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Determination of Hydrodynamic Forces in Rectangular Storage Tanks under Seismic Action

Michel Faridkhouri[1]* and Wassim Joseph Elias [2]

[1] Department of Civil Engineering, Branch II, Lebanese University, Roumieh, Lebanon
[2] Research Engineer at Optimal Engineering Consulting & Contracting, Roumieh, Lebanon

ABSTRACT

In designing reinforced concrete rectangular storage tanks, it is important to evaluate the hydrodynamic fluid pressures generated during earthquakes. Codes and investigators suggested procedures to evaluate hydrodynamic effects on rectangular tank structures; the complications of these procedures and the factors involved make them difficult to use. This article provides a simplified method to solve wall forces in rectangular storage tanks subjected to earthquake loading. Comparing the suggested procedure with Housner's and ACI methods, it is observed that the results of these equations are accurate and reliable. Consequently, a step-by-step procedure that is very useful for structural engineers and designers is presented to evaluate wall forces.

KEYWORDS: Rectangular tanks, convective component, impulsive component, liquid-containing storage tanks, zone acceleration factor.

*Corresponding Author:

Michel Farid Khouri
Department of Civil Engineering, Branch II,
 Lebanese University, Roumieh, Lebanon
Email: mkhuri@ul.edu.lb, wassim.elias@optimal-eng.com

1. INTRODUCTION

To calculate the hydrodynamic forces in liquid-containing tanks during lateral seismic excitation, a reduced model is used that is composed of a spring-mass system shown in Figure 1 which simulates the impulsive and convective modes of vibration of a tank-fluid system.
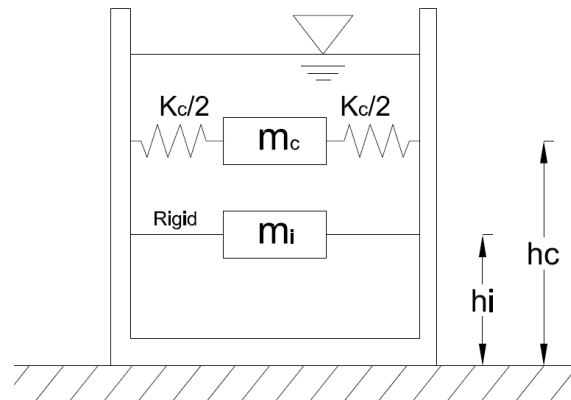

Figure *1*.Spring-mass Model.

Under earthquake loading, tank walls and liquid are subjected to horizontal acceleration. In the lower region of the tank, the liquid behaves like a mass that is rigidly connected to the tank walls. This mass is termed as *impulsive liquid* mass which accelerates along with the wall and induces impulsive hydrodynamic pressure on the tank walls and similarly on the base[1]. In the upper region of the tank, the liquid mass undergoes sloshing motion. This mass is termed as *convective liquid* mass and exerts convective hydrodynamic pressure on the tank walls and base[1].

Consequently, the total liquid mass gets divided into two parts, impulsive mass and convective mass. In the spring-mass model of the tank-liquid system presented in Figure 1, these two liquid masses are to be properly represented. The dynamic movement induces impulsive component where the mass generates pressure and suction; these effects will produce the impulsive force P_i applied at height h_i from the bottom of the tank, whereas the convective pressures on the tank walls will induce the convective force P_C applied at height h_C.

The main parameters that affect the response of the liquid containing concrete rectangular tanks arepresented in Figures 2.a &2.b; these are:

B = Width of the rectangulartank,

L = Half Length of the rectangulartank and

H = Height of the water in the tank.

Notice that B is less than L, and if L is less then B a permutation procedure between L and B is adopted in all the equations.
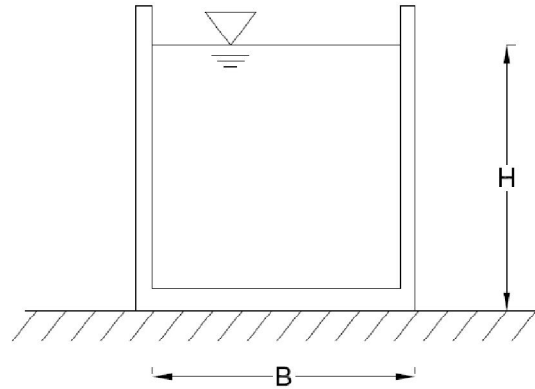

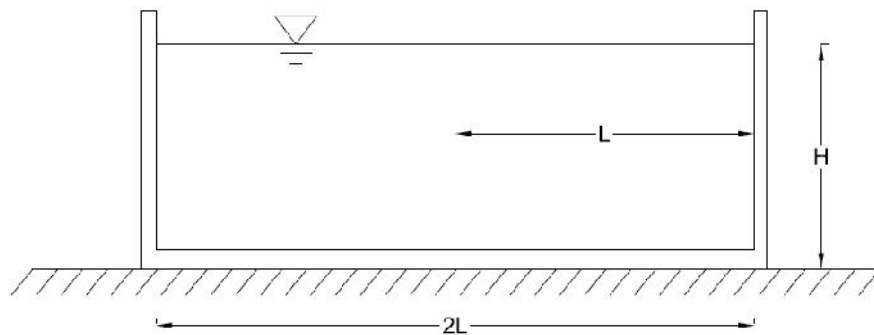Figure *2*.a Geometry of the Rectangular Tank.

Figure *2*.b Geometry of the Rectangular Tank.

Usually, when (L/H) is greater than 0.66,the tank is considered a shallow tanks. If (L/H) is smallerthan 0.66 the tank is considered a deep tank. The authors have treated a similar problem for shallow circular storage tanks[2]. On the other hand, for deep tank structures that contain solid material, a different concept is applied and the tank is treated like a silo[3].

2. BACKGROUND

Hydrodynamic forces in liquid storage tanks have been considered by various investigators. Housner et al.[4] have developed a system of equations that can be used to determine these forces. Also, a report by NSF Report[5]presents the earthquake design criteria.

In addition, ACI 350-01[1] presents a seismic design of liquid containing concrete structures which contain the seismic zone factor in addition to many other factors that affect the design.The parameters of the spring mass model depend on tank geometry and were originally derived by Housner[6], and used in the ACI code[1].

Similar to ACI[1], a comprehensive study was made by NZC 3106[7]; it is based on assuming that the tank is rigid. Another method generated by TID 7024[8]; it is based on an average velocity response spectrum.

The above methods are a bit complicated to apply. They suggest equations that require various data and constants that may be complicated to get and/or apply.

There are many other dimensions to the problem where many investigators have worked in this area such as Zhou et al.[9] who studied the effect of impact loading on liquid tanks as compared to seismic loading. Also, Sezena et al.[10] studied the supporting system using a simplified three-mass model along with a finite element model. Chen et al.[11] on the other hand, proposed a model that considers the effect of the flexibility of tank walls on hydrodynamic forces, and Ghammaghami et al.[12]used the finite element method to study the effect of wall flexibility on the dynamic response using the scaling of El-Centro earthquake, while Cheng et al.[13]derived dynamic fluid pressure formulas based on the velocity potential function of liquid movement.

The objective of this work is to make use of the above models and codes to generate a sequential system of equations that estimate forces and components needed in the design of rectangular tank walls; these components are then used to come up with a simplified design method that can be used by engineers and designers.

3. ANALYSIS

There are various important factors that need to be determined when performing the analysis. These factors are the impulsive and convective components, frequency of liquid oscillation, wave amplitude, base shear and moments. These are determined in the following sections.

3.1 Impulsive component

As far as the vibration of the tank walls,our interest is with the first mode which is activated by the impulsive mass of the liquid and the geometry of the tank.

The circular frequency of the impulsive mode of vibration ω_i is calculated using equation (1),

$$\omega_i = \sqrt{\frac{k}{m}} \tag{1}$$

where the stiffness k and mass m are given by:

$$k = \frac{E_c}{48} \times \left(\frac{t_w}{h}\right)^3 \tag{2}$$

$$m = m_w + m_i \tag{3}$$

E_c= modulus of elasticity of concrete (MPA),

t_w= average wall thickness (mm),

h = height of the wall (mm),

m_w= mass of the tank wall (T), and

m_i = impulsive mass of contained liquid of a circular tank (T).

It is assumed that the tank walls are rigid and it is important to note that wall flexibility does not affect convective pressure distribution, but can have substantial influence on impulsive pressure distribution in tall tanks.

3.1.1 Equivalent mass

The equation that relates m_i/m_L to B.L/H is given by:

$$\frac{m_i}{m_L} = 1.1487. \, e^{-0.124.(\frac{B.L}{H})} \quad \text{where,} \tag{4}$$

m_L= Total mass of the stored liquid, and

m_i= Equivalent mass of the impulsive component stored liquid.

The graph relating m_i/m_L to BxL/H is presented in Figure 3. Notice that the effect of the impulsive seismic mass decreases as BxL/H increases. This is due to the fact that when the tank becomes deeper (BxL/Hdecreases) the fluid in the tank behaves like it is rigidly connected to the tank wall so the fluid mass m_i moves with the tank walls as they respond to the ground motion and the impulsive mass controls the behavior.
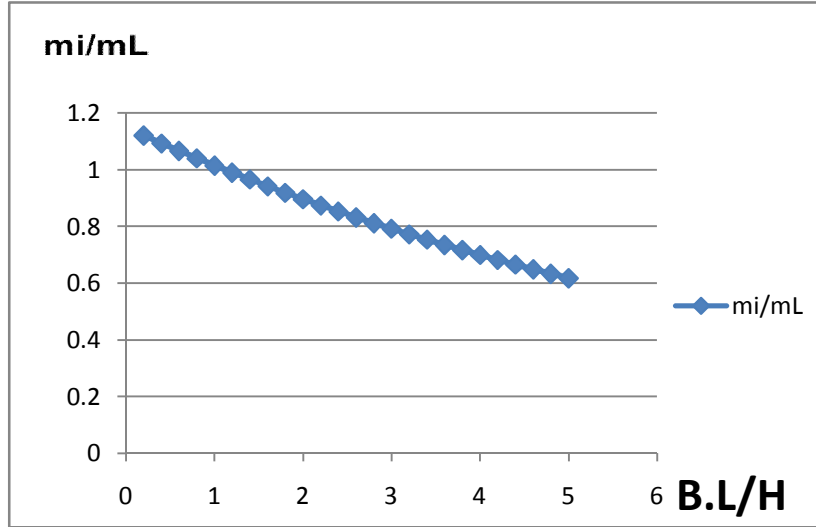
Figure 3. Equivalent mass of the impulsive component stored liquid versus BxL/H.

3.1.2 Load application level

For the load application levels, formulas (5) and (6) give hi/H with Excluding Base Pressure (EBP) and Including Base Pressure (IBP) as a function of L/H, respectively.

These equationsare:

$$\frac{h_i}{H}(EBP) = 0.4951. e^{-0.379.(\frac{L}{H})} \tag{5}$$

$$\frac{h_i}{H}(IBP) = 0.6976.\left(\frac{L}{H}\right)^2 - 0.334.\left(\frac{L}{H}\right) + 0.486 \quad \text{5)}$$

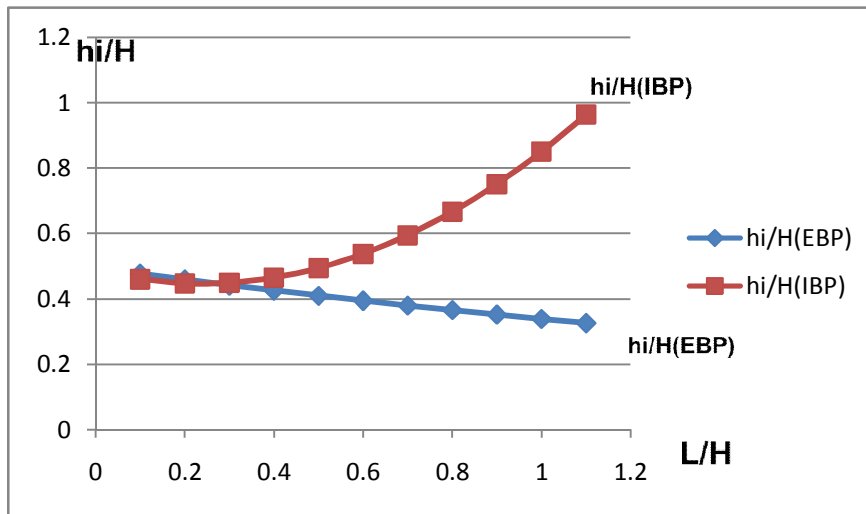*Figure 4.* Height of the impulsive seismic forces versus L/H with H = 1m&a_z= 0.3.

In Figure 4, an example is presented for H=1m and zone acceleration factora_z=0.3 with variable L shows the application level of the impulsive component including and excluding base pressure. It increases with L/H for (IBP) and decreases somewhat for (EBP) due to the reduction of the impulsive mass relative to the static fluid mass. It is more critical to design the tank wall when excluding base pressure (EBP).

On the other hand, including base pressure is very significant for the base and foundation design because it will change the location of the impulsive resultant force and tends to increase with increasing L/H as presented in Figure 4.

3.1.3 Lateral pressure

During an earthquake, the force $P_i$applied at $h_i$changes direction several times per second, corresponding to the change in direction of the base acceleration, this causes the overturning moment generated by P_i to be basically ineffective in tending to overturn the tank.

The equation that relates P_i to LxB and H is given in Tons by:

$$P_i = a_z . \frac{10}{3} . [\alpha . \ln(L.B) + \beta] \tag{7}$$

P_i = Total lateral impulsive force associated with m_i

a_z= Zone acceleration factor,Sezena, H., Livaoglu. et al.

α and β are coefficients that depend on B & H;they are calculated using equations 8 and 9.

$$\alpha = \alpha_1 . e^{\alpha_2 . H} \tag{8}$$

$$\beta = \beta_1 . H^2 + \beta_2 . H + \beta_3 \tag{9}$$

Where the coefficients α_1, α_2, β_1, β_2 and $\beta_3$are calculated using equation (10).

$$\alpha_1 = 0.0792 . B^2 - 0.3848 . B + 1.2462$$

$$\alpha_2 = 0.0069 . B^3 - 0.0967 . B^2 + 0.4026 . B + 0.0271$$

$$\beta_1 = -0.4773 . B^3 + 5.6997 . B^2 - 22.423 . B + 24.244$$

$$\beta_2 = 2.0378 . B^4 - 28.197 . B^3 + 134.56 . B^2 - 248.64 . B + 159.84 \tag{10}$$

$$\beta_3 = -4.3741 . B^4 + 60.575 . B^3 - 290.62 . B^2 + 546.69 . B - 353.05$$

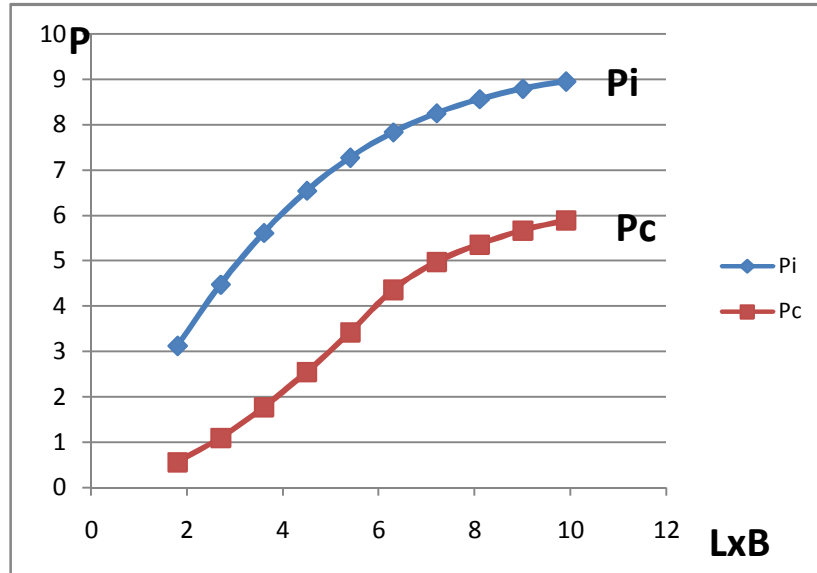*Figure 5*. Total impulsive and convective forces associated with m_i versus L x B with H = 3m&a_z=0.3.

Figure 5 relates P to LxBwhere the zone acceleration factor istaken equal to 0.3 and H=3m; notice that the impulsive force and the convective force increases with LxB and the latter curves up higher than the former which significantly influences the design.

3.2 Convective Component

3.2.1 Circular frequency

The circular frequency$\omega_c$of the first convective mode in rad/s of the spring-mass system shown in Figure 1 is given by:

$$\omega_c = a.\ln\left(\frac{L}{H}\right) + b \qquad (11)$$

The coefficients a and b of equation (11) are calculated using equation (12).

$$a = -0.0399.H^3 + 0.5943.H^2 - 2.6395.H + 1.8987$$

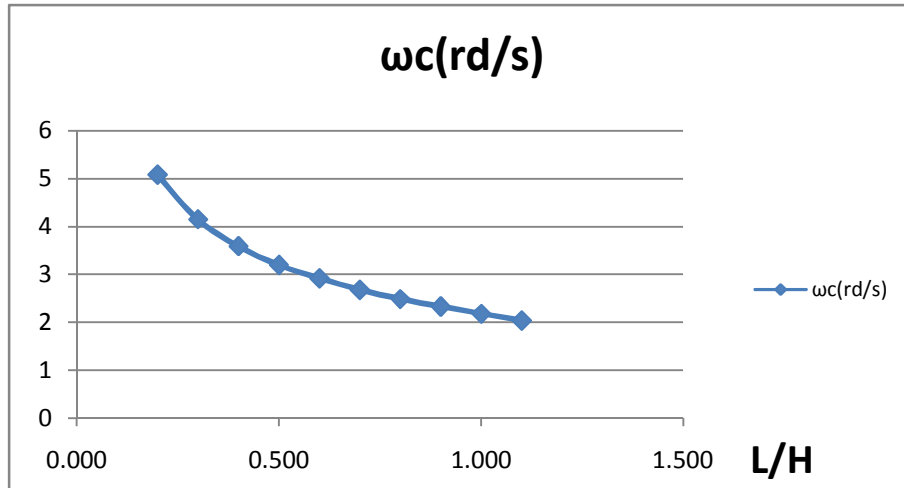$$b = -0.0182.H^3 + 0.306.H^2 - 1.8063.H + 5.2591 \qquad (12)$$

*Figure 6.*Circular frequency of oscillation of the first convective mode versus L/H.

Figure 6shows thatω_c decreases as L/H increases so that the wave oscillation tends to be slower as L/H increases.

3.2.2 Equivalent mass

The convective pressures on the tank walls with resultant force P_c that acts at a height h_c above the tank bottom is produced by the equivalent mass $m_C$of the oscillating fluid. In the model, the force P_c fluctuates sinusoidally with a period of vibration that depends on the dimensions of the tank and can be several seconds or longer. The mass m_c is considered to be fastened to the tank walls by springs that produce a period of vibration corresponding to the period of the vibrating liquid as presented in Figure 1. The equation that relates m_c/m_L to R/H is given by:

$$\frac{m_c}{m_L} = 0.2503 . \ln\left(\frac{B.L}{H}\right) + 0.0167 \tag{13}$$

A graph relating m_C/m_L to BxL/H is presented in Figure 7. Notice the convective seismic mass increases with BxL/H opposite to what is shown in Figure 3 where the impulsive seismic mass decreases with BxL/H. This is due to the geometry of the tank, when the tank becomes shallower (BxL/H increases), the fluid in the tank becomes less connected to the tank walls so the fluid will oscillate easily and the convective mass controls the forces.
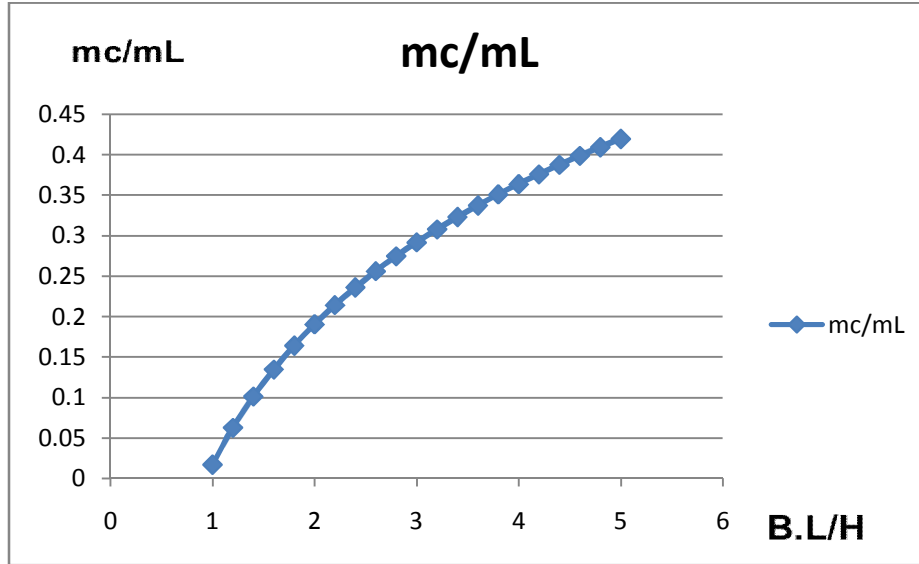
Figure 7. Equivalent mass of the convective component stored liquid versus B.L/H.

3.2.3 Load application level

The equations that relate h_c/H to (L/H) are:

$$\frac{h_c}{H}(EBP) = 0.5886 \cdot \left(\frac{L}{H}\right)^{-0.257}$$

(14)

$$\frac{h_c}{H}(IBP) = 0.9012 \cdot \left(\frac{L}{H}\right)^2 - 1.0908 \cdot \left(\frac{L}{H}\right) + 1.0576$$
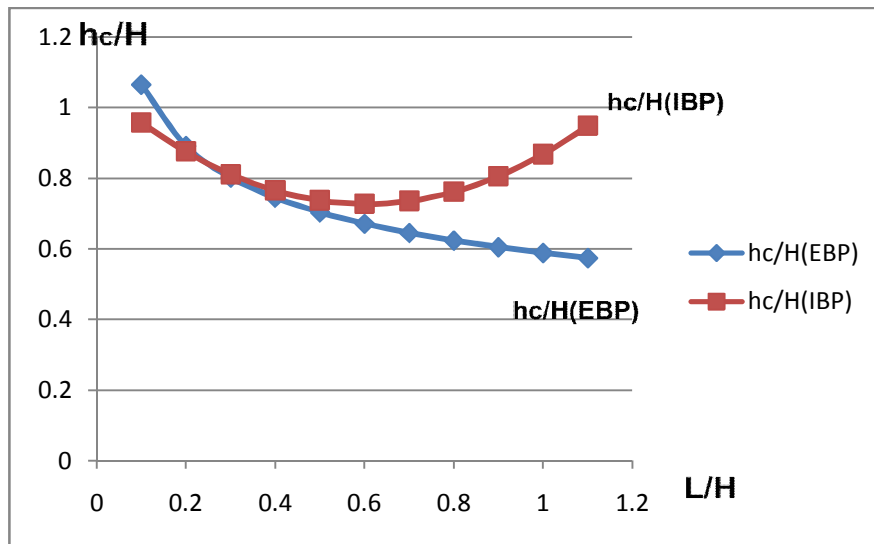
(15)

*Figure 8.*Height of the applied convective seismic forces versus L/H.

Figure 8 shows the application level of the impulsive component including and excluding the base pressure.Notice that h_c/H(EBP) decreases with L/H while h_c/H(IBP) behaves in a parabolic manner decreasing with L/H (minimum about L/H=0.66) and then increasing as L/H increases. This is because when including base pressure, the rectangular tank is considered as a rigid and stiff structure when L/H is too small so that the impulsive mass mi controls the design while the convective mass generates a bending moment on the rectangular tank walls different to the behavior seen in circular walls tank, Khouri, M. F. et al, where the internal circular walls forces are axial loads; that is why hc/H curve decreases while L/H increases. Notice that when the value of L/H becomes greater than 0.66, the height h_c of the convective force begins to increase while L/H increases as a result of the oscillating convective mass mc.

3.2.4 Lateral pressure

The total lateral convective force$P_c$that is associated with $m_c$can be represented as a function of LxB and H as follows:

$$P_c = a_z \cdot \frac{10}{3} \cdot [\gamma \cdot \ln(L \cdot B) + \delta] \tag{16}$$

where P_c is in Tons, and the coefficients γ and δthat depend on H and B are given as follows:

$$\gamma = \gamma_1 \cdot H^{\gamma_2} \tag{17}$$

$$\delta = \delta_1 \cdot H + \delta_2 \tag{18}$$

The coefficients $\gamma_1, \gamma_2, \delta_1$ and $\delta_2$are calculated using the following equations:

$$\gamma_1 = -0.0489 \cdot B^3 + 0.5968 \cdot B^2 - 1.759 \cdot B + 2.4587$$

$$\gamma_2 = -0.0509 \cdot B^4 + 0.8634 \cdot B^3 - 5.2655 \cdot B^2 + 13.521 \cdot B - 11.26$$

$$\delta_1 = 0.1112 \cdot B^4 - 1.8801 \cdot B^3 + 11.386 \cdot B^2 - 29.786 \cdot B + 26.514$$

$$\delta_2 = -0.3075 \cdot B^4 + 5.169 \cdot B^3 - 31.229 \cdot B^2 + 79.894 \cdot B - 69.421$$

$$\tag{19}$$

In plotting P versus BxLfor the convective force, a pattern similar to the pattern shown in Figure 5 can be observed.

3.3 Wave oscillation

The contained fluid vibrates to a maximum amplitude d_{max} above the rest fluid flat surface level when subjected to horizontal earthquake acceleration. In order not to allow any over spills during earthquakes,this amplitude should stay below the unfilled free height of the tank.
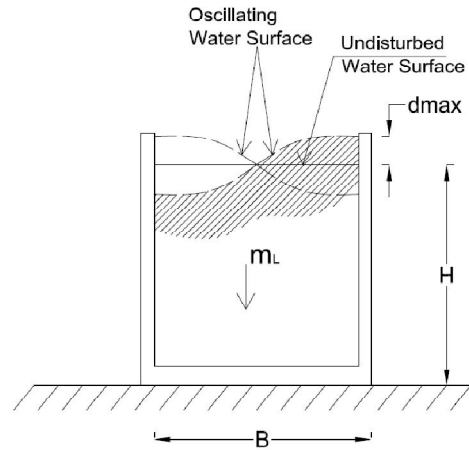

Figure 9.Maximum wave amplitude.

The maximum wave amplitude d_{max} shown in Figure 9 due to the earthquake acceleration is calculated in meter using the following equation:

$$d_{max} = a_z \cdot \left(\frac{10}{3}\right) \cdot \left\lceil c \cdot \left(\frac{L}{H}\right)^2 + d \cdot \left(\frac{L}{H}\right) + e \right\rceil \tag{20}$$

The coefficients c,d and e are calculated using the following equations:

$$c = -0.0133.H^3 + 0.2471.H^2 - 1.3458.H + 1.2819$$

$$d = 0.0092.H^3 - 0.1941.H^2 + 1.0198.H - 0.0137$$

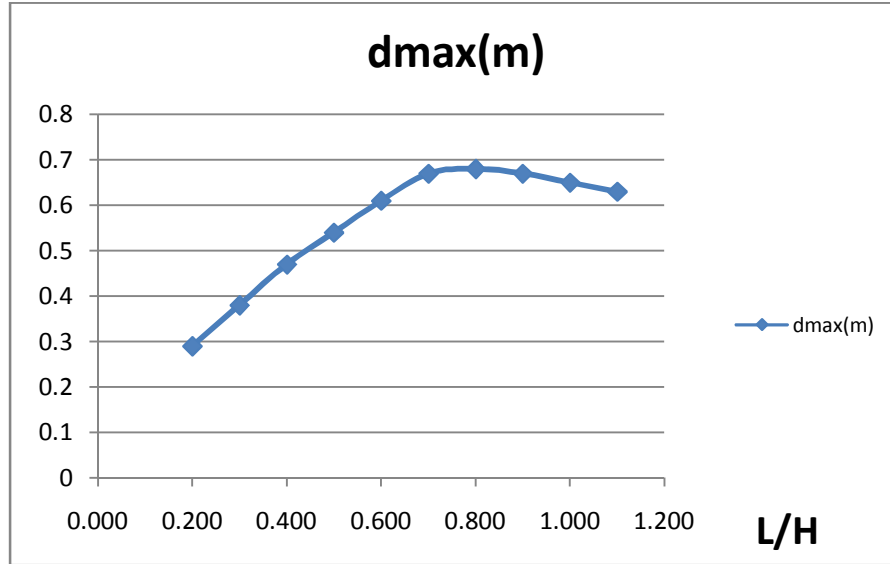$$e = 0.0039.H^3 - 0.0507.H^2 + 0.3129.H - 0.5855 \tag{21}$$

Figure 10.Maximum wave amplitude in terms of (L/H).

A graph relating d_{max} to L/H is presented in Figure 10, where the wave oscillation increases as L/H increases from 0.2 to 0.80; when L/H passes the value of 0.80 the wave oscillation decreases due to the fact that as the length of the tank increases, the wave is dampened due to the longer travel distance which will decrease the amplitude of the oscillating wave.

3.4 Base shear and moment

The base shear V of the seismic forces, due to liquid effects applied at the bottom of the tank wall, shall be determined by combining the base shear in impulsive and convective modes by Square Root of Sum of Squares (SRSS) and is given in Tons as follows:

$$V = \sqrt{V_i^2 + V_c^2} \tag{22}$$

V_i = P_i = Base shear in impulsive mode   (23)

V_c = P_c = Base shear in convective mode   (24)

The base bending moment on the entire tank cross section M just above the base of the tank wall excluding base pressure shall be determined by combining the base moment in impulsive and convective mode; it is given in Ton-meter as follows:

$$M = \sqrt{M_i^2 + M_c^2} \tag{25}$$

$$M_i = P_i \times h_i(EBP) \tag{26}$$

$$M_c = P_c \times h_c(EBP) \tag{27}$$

The overturning moment at the base of the tank M* just above the base of the tank wall shall be determined by combining the base moment in impulsive and convective modes (IBP) is given in Ton-meter as follows:

$$M^* = \sqrt{M_i^{*2} + M_c^{*2}} \tag{28}$$

$$M_i^* = P_i \times h_i(IBP) \tag{29}$$
$$M_c^* = P_c \times h_c(IBP) \tag{30}$$

4. DESIGN PROCEDURE

Given rectangular storage tank with L/H ratio the tank is to contain a certain fluid volume along with a zone acceleration factor a_z, a design procedure would look as follow:

Step 1: Find the impulsive force P_i and the corresponding application positions (EBP) and (IBP) from the eq (7), eq (5) and eq(6) respectively after finding the coefficients from the eq (10), eq (8) and eq (9) respectively.

Step 2: Find the convective force P_c and the corresponding application positions (EBP) and (IBP) from the eq (16), eq (14) and eq (15) respectively after finding the coefficients from the eq (19), eq (17) and eq (18) respectively.

Step 3: To design the tank wall against the bending moment, find the base moments in impulsive and convective modes (EBP) from eq (26) and eq (27) respectively, and then calculate the base bending moment M just above the base of the tank wall using eq (25).

Step 4: Find the base moments in impulsive and convective modes (IBP) from eq (29) and eq (30) respectively, and then calculate the overturning moment M* just above the base of the tank using eq (28).

Step 5: To calculate the base shear, find the shear in impulsive and convective modes from eq (23) and eq (24) respectively, and then calculate the base shear due to seismic forces using eq (22).

Step 6: To determine the required top unfilled free height of the tank wall, find the maximum wave

amplitude for a given site zone acceleration factor as, use eq (20) after finding the coefficients from the eq (21).

Notice that the force pressures applied on dimension B of the rectangular tank walls perpendicular to the direction of the earthquake are calculated using the above design procedure, while the force pressures applied on dimension L are calculated using the same design procedure with permuting B with L and vice versa during the calculations.

5. COMPARISON

Table 1.Comparison of the results obtained from the suggested equations with ACI and Housner.

Dimensions	Codes	Housner	ACI 350-01	Khouri& Elias
B = 3 m H = 4 m L = 4 m a_z = 0.2	Impulsive Force	Pi = 10.76 (T)	Pi = 10.40 (T)	Pi = 8.36 (T)
	Convective Force	Pc = 4.59 (T)	Pc = 5.04 (T)	Pc = 6.32 (T)
	Convective Circular Frequency	ω_c= 2.05 (rd/s)	ω_c= 1.89 (rd/s)	ω_c= 1.77 (rd/s)
	Maximum wave amplitude	d_{max}= 1.13 (m)	d_{max}= 0.43 (m)	d_{max}= 0.65 (m)
B = 4 m H = 4 m L = 3 m a_z = 0.3	Impulsive Force	Pi = 16.15 (T)	Pi = 15.61 (T)	Pi = 12.54 (T)
	Convective Force	Pc = 7.45 (T)	Pc = 7.56 (T)	Pc = 9.48 (T)
	Convective Circular Frequency	ω_c= 2.05 (rd/s)	ω_c= 1.89 (rd/s)	ω_c= 1.77 (rd/s)
	Maximum wave amplitude	d_{max}= 1.13 (m)	d_{max}= 0.65 (m)	d_{max}= 0.98 (m)
B = 4 m H = 6 m L = 8 m a_z = 0.2	Impulsive Force	Pi = 33.80 (T)	Pi = 27.25 (T)	Pi = 22.40 (T)
	Convective Force	Pc = 8.58 (T)	Pc = 7.69 (T)	Pc = 8.51 (T)
	Convective Circular Frequency	ω_c= 1.61 (rd/s)	ω_c= 1.27 (rd/s)	ω_c= 1.17 (rd/s)
	Maximum wave amplitude	d_{max}= 0.706 (m)	d_{max}= 0.39 (m)	d_{max}= 0.28 (m)

Table 1 shows an example that compares the values of the impulsiveseismic force, convective seismic force,convective circular frequency and maximum wave amplitudeusing different codes. Various dimensions for shallow tanks and zone acceleration factors are analyzed using the suggested equations in this article and compared to the method presented by ACI and Housner. It can be observed that the valuesare very close for similar tank dimensions and Zone acceleration factor.

It is important to realize the simplicity of the application of the equations as well as the design procedure which can be used by engineers and designers of liquid containing storage tanks.

6. CONCLUSION

In analyzing rectangular tanks, it is important to observe the length to height ratio (L/H). For L/H<0.66 the rectangular tank behaves like deep tank that vibrates like a cantilever beam with impulsive components controlling the movement taking into consideration that B is always less than L.

On the other hand, if L/H>0.66, the tank is treated as shallow tank and controlled by convective components action.

The impulsive pressures are generated by the seismic accelerations of the tank walls so that the force $P_i$is applied as a pressure force on the wall accelerating the fluid into the wall, and a suction force accelerating the fluid away from the wall. During an earthquake, the force P_i changes direction several times per second, corresponding to a change in the direction of the base acceleration; this causes the overturning moment generated by P_i to be basically in-effective in tending to overturn the tank.

In evaluating the equations presented above for rectangular tanks, the behavior can be summarized by the fact that seismic forces dominate the design of liquid tanks. Also seismic forces are affected by two main components which are the dimensions of the tank and the zone acceleration factor a_z.

Moreover, the results obtained by the suggested equations match very well with those obtained using Housner and ACI350 in any seismic zone.

Finally, the simplicity of the equations in this design procedure make them very easy to use or program by engineers and designers who are working in this field.

ACKOWLEDGEMENTS

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