFint(f(Nint(A))) \leq NFint(f(A))$, $f(Nint(A)) \leq NFint(f(A))$. This proves (ii).

(ii)⇒ (iii): Let $x_{(\alpha,\beta,\gamma)} \in X$ and U be any arbitrary neutrosophic neighbourhood of $x_{(\alpha,\beta,\gamma)}$ in X. Then there exists a neutrosophic open set G such that $x_{(\alpha,\beta,\gamma)} \in G \leq U$. By (ii), $f(G)$=$f(Nint(G)) \leq NFint(f(G))$. But, $NFint(f(G)) \leq f(G)$. Therefore, $NFint(f(G)) = f(G)$ and hence $f(G)$ is neutrosophic feebly open in Y.Since $x_{(\alpha,\beta,\gamma)} \in G \leq U$, $f(x_{(\alpha,\beta,\gamma)}) \in f(G) \leq f(U)$ and so (iii) holds, by taking $W = f(G)$.

(iii) ⇒(i): Let U be any neutrosophic open set in X. Let $x_{(\alpha,\beta,\gamma)} \in U$ and $f(x_{(\alpha,\beta,\gamma)}) = y_{(r,t,s)}$. Then for each $x_{(\alpha,\beta,\gamma)} \in U$, $y \in f(U)$, by assumption there exists a neutrosophic feebly neighbourhood $W(y_{(r,t,s)})$ of $y_{(r,t,s)}$ in Y such that $W(y_{(r,t,s)}) \leq f(U)$. Since $W(y_{(r,t,s)})$ is a neutrosophic feebly neighbourhood of $y_{(r,t,s)}$, there exists a neutrosophic feebly open set $V(y_{(r,t,s)})$ in Y such that

$y_{(r,t,s)} \in V(y_{(r,t,s)}) \leq W(y_{(r,t,s)})$. Therefore, $f(U)$=$\vee\{V(y_{(r,t,s)})/y_{(r,t,s)} \in f(U)\}$. Since the union of neutrosophic feebly open sets is neutrosophic feebly open, $f(U)$ is a neutrosophic feebly open set in Y. Thus, f is a neutrosophic feebly open map.

Theorem 5.12. For any bijective map $f: X \to Y$ the following statements are equivalent:

(i) f^{-1}:$Y \to X$ is neutrosophic feebly continuous.

(ii) $f$isneutrosophic feebly open.

(iii) $f$isneutrosophic feebly closed.

Proof. (i) → (ii): Let U be a neutrosophic open set in X. By assumption, $(f^{-1})^{-1}(U) = f(U)$ is neutrosophic feebly open in $Y$and so f is neutrosophic feebly open.

(ii) → (iii): Let F be a neutrosophic closed set of X. Then $F^c$is a neutrosophic open set in X. By assumption $f(F^c)$ is neutrosophic feebly open in Y. But $f(F^c) = (f(F))^c$ .Therefore $f(F)$ is neutrosophic feebly closed in Y. Hence, f is almost neutrosophic feebly closed.

(iii) → (i): Let F be a neutrosophic feebly closed set of X. By assumption, $f(F)$ is neutrosophic feebly closed set in Y. But $f(F)$=$(f^{-1})^{-1}(F)$ and therefore by Definition 2.14, f^{-1} is neutrosophic feebly continuous.

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