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K-Contra Harmonic Mean Labeling of Some Graphs

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ABSTRACT

Let G be a (p, q) graph. A function f is called a k-contra harmonic mean labelling of a graph G if f:V (G) → {k, k+1, k+2, ..., k + q} in such a way that the function

$f^*: E(G) \rightarrow \{k, k+1, k+2, \ldots, k+q-1\}$defined as,

$f^*(e = uv) = \left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$edge labels. The graph which admits k-contra harmonic mean labelling is called k-Contra harmonic mean graph.

KEYWORDS :k-Contra Harmonic mean labeling, K-Contra Harmonic mean graphs, Path, Cycle, Comb, etc.

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1. INTRODUCTION

By a graph *G* = (*V* (*G*), *E* (*G*)) with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [2].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in [1]. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

All graphs in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harary [2]. S. Somasundaram and R. Ponraj introduced mean labeling for some standard graphs in 2013. S.S. Sandhya and S. Somasundaram introduced Harmonic mean labeling of graph. S. S. Sandhya, S. Somasundaram and J. Rajeshni Golda introduced Contra Harmonic mean labeling of graphs[9].

We have introduced K- Contra Harmonic mean labeling.In this paper we investigate the k-Contra Harmonic mean labeling behaviour of some special graphs. The following definitions are useful for our present study.

Definition 1.1 Let G be a (p, q) graph. A function f is called a k-contra harmonic mean labelling of a graph G if $f: V(G) \rightarrow \{k, k+1, k+2, \ldots, k+q\}$ in such a way that the function$f^*: E(G) \rightarrow \{k, k+1, k+2, \ldots, k+q-1\}$ defined as

$f^*(e = uv) = \left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$with distinct edge labels. The graph which admits k-contra harmonic mean labeling is called k-contra harmonic mean graph.

Definition 1.2 The union of two graphs $G1=(V_1,E_1)$ and $G2=(V_2,E_2)$ is a graph $G=G_1 \cup G_2$ with vertex set $V= V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.

Definition 1.3 The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th}vertex of G_1 is adjacent to every vertex in the i^{th}copy of G_2

Definition 1.4A Triangular ladder $TL_n, n \geq 2$is a graph obtained from a ladder $L_n$by addingthe edges $u_i v_{i+1}$, for $1 \leq i \leq n-1$ whereu_i and v_i for$1 \leq i \leq n$, are the vertices of L_nSuchthat $u_1, u_2, \ldots, u_n$and $v_1, v_2, \ldots, v_n$are two paths of length n inL_n.

Definition 1.5 An (m, n) kite graph consists of cycle of length m with nedges path attached to one vertex of a cycle.

Definition 1.6 Comb is a graph obtained by joining a single pendant edge to each vertex of a path.

2.MAIN RESULTS

Theorem 2.1. The path P_n is a k-contra harmonic mean graph for all k and $n \geq 2$.

Proof: Let $V(P_n) = \{v_i \backslash 1 \leq i \leq n\}$ and $E(P_n) = \{e = v_i v_{i+1} \backslash 1 \leq i \leq n-1\}$

Define a function $f : V(G) \to \{k, k+1, k+2, \dots, k+q\}$ by

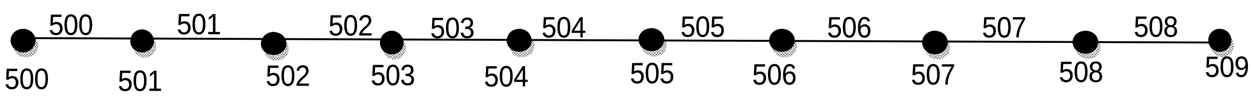
$f(v_i) = k + i - 1, 1 \leq i \leq n$

Then the induced edge labels are

$f^*(e_i) = k + i - 1, 1 \leq i \leq n - 1$

The above defined function f provides k- contra harmonic mean labeling of the graph. Hence P_n is a k- contra harmonic mean graph.

Example 2.2



500-harmonic mean labeling of P_{10}

Theorem 2.3 The cycle graph C_n is a k-contra harmonic mean graph.

Proof: Let $u_1, u_2, \dots, u_n, u_1$ be the given cycle of length n.

Define a function $f : V(G) \to \{k, k+1, k+2, \dots, k+q\}$ by

$f(u_i) = k + i - 1, for\ 1 \leq i \leq n-1,$

$f(u_i) = k + q, \quad for\ i = n.$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + i - 1, for\ 1 \leq i \leq n-2$

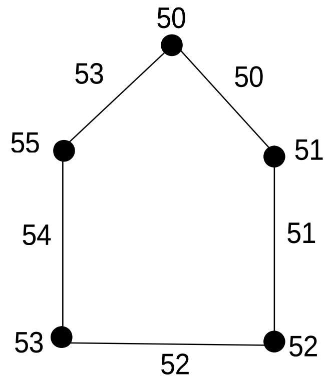
$f^*(u_i u_{i+1}) = k + q - 1, for\ i = n - 1$

$f^*(u_1 u_i) = k + q - 2, for\ i = n$

The above defined function *f* provides k- contra harmonic mean labeling of the graph.

Hence C_n is a k- contra harmonic mean graph.

Example 2.3



50-contra harmonic mean labeling of C_5

Theorem 2.4 The Triangular ladder TL_n is k- contra harmonic mean graph for all k and n ≥ 2.

Proof: Let $V\ (TL_n) = \{ u_i, v_i \setminus 1 \le i \le n\}$ and

$E\ (TL_n) = \{ u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} \setminus 1 \le i \le n-1\} \cup \{ u_i v_i \setminus 1 \le i \le n\}$.

First we label the vertices as follows

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, k+q\}$ by

$f(u_i) = k + 4i - 3, for\ 1 \le i \le n$

$f(v_1) = k$

$f(v_i) = k + 4i - 5, for\ 2 \le i \le n$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + 4i - 1, for\ 1 \le i \le n - 1$

$f^*(v_i v_{i+1}) = k + 4i - 3, for\ 1 \le i \le n - 1$

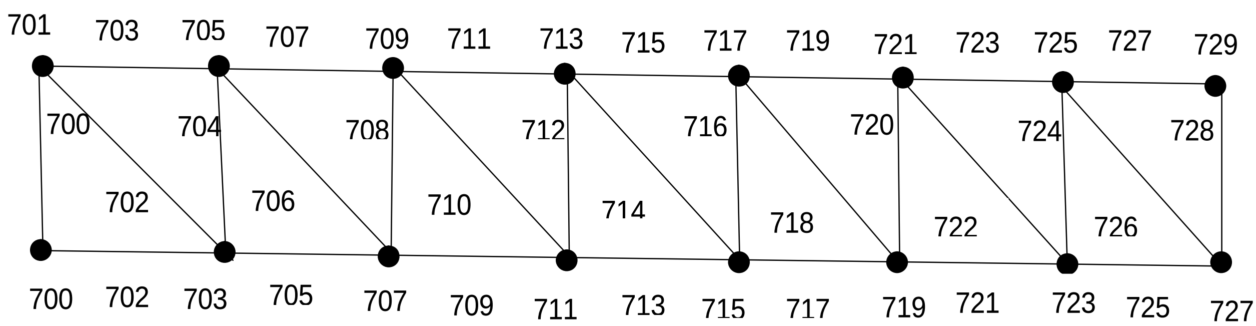
$f^*(u_i v_i) = k + 4i - 4, for\ 1 \le i \le n$

$f^*(u_i v_{i+1}) = k + 4i - 2, for\ 1 \le i \le n - 1$

The above defined function *f* provides k- contra harmonic mean labeling of the graph.

Hence TL_n is a k- contra harmonic mean graph.

Example: 2.4



700 -Contra harmonic mean labeling of TL_8

Theorem 2.5A graph obtained by attaching a triangle at each pendent vertex of a comb is k- Contra harmonic mean graph for all k.

Proof: Let G be a graph obtained by attaching a triangle K_3 at each pendentvertex of $P_n \odot K_1$. Let u_i, v_ibe the vertices of the comb $P_n \odot K_1$ in which $v_i$is joined withthe vertex $u_i$of P_n.Let x_i, y_i, z_ibe the vertices of i^{th}copy ofK_3. Identify $z_i$withv_i,$1 \le i \le n$.

The resultant graph is G whose edge set is

$E = \{u_iu_{i+1} \setminus 1 \le i \le n-1\} U \{u_iv_i, v_ix_i, v_iy_i, x_iy_i \setminus 1 \le i \le n\}$.

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, k+q\}$ by

$f(u_i) = k + 5i - 3, \quad for\ 1 \le i \le n$

$f(v_i) = k + 5i - 2, for\ 1 \le i \le n$

$f(x_i) = k + 5i - 5, for\ 1 \le i \le n$

$f(y_i) = k + 5i - 4, for\ 1 \le i \le n$

Then the induced edge labels are

$f^*(u_iu_{i+1}) = k + 5i - 1, for\ 1 \le i \le n-1$

$f^*(u_iv_i) = k + 5i - 2, for\ 1 \le i \le n$

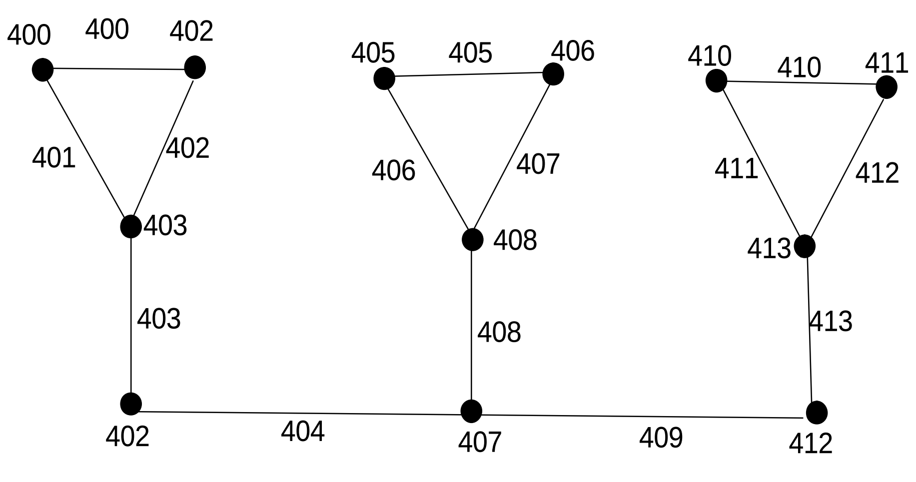
$f^*(v_ix_i) = k + 5i - 4, for\ 1 \le i \le n$

$f^*(v_iy_i) = k + 5i - 3, for\ 1 \le i \le n$

$f^*(x_iy_i) = k + 5i - 5, for\ 1 \le i \le n$

The above defined function *f* provides k-contra harmonic mean labeling of the graph. Hence the graph G is k- contra harmonic mean graph.

Example: 2.6



400 -Contra harmonic mean labeling of G

Theorem 2.7$P_n \odot K_1$is k- contra harmonic mean labelling

Proof: Let $v_1, v_2, \ldots, v_n$ be the path P_n.Let w_i be the vertices which is joined to the vertex v_i, $1 \le i \le n$ of the path P_n.The resultant graph is $P_n \odot K_1$.

Let $G = P_n \odot K_1$.Define a function $f : V(G) \to \{k, k+1, k+2, \ldots, k+q\}$ by

$f(v_i) = k + 2i - 2$ for $1 \le i \le n$

$f w_i) = k + 2i - 1$ for $1 \le i \le n$

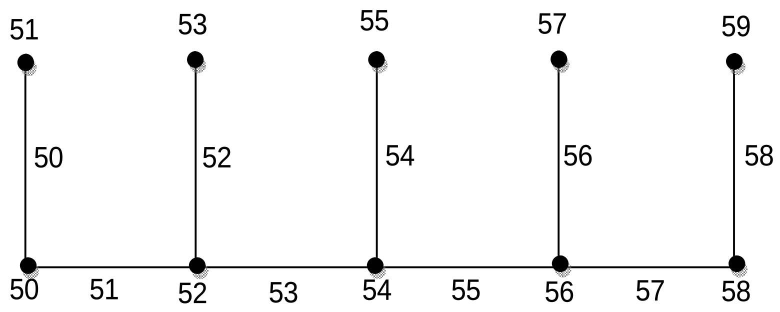
Then the distinct edge labels are

$f^*(v_i v_{i+1}) = k + 2i - 1, for\ 1 \le i \le n - 1$

$f^*(v_i w_i) = k + 2i - 2, for\ 1 \le i \le n$

The above defined function f provides k-contra harmonic mean labelling of the graph. Hence $P_n \odot K_1$ is k- contra harmonic mean labelling.

Example: 2.8



50-contra harmonic mean labelling of $P_5 \odot K_1$

Theorem: 2.9 A Triangular snake $T_n (n \ge 2)$ is k -contra harmonic mean graph $\forall\, k \ge 2$.

Proof: Let $V(T_n) = \{u_i \backslash 1 \le i \le n\} \cup \{v_i \backslash 1 \le i \le n - 1\}$ and

$E(T_n) = \{u_i u_{i+1}, u_{i+1} v_i, u_i v_i \backslash 1 \le i \le n - 1\}$.

First we label the vertices as follows.

Define a function $f : V(T_n) \to \{k, k+1, k+2, \ldots, k+q\}$ by

$f(u_i) = k + 3i - 3$, $for\ 1 \le i \le n$

$f(v_1) = k + 1$

$f(v_i) = k + 3i - 2$, $for\ 2 \le i \le n - 1$

Then the induced edge labels are

$f^*(u_1 u_2) = k + 1$

$f^*(u_i u_{i+1}) = k + 3i + 1$, $for\ 2 \le i \le n - 1$

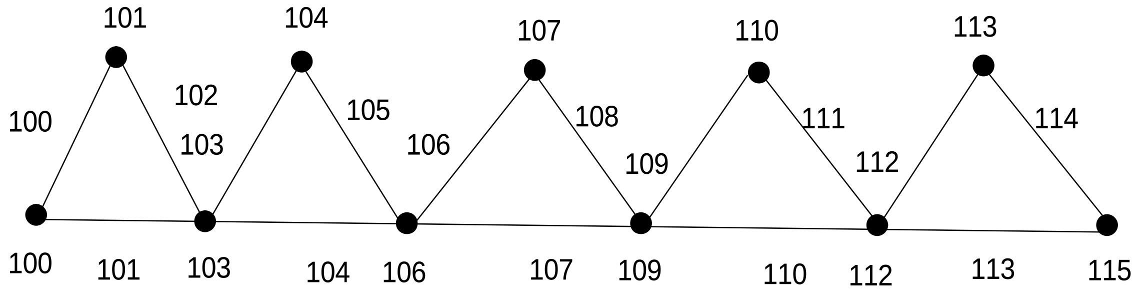
$f^*(u_i v_i) = k + 3i - 3$, $for\ 1 \le i \le n - 1$

$f^*(u_{i+1} v_i) = k + 3i - 1$, $for\ 2 \le i \le n - 1$

$f^*(u_2 v_1) = k + 2$

The above defined function f provides k-contra harmonic mean labeling of the graph. Hence T_n is a k– Contra harmonic mean graph.

Example 2.10



100– Contra harmonic mean graph of T_6

Theorem 2.11 A (m, n) kite graph G is a k-contra harmonic mean graph.

Proof: Let $u_1, u_2, \ldots, u_m, u_1$ be the given cycle of length m and $v_1, v_2, \ldots, v_n$ be the given path of length n.

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, k+q\}$ by

$f(u_i) = k + i - 1, for\ 1 \leq i \leq m,$

$f(v_i) = k + i + 5, for\ 1 \leq i \leq n.$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + i - 1\ , for\ 1 \leq i \leq n - 2$

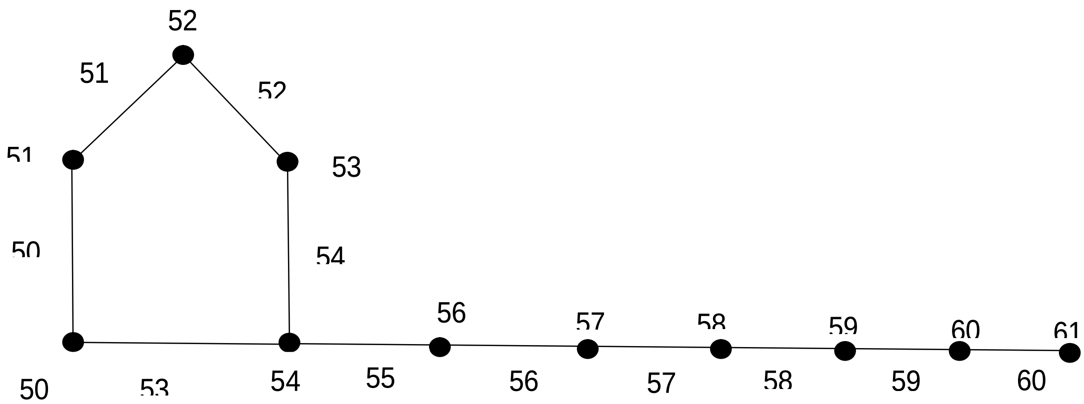
$f^*(u_m u_{m-1}) = k + m - 1$

$f^*(u_1 u_m) = k + 3$

and the edge labels of the path are $\{k + m + 1, k + m + 2, \ldots, k + m + n - 1\}$. The above defined function f provides k-contra harmonic mean labeling of the graph.

Hence the (m, n) kite graph is a k-contra harmonic mean graph.

Example 2.12



50 -contra harmonic mean labeling of $(5,6)$ kite graph

Theorem 2.13 Let $P_n$be the path and G be the graph obtained from $P_n$by attachingC_3 in both the end edges of P_n.Then G is a k-contra harmonic mean graph.

Proof: Let $P_n$be the path $u_1, u_2, \ldots, u_n$and $v_1u_1u_2, v_2u_{n-1}u_n$be the triangles at the end.

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, k + q\}$by

$f(u_i) = k + i \text{ , for } 1 \leq i \leq n,$

$f(v_1) = k \; ; \; f(v_2) = k + q.$

Then the induced edge labels are

$f^*(u_iu_{i+1}) = k + i + 1 \text{ , for } 1 \leq i \leq n - 1$

$f^*(u_1v_1) = k$

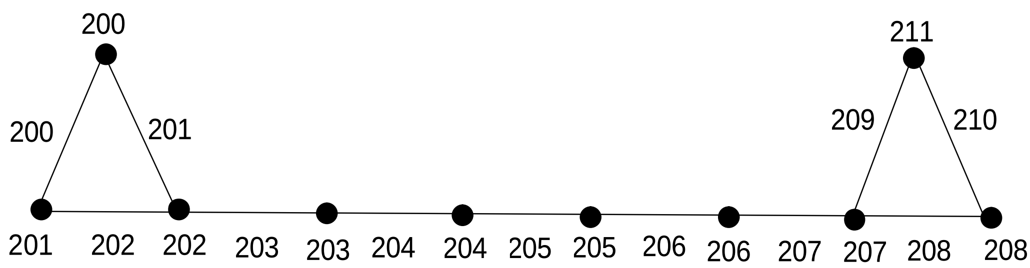
$f^*(u_2v_1) = k + 1$

$f^*(u_{n-1}v_2) = k + n + 1$

$f^*(u_nv_2) = k + n + 2$

The above defined function f provides k-contra harmonic mean labelling of the graph Hence G is a k-contra harmonic mean graph.

Example 2.14: A k-contra harmonic mean labelling of G obtained from P_8 is



200-contra harmonic mean labelling of G

3. CONCLUSION

The Study of labelled graph is important due to its diversified applications. It is very interesting to investigate graphs which admit k-Contra Harmonic Mean Labelling. In this paper, we proved that Path, Triangular Ladder TL_n,a graph obtained by attaching a triangle at each pendent vertex of a comb, Comb, Triangular Snake, (m,n)Kite graph, the graph obtained from $P_n$by attaching C_3 in both the end edges of $P_n$are k-Contra Harmonic Mean Graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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