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Single Species Discrete Age-Structured Population with Variable Fraternity and Age-Dependent Harvesting in Unreserved Area

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ABSTRACT

Asingle species age-structured fishery model is formulated using Ricker model relationship under age dependent harvesting condition using reserved-unreserved area fisheries technique. Here, atthe initial period during which the growth of the species is not much, harvesting is performed as the species is inside reserved area. After a certain age the species infiltrate into the unreserved area, where harvesting is permitted. Here, harvesting rate is assumed to be proportional to the available bio-mass (number of species) of different age group population and decreases with the age of the species. The modified Leslie matrix for the present models is derived. Stability of the system is studied from the eigen values of the modified Leslie matrix.

KEYWORDS: Single Species, Reserved-Unreserved area, Age-Dependant harvesting, Age-structure, Variable fraternity.

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INTRODUCTION

In reality, most natural biological populations are subjected to complex dynamic processes that cannot be described by simple continuous time models. In fish population, for example, recruitment to the fishable stock may only occur several years after spawning of the existing adult population. Furthermore, the entire life history of fish and other organism is generally subjected to strong seasonal or periodic influence. Discrete single species insect population model was developed by Nicholson and Baily[1] and after that another single species discrete population was studied by Maynard Smith[2, 3], May[4]etc. The number of individuals within a population is often kept under control by the changes in the maternity function of the females. These changes may occur in response to the values of some demographic parameters, total size of the population, birth rate, cohort density, the ratio of older and younger females, the ratio of males to females and so forth. Another factor is also due to the change in the mortality of the population. So, discrete age structure model is very much important to discuss the real phenomena. The recruitment of fish is very much important in age structure fish population model. Some researcher considered the rate of recruitment of fish in different age class as constant. One may refer to Gurtin & Macany[5], De Angelis[6], Dekker[7], Landel and Hersen[8]. Kapur[9]etc. But in reality, the rate of recruitment of fish is not constant and it is population dependant. Ricker[10,11] developed a model known as Ricker spawner-recruitment model and it has an important role during many years in the area of fishery science. But in this model, there are several restrictions. Neave[12],Clark[13, 14], Gulland[15] and May[16] observed that the possibility of dispensatory stock-recruitment relationships has an important role in fishery management and this factor was not considered by Ricker[10,11].

Ricker model is simply density dependant population model in the fishery management. Another recruitment functions in fish population model are discussed by May[17], May and Oster[18] and Oster[19]. Levin and May[20] and Clark[21] also developed fishery model considering the interaction of density dependence with age structure. After that there was necessary to develop an ecologically acceptable strategy for harvesting of a renewable resource such as fish, animals etc. and also the optimum strategy for maximum sustainable yield with the minimum effort. M. B. Schaefer[22] first considered the above economic factor and formulated a model with logistic growth. He considered the catch-per-unit effort hypothesis to represent the rate of harvesting. Clark[14] developed several population models considering different forms of harvesting. Rotenberg[23] also considered the logistic model with harvesting. Later on, Goodyear[24],Levin and Goodyear[25] introduced an age-structured fishery model introducing the reproductive delay with deferred reproduction and the truncated delay associated with an eventual levelling off of fecundity in latter age classes. They studied the stability of the system under above two delays separately and combining. But, in their analysis, the harvesting which is an important part in fish population was ignored.

In this paper, we have extended the work of Levin and Goodyear[25]introducing areal-life form of harvesting and presenting the results in detail under the reproductive delay, during this period of time the species are in reserved area, where no harvesting can take place. Hence a discrete age-structured single species population model is formulated under age dependent harvesting condition of mature species. In this model, we consider the rate of stock recruitment as Ricker spawner-recruit function in different age group. In this formulation, some realistic conditions on harvesting are incorporated. In some countries, governments impose restriction on the harvesting of immature population of some species such as fishes. Generally, fishermen throw the smaller fishes back into the water. It is fact that withdrawal of a species decreases with the age of the species. Hence, in the present model, the species of age group 0 to 2 is not considered for harvesting and after that harvesting is performed. Its rate is directly proportional to the available biomass of different age group population and decreases with the age of the species. The modified Leslie matrix for the model is derived. Stability of the system is discussed by Perror-Frobenius theory. A simple approximation of this model considering two age classes is formulated and its stability is discussed. Also, its stability diagram is presented through numerical values. Another simple approximation of this model is also developed considering two stage delay production and its stability is also discussed.

BASIC MODEL

Let us consider a fish population which are divided into different classes according to their ages in years, as $x_1(t), x_2(t), x_3(t), \cdots, x_n(t)$.Also, we assume that each class can give birth. Here we assume that the model is one sex model as we take into account only the changes in female populations. Now for each such age class there is a maternity, the number of new born next year spawned by an average individual of age $m_i$this year, because of density dependence $m_i$is not constant. Now we consider the rate of harvesting $h_2, \cdots, h_{n-1}$of the population$x_2(t), x_3(t), \cdots, x_n(t)$ respectively, as in this paper we have consider that first two age groups are inside the marine protected area, where harvesting is not allowed.

And let $p_i, (i = 1, 2, \cdots, n-1; 0 < p_i \leq 1)$ Be the proportion of females of the $i-th$a Agegroup at time t, who are surviving to become females of $(i+1) -th$ age group at time $(t + 1)$.

Let, K_i =Average number of eggs of females of age class i, in any year, and indicates the parental egg production for that year. Therefore

$$P = \sum_{i=1}^{n} x_i K_i \quad (1)$$

Now from the Ricker relationship, the number of recruits for the following year i.e. the new value of x_i is

$$x_i' = \hat{\alpha}\, P\, exp(-\beta P) \tag{2}$$

where $\hat{\alpha}$ = density-independent probability of surviving from egg to age 1 and β = coefficient of density dependent mortality. With this we can write,

$$m_i = \hat{\alpha} K_i\, exp(-\beta P) \tag{3}$$

Therefore, the model can be expressed as

$$\left.\begin{aligned} x_1(t+1) &= m_1x_1(t) + m_2x_2(t) + m_3x_3(t) + \cdots + m_nx_n(t) \\ x_2(t+1) &= p_1x_1(t) \\ x_3(t+1) &= p_2x_2(t) - h_2x_3(t) \\ &\vdots \\ x_{r+1}(t+1) &= p_rx_r(t) - h_rx_{r+1}(t) \\ &\vdots \\ x_{n-1}(t+1) &= p_{n-2}x_{n-2}(t) - h_{n-2}x_{n-1}(t) \\ x_n(t+1) &= p_{n-1}x_{n-1}(t) - h_{n-1}x_n(t) \end{aligned}\right\}$$

(4)

Now let us consider $h_2 = h, h_i = Rh_{i-1}, i \geq 3$ with $R < 1$. So, model (4) is of the form

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ \vdots \\ x_{r+1}(t+1) \\ \vdots \\ x_{n-1}(t+1) \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 & \cdots & m_r & m_{r+1} & \cdots & m_{n-1} & m_n \\ p_1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & -h_2 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p_r & -h_r & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -h_{n-2} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & p_{n-1} & -h_{n-1} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_{r+1}(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} \tag{5}$$

Or, $$X(t+1) = AX(t)$$

(6)

where, $A = \begin{bmatrix} m_1 & m_2 & m_3 & \cdots & m_r & m_{r+1} & \cdots & m_{n-1} & m_n \\ p_1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & -h & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p_r & -R^{r-2}h & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -R^{n-4}h & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & p_{n-1} & -R^{n-3}h \end{bmatrix}$ and $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_{r+1}(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}$

(7)

For $r = 1$, system (6) reduces to

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ \vdots \\ x_{n-1}(t+1) \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 \ldots & m_{n-1} & m_n \\ p_1 & 0 & 0 \ldots & 0 & 0 \\ 0 & p_2 & -h \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \;\vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots & -R^{n-4}h & 0 \\ 0 & 0 & 0 \cdots & p_{n-1} & -R^{n-3}h \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} \quad (8)$$

where $\hat{\alpha}$ relates to α of Ricker model through the relation,

$$\hat{\alpha} = \frac{\alpha}{H_s}$$

(9)

where H_s, the stock value of an age 1 recruit, is

$$H_s = \sum_{i=1}^{n} l_i K_i \quad (10)$$

Here l_i = survival probability from age class 1 to age class i and therefore

$$l_i = \frac{p_1 p_2 \cdots p_{i-1}}{(1+h)(1+Rh)\cdots(1+R^{i-3}h)} \quad (11)$$

Now we take $l_1 = 1$.

The life-history strategy of a population is encapsulated in its reproductive function, that is the distribution of $a_i = (l_i m_i)$ with respect to i. In our model,

$$a_i = l_i m_i = \frac{l_i K_i}{\sum l_i K_i} \alpha\, exp(-\beta P)$$

(12)

Hence, in any year, the shape of $l_i m_i$ distribution is identical to that of $l_i K_i$ at equilibrium,

$$a_i = l_i m_i = \frac{l_i K_i}{\sum l_i K_i} \quad (13)$$

and so

$$\sum a_i = 1 \quad (14)$$

At equilibrium, the matrix A of (7) can be written as

$$J = \begin{bmatrix} \mu_1 & \mu_2 & \mu_3 \ldots & \mu_{n-1} & \mu_n \\ p_1 & 0 & 0 \ldots & 0 & 0 \\ 0 & p_2 & -h \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \;\vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots & -R^{n-4}h & 0 \\ 0 & 0 & 0 \cdots & p_{n-1} & -R^{n-3}h \end{bmatrix} \quad (15)$$

in which

$$\mu_i = \left[\frac{\partial}{\partial x_i}(m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n)\right]_{X=\bar{X}} = \left[\frac{\partial}{\partial P}\left(\hat{\alpha}\, P\, exp(-\beta P)\right)\right]_{X=\bar{X}} \cdot \frac{\partial P}{\partial x_i}$$
$$= \hat{\alpha} K_i (1 - \beta \bar{P}) exp(-\beta P)$$

(16)

Recalling $\bar{P} = \frac{\ln \alpha}{\beta}$, we get

$\mu_i = \frac{\hat{\alpha}}{\alpha}(1-\ln\alpha)K_i = \frac{(1-\ln\alpha)K_i}{\sum_{i=1}^{n} l_i K_i} \Rightarrow \frac{(1-\ln\alpha)a_i}{p_1 p_2 \cdots p_{i-1}}\left[(1+h)(1+Rh)\cdots(1+R^{i-3}h)\right]$

(17)

The eigen values of J are the roots of the equation

$\phi(\lambda) \equiv$

$\lambda^n - \{\mu_1 - (h_2 + h_3 + \cdots + h_{n-1})\}\lambda^{n-1} - \{\mu_1 h_{n-1} + \mu_2 p_1 - (h_2 h_3 + \cdots + h_{n-2}h_{n-1})\}\lambda^{n-2} - \{\mu_1 h_1 h_2 + \mu_2 p_1 h_2 + \mu_3 p_1 p_2 - (h_2 h_3 h_4 + \cdots + h_{n-3}h_{n-2}h_{n-1})\}\lambda^{n-3} - \{\mu_2 p_1 h_2 h_3 + \mu_3 p_1 p_2 h_3 + \mu_4 p_1 p_2 p_3 - (h_1 h_2 h_3 h_4 + h_2 h_3 h_4 h_5 + \cdots + h_{n-4}h_{n-3}h_{n-2}h_{n-1})\}\lambda^{n-4} - \cdots - \{\mu_2 p_1 h_2 h_3 \cdots h_{n-2} + \mu_3 p_1 p_2 h_3 h_4 \cdots h_{n-2} + \cdots + \mu_{n-1} p_1 p_2 p_3 \cdots p_{n-2}\}\lambda - \{\mu_2 p_1 h_2 h_3 \cdots h_{n-1} + \mu_3 p_1 p_2 h_3 h_4 \cdots h_{n-1} + \cdots + \mu_{n-1} p_1 p_2 p_3 \cdots p_{n-2} h_{n-1} + \mu_n p_1 p_2 p_3 \cdots p_{n-1}\} = 0$

(18)

Therefore, with the help of (13) and (17) we get equation (18) as

$\phi(\lambda) \equiv \lambda^n - \left\{a_1(1-\ln\alpha) - \left(h\frac{1-R^{n-2}}{1-R}\right)\right\}\lambda^{n-1} - \left\{a_1(1-\ln\alpha)hR^{n-3} + a_2(1-\ln\alpha) - \left(h^2 R\frac{1-R^{2n-6}}{1-R^2}\right)\right\}\lambda^{n-2} - \left\{a_2(1-\ln\alpha)h + a_3(1-\ln\alpha)(1+h) - \left(h^3 R^3\frac{1-R^{3n-12}}{1-R^3}\right)\right\}\lambda^{n-3} - \left\{a_2(1-\ln\alpha)h^2 R + a_3(1-\ln\alpha)(1+h)hR + a_4(1-\ln\alpha)(1+h)(1+Rh) - \left(h^4 R^6\frac{1-R^{4n-20}}{1-R^4}\right)\right\}\lambda^{n-4} - \cdots - \left\{a_2(1-\ln\alpha)h^{n-3}R^{\frac{(n-3)(n-4)}{2}} + a_3(1-\ln\alpha)(1+h)h^{n-4}R^{\frac{(n-5)(n-4)}{2}} + \cdots + a_{n-1}(1-\ln\alpha)(1+h)\ldots(1+R^{n-4}h)R^{\frac{(n-1)(n-2)}{2}}\right\}\lambda - \left\{a_2(1-\ln\alpha)h^{n-2}R^{\frac{(n-3)(n-2)}{2}} + a_3(1-\ln\alpha)(1+h)h^{n-3}R^{\frac{(n-4)(n-5)}{2}} + \cdots + a_{n-1}(1-\ln\alpha)(1+h)\ldots(1+R^{n-4}h)hR^{n-3} + a_n(1-\ln\alpha)(1+h)\ldots(1+R^{n-3}h)\right\} = 0 \qquad (19)$

Equation (19) can be written as

$$\phi(\lambda) \equiv \frac{\lambda^n}{(1-\ln\alpha)} - \left\{a_1 - \frac{1}{(1-\ln\alpha)}\left(h\frac{1-R^{n-1}}{1-R}\right)\right\}\lambda^{n-1}$$
$$-\left\{a_1 hR^{n-3} + a_2 - \frac{1}{(1-\ln\alpha)}\left(h^2 R\frac{1-R^{2n-6}}{1-R^2}\right)\right\}\lambda^{n-2}$$
$$-\left\{a_2 h + a_3(1+h) - \frac{1}{(1-\ln\alpha)}\left(h^3 R^3\frac{1-R^{3n-12}}{1-R^3}\right)\right\}\lambda^{n-3}$$
$$-\left\{a_2 h^2 R + a_3(1+h)hR + a_4(1+h)(1+Rh) - \frac{1}{(1-\ln\alpha)}\left(h^4 R^6\frac{1-R^{4n-20}}{1-R^4}\right)\right\}\lambda^{n-4}$$

$- \cdots - \left\{a_2 h^{n-3}R^{\frac{(n-3)(n-4)}{2}} + a_3(1+h)h^{n-4}R^{\frac{(n-5)(n-4)}{2}} + \cdots + a_{n-1}(1+h)\ldots(1+R^{n-4}h)\right\}\lambda - \left\{a_2 h^{n-2}R^{\frac{(n-3)(n-2)}{2}} + a_3(1+h)h^{n-3}R^{\frac{(n-4)(n-5)}{2}} + \cdots + a_{n-1}(1+h)\ldots(1+R^{n-4}h)hR^{n-3} + \right.$

$a_n(1 + h) \dots (1 + R^{n-3}h)\big\} = 0$ (20)

Since $R < 1$, so if $1 < ln\,\alpha$, all of the coefficients in (20) are of the same signand so by Descarte's rule, there are no positive real roots. If $0 < \ln\alpha < 1$, and $a_1(1 - \ln\alpha) > \left(h\frac{1-R^{n-1}}{1-R}\right)$, then by the Perror-Frobenius theory, the dominant eigenvalue is real, positive and less than 1. Hence in this case all eigenvalues lie within the unit circle. As it is well known that equilibrium will be stable to small perturbations provided all roots of (19) lie within the unit circle in the complex plane and unstable if any one lies outside that circle. The upper boundary of the region of stability is thus obtained by finding the largest α for which the all roots of (19) lie within the unit circle. For all possible choice of $a_1, a_2, \dots, a_n$, the stability diagram will then typically involve a surface in $(n + 1)$ −dimensional space, separating the stable region from the unstable one. The largest value ofαanywhere on the surface is simply the maximumαsubject to the constraints that the roots of (20) lie within the unit circle and that$a_1 > 0, a_2 \geq 0, \dots, a_n \geq 0, \sum a_i = 1$.

A simple approximation

For this simple model we take$K_1 = 1, R << 1$ and$K_i = K > 1$,for$i > 1$ and$p_i = p$ for all$i$.To describe this model we need only two state variables, since the age classes after the first may be lumped into the single descriptorx_2.The model thus takes the form

$$\left.\begin{aligned} x_1' &= \hat{\alpha}\, P\, exp(-\beta P) \\ x_2' &= p x_1 \end{aligned}\right\}$$

(21)

In which $P = x_1 + Kx_2$

and

$$\begin{aligned} \alpha &= \hat{\alpha}\left[1 + K\left(p + \frac{p^2}{(1+h)} + \frac{p^3}{(1+h)(1+Rh)} + \cdots\right)\right] \\ &= \hat{\alpha}\left[1 + Kp + \frac{Kp^2}{(1+h)} + \left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right] \end{aligned} \qquad (22)$$

The stability of the system is governed by the eigen value of the matrix

$$M = \begin{bmatrix} \frac{1-\ln\alpha}{1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)} & \frac{K(1-\ln\alpha)}{1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)} \\ p & 0 \end{bmatrix}$$

(23)

Therefore the characteristic equation of M is

$$\lambda^2 + \frac{(ln\alpha-1)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]}\lambda + \frac{Kp(ln\alpha-1)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} = 0. \quad (24)$$

Let us consider

$$A_1 = \frac{p(1-ln\alpha)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} \; and \; A_2 = \frac{p(1-ln\alpha)(h+Kp)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]}$$

As $\ln\alpha > 1$, so both $A_1 > 0$ and $A_2 > 0$.

Then $\ln\alpha > 1$ is the necessary and sufficient conditions for stability by Routh-Hurwitz criteria.

MODEL WITH DELAYED REPRODUCTION

For a more transparent demonstration, suppose that the example of previous section is modified so that

$$K_1 = 0, \quad K_2 = 1$$

and

$K_i = K > 1 \, for \, i \geq 3$

(25)

For this case, the matrix M of (23) becomes

$$M = \begin{bmatrix} 0 & \frac{1-\ln\alpha}{p\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} & \frac{K(1-\ln\alpha)}{p\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} \\ p & 0 & 0 \\ 0 & p & -h \end{bmatrix}$$

(26)

Therefore the characteristic equation of M is

$$\lambda^3 + \lambda^2 h + \frac{p(1-ln\alpha)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]}\lambda + \frac{p(1-ln\alpha)(h+Kp)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} = 0. \quad (27)$$

Setting,

$$A_1 = h, \; A_2 = \frac{p(1-ln\alpha)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} \; and \; A_3 = \frac{p(1-ln\alpha)(h+Kp)}{\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]} \quad (28)$$

Since $A_1 = h > 0$, so, by the application of the Routh-Hurwitz conditionthe stability region for the system is given by

$$\begin{vmatrix} A_1 & A_3 \\ 1 & A_2 \end{vmatrix} > 0, \; i.e. \; A_1A_2 - A_3 > 0. \quad (29)$$

From the condition of (30) we have

$$\ln\alpha > \frac{1}{Kp}\left[1+Kp+\frac{Kp^2}{(1+h)}+\left(\frac{Kp^3}{(1+h)(1+Rh)}\right)\left(\frac{1}{(1-p)}\right)\right]+1 \qquad (30)$$

NUMERICAL ILLUSTRATION

Considering the annual mortality rate $z = -\ln p$ and taking $K = 10, R = 0.001 \; with \; h = 0.5 \; \& \; 0$ we present stability region for (30) in Fig-1.

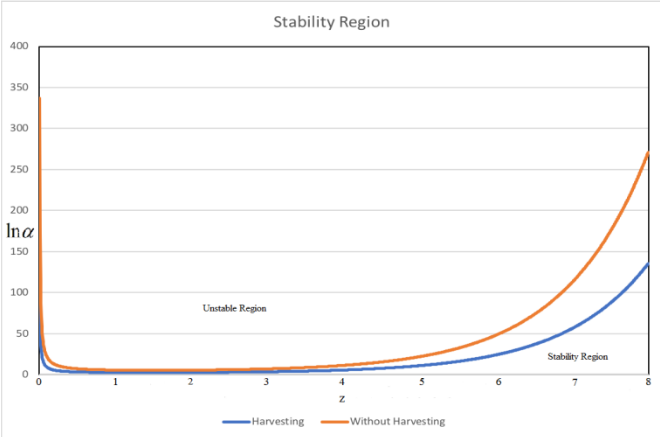

Figure-1: Stability diagram for h = 0.5& 0

CONCLUSION

It is interesting to note that the stability region in Fig-1 for the system without harvesting is larger than the corresponding region in Fig-1 for the system with harvesting. This observation agrees with the reality.When the harvesting is withdrawn from the system, the total population increases and hence the stability region with respect to the survival probability is more. Moreover the consideration of a system without harvesting is an unrealistic one, rather harvesting is a natural phenomenon in a biological species system like fish,etc. for which the stability region is presented in Fig-1.

Here, the Ricker Stock-recruitment relation with age dependent harvesting make a good example of application of the proposed model in the fisheries in India and Bangladesh. Now-a-days, both Governments have banned the harvesting of hilsa at the juvenile stage of hilsa (jatka) and harvesting is permitted after a certain period of growth. During the initial period of growth, Government declare a certain area as reserved area (protected area) for a fish species, where harvesting is ban up-to a limited time span and after that harvesting is open for few month. In this way a heavy bio-mass of some species can maintain for long run. Hence, the proposed model can be used for some practicalcase studies in fish cultivation.

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