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Integer Cordial Labeling of Triangular Snake Graph

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ABSTRACT

A graph $G=(V,E)$ with $|V|=p$ is called integer cordial labeled graph if it has an injective map $f:V\to\left[-\frac{p}{2},\dots,\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor,\dots,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, which includes an edge labeling $f^*:E\to\{0,1\}$ defined by $f^*(e=uv)=1$ if $f(u)+f(v)\ge 0$ and 0 otherwise such that $|e_f(0)-e_f(1)|\le 1$. In this paper we discuss Integer cordial labeling of triangular snake graph T_n, double triangular snake graph DT_n, triple triangular snake graph TT_n and alternate triangular snake graph AT_n.

KEYWORDS: Integer cordial labeling, Triangular Snake graphs

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INTRODUCTION

In this paper, we consider finite, connected and undirected graph. A graph $G=(V(G),E(G))$ having set of vertices $V(G)$ and set of edges $E(G)$. For the standard notation, we refer Gross and Yellen.[2] The concept of cordial labeling was introduced by I. Cahit[3] in 1987.

Definition-1.1: If the vertices or edges of graph are assigned values or label to certain conditions is known as graph labeling.

Definition-1.2: A labeling of a graph G is said to be cordial labeling if $|v_f(0)-v_f(1)|\le 1$ & $|e_f(0)-e_f(1)|\le 1$, where $v_f(i)$ and $e_f(i)$ is the numbers of vertices and edges of graph G having labeled i respectively for $i=0,1$. A graph which admits cordial labeling is called cordial graph.

Different types of cordial labeling are introduced and explored by many researchers. For detailed survey on graph labeling we refer to a dynamic survey on graph labeling by Gallian.[4]

Definition-1.3: A simple connected graph $G=(V,E)$ with $|V|=p$. Let $f:V\to\left[-\frac{p}{2},....,\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor,....,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd be an injective map, which includes an edge labeling $f^*:E\to\{0,1\}$ defined by $f^*(e=uv)=1$ if $f(u)+f(v)\ge 0$ and 0 otherwise then f is said to be integer cordial if $|e_f(0)-e_f(1)|\le 1$. Where $e_f(i)$ is the numbers of edges of graph G having label i for $i=0,1$. A graph is called integer cordial graph if it admits an integer cordial labeling. Where $[-t,...,t]=\{x\,|\,x \text{ is an integer } \& |x|\le t\}$ and $[-t,...,t]^*=[-t,...,t]-\{0\}$.

- T. Nicholas and P. Maya[6] have proved following result:

(i) Complete graph K_n is not integer cordial graph, $n > 3$.

(ii) Star graph $K_{1,n}$ is integer cordial.

(iii) Helm graph H_n is integer cordial.

(iv) Closed Helm graph CH_n is integer cordial.

(v) Complete bipartite graph $K_{n,n}$ is integer cordial iff n is even.

(vi) Graph $K_{n,n}\setminus M$ is an integer cordial, where M is a perfect matching.

Definition-1.4: A Triangular snake graph T_n is obtained from a path $u_1,u_2,....,u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1\le i\le n$, that is every edge of a path is replaced by a triangle.

Definition-1.5: Double Triangular Snake graph DT_n consists of two Triangular snakes that have a common path.

Definition-1.6: Triple Triangular Snake graph TT_n consists of three Triangular snakes that have a common path.

Definition-1.7: An Alternate Triangular Snake graph AT_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} alternatively $(i = 1, 3, 5, ...)$ to a new vertex v_i. That is every alternate edge of a path is replaced by C_3.

MAIN RESULTS

Theorem-2.1: The Triangular snake graph T_n is integer cordial graph, $n \geq 2$.

Proof: Let $u_1, u_2, ..., u_n$ be the n vertices and joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$ .Hence total no. of vertices in $T_n = p = 2n-1$ and number of edges in $T_n = q = 3(n-1)$.

There are two cases for the value of *n*.

Case-1: n is even

When *n* is even then *p* is odd.

We define $f : V \rightarrow \left[-\left\lfloor \frac{p}{2} \right\rfloor, ..., \left\lfloor \frac{p}{2} \right\rfloor \right]$ as follows:

$$f(u_i) = \begin{cases} i - \dfrac{n+2}{2}; & 1 \leq i \leq \dfrac{n}{2} \\ i - \dfrac{n}{2} & ; \dfrac{n}{2} < i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} i - n; & 1 \leq i < \dfrac{n}{2} \\ 0 & ; i = \dfrac{n}{2} \\ i & ; \dfrac{n}{2} < i \leq n-1 \end{cases}$$

Case-2: n is odd

When *n* is odd then *p* is odd.

We define $f : V \rightarrow \left[-\left\lfloor \frac{p}{2} \right\rfloor, ..., \left\lfloor \frac{p}{2} \right\rfloor \right]$ as follows:

$$f(u_i) = i - \frac{n+1}{2}; \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} i - n; & 1 \leq i \leq \dfrac{n-1}{2} \\ i & ; \dfrac{n-1}{2} < i \leq n-1 \end{cases}$$

Table – 1 "edge condition for T_n"

Case No.	Value of n	Value of p	Edge condition
1	n is even	p is odd	$e_f(0)=\left\lfloor \frac{3(n-1)}{2} \right\rfloor$ and $e_f(1)=\left\lceil \frac{3(n-1)}{2} \right\rceil$
2	n is odd	p is odd	$e_f(0)=\frac{3(n-1)}{2}$ and $e_f(1)=\frac{3(n-1)}{2}$

Thus, in each case we get $|e_f(0)-e_f(1)|\le 1$.

Hence Triangular snake graph T_n is integer cordial.

Example-2.2: An integer cordial labeling of T_7 is shown in Figure-1.

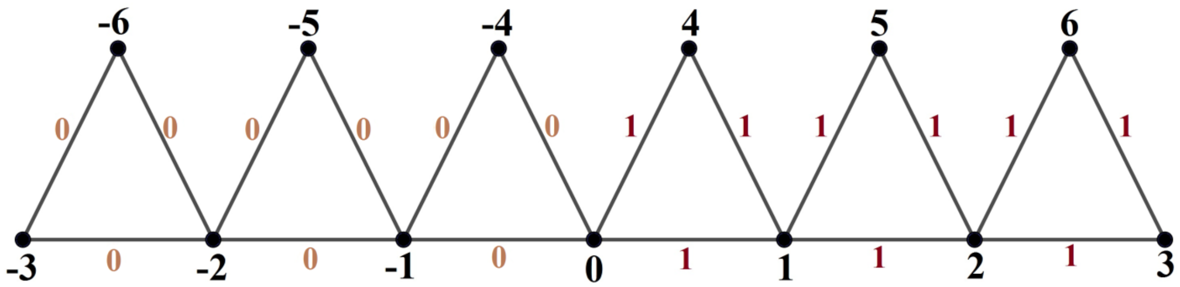

Figure – 1 "triangular snake graph with 7 vertices (T_7)"

Theorem-2.3: The Double Triangular snake graph DT_n is integer cordial graph, $n\ge 2$.

Proof: Let $u_1,u_2,....,u_n$ be the n vertices and joining u_i and u_{i+1} to a new vertex v_i and v_i' for $1\le i\le n-1$. Total no. of vertices in $DT_n = p = 3n-2$ and number of edges in $DT_n = q = 5(n-1)$.

Case-1: n is even

When n is even then p is also even.

We define $f:V\to\left[-\frac{p}{2},...,\frac{p}{2}\right]^*$ as follows:

$$f(u_i)=\begin{cases} i-\frac{3n}{2}\ ; & 1\le i\le \frac{n}{2} \\ i+\frac{n-2}{2}\ ; & \frac{n}{2}<i\le n \end{cases}$$

$$f(v_i)=i;\quad 1\le i\le n-1$$

$$f(v_i')=-i;\quad 1\le i\le n-1$$

Case-2: n is odd

When n is odd then p is also an odd.

We define $f:V\to\left[-\left\lfloor\frac{p}{2}\right\rfloor,....,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$$f(u_i)=\begin{cases} i-\left(\dfrac{3n-1}{2}\right); & 1\le i<\dfrac{n+1}{2} \\ 0; & i=\dfrac{n+1}{2} \\ i+\dfrac{n-3}{2}; & \dfrac{n+1}{2}<i\le n \end{cases}$$

$f(v_i)=i;\quad 1\le i\le n-1$

$f(v_i')=-i;\quad 1\le i\le n-1$

Table – 2 "edge condition for DT_n"

Case No.	Value of n	Value of p	Edge condition
1	n is even	p is even	$e_f(0)=\left\lfloor\frac{5(n-1)}{2}\right\rfloor$ and $e_f(1)=\left\lceil\frac{5(n-1)}{2}\right\rceil$
2	n is odd	p is odd	$e_f(0)=\frac{5(n-1)}{2}$ and $e_f(1)=\frac{5(n-1)}{2}$

Thus, in each case we get $|e_f(0)-e_f(1)|\le 1$.

Hence, Double Triangular snake graph DT_n is integer cordial.

Example-2.4: An integer cordial labeling of DT_6 is shown in Figure-2.

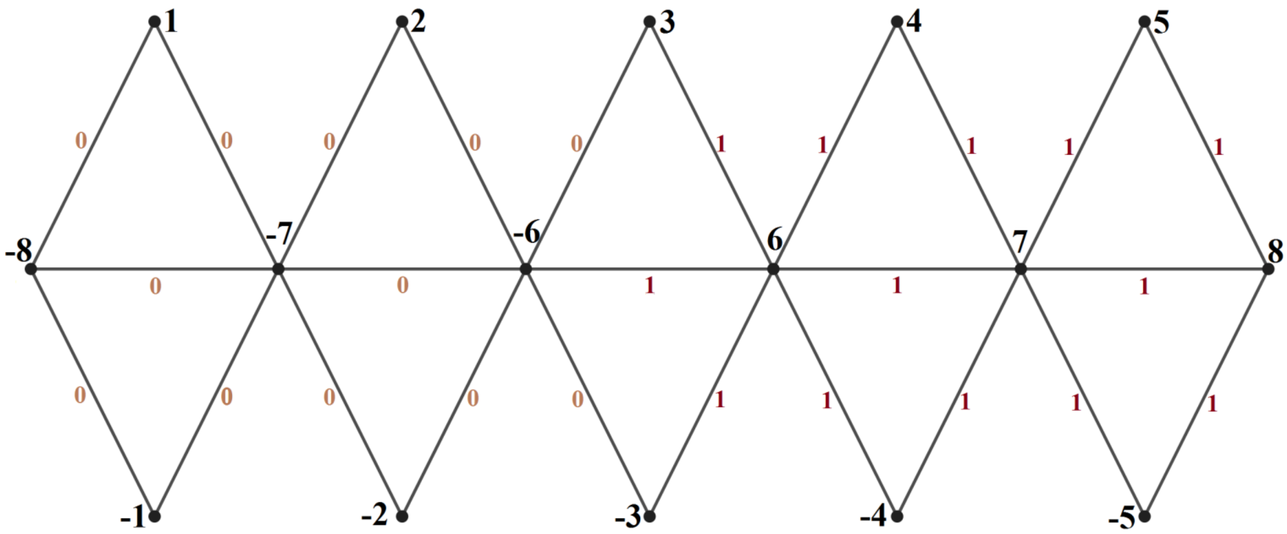

Figure – 2 "double triangular snake graph with 6 vertices (DT_6)"

Theorem-2.5: The Triple Triangular snake graph TT_n is integer cordial graph, $n\ge 2$.

Proof: Let $u_1,u_2,....,u_n$ be the n vertices and joining u_i and u_{i+1} to a new vertex v_i,v_i' and v_i'' for $1\le i\le n-1$. Total no. of vertices in $TT_n=p=4n-3$ and number of edges in $TT_n=q=7(n-1)$.

Case-1: n is even.

When n is even then p is odd.

We define $f : V \rightarrow \left[-\left\lfloor \frac{p}{2} \right\rfloor, ..., \left\lfloor \frac{p}{2} \right\rfloor \right]$ as follows:

$$f(u_i) = \begin{cases} i - \frac{3n}{2} \ ; & 1 \le i \le \frac{n}{2} \\ i + \frac{n-2}{2} \ ; & \frac{n}{2} < i \le n \end{cases}$$

$$f(v_i) = i \ ; \quad 1 \le i \le n-1$$

$$f(v_i') = \begin{cases} i - (2n-1) \ ; & 1 \le i < \frac{n}{2} \\ 0 \ ; & i = \frac{n}{2} \\ i + (n-1) \ ; & \frac{n}{2} < i \le n-1 \end{cases}$$

$$f(v_i'') = -i ; \quad 1 \le i \le n-1$$

Case-2: n is odd

When n is odd then p is also an odd.

We define $f : V \rightarrow \left[-\left\lfloor \frac{p}{2} \right\rfloor, ..., \left\lfloor \frac{p}{2} \right\rfloor \right]$ as follows:

$$f(u_i) = \begin{cases} i - \left(\frac{3n-1}{2} \right) \ ; & 1 \le i < \frac{n+1}{2} \\ 0 \ ; & i = \frac{n+1}{2} \\ i + \frac{n-3}{2} \ ; & \frac{n+1}{2} < i \le n \end{cases}$$

$$f(v_i) = i \ ; \quad 1 \le i \le n-1$$

$$f(v_i') = \begin{cases} i - (2n-1) \ ; & 1 \le i \le \frac{n-1}{2} \\ i + (n-1) \ ; & \frac{n-1}{2} < i \le n-1 \end{cases}$$

$$f(v_i'') = -i ; \quad 1 \le i \le n-1$$

Table – 3 "edge condition for TT_n"

Case No.	Value of n	Value of p	Edge condition
1	n is even	p is odd	$e_f(0) = \left\lfloor \frac{7(n-1)}{2} \right\rfloor$ and $e_f(1) = \left\lceil \frac{7(n-1)}{2} \right\rceil$
2	n is odd	p is odd	$e_f(0) = \frac{7(n-1)}{2}$ and $e_f(1) = \frac{7(n-1)}{2}$

Thus, in each case we get $\left|e_f(0)-e_f(1)\right|\le 1$.

Hence, Triple Triangular snake graph TT_n is integer cordial.

Example-2.6: An integer cordial labeling of TT_6 is shown in Figure-3.

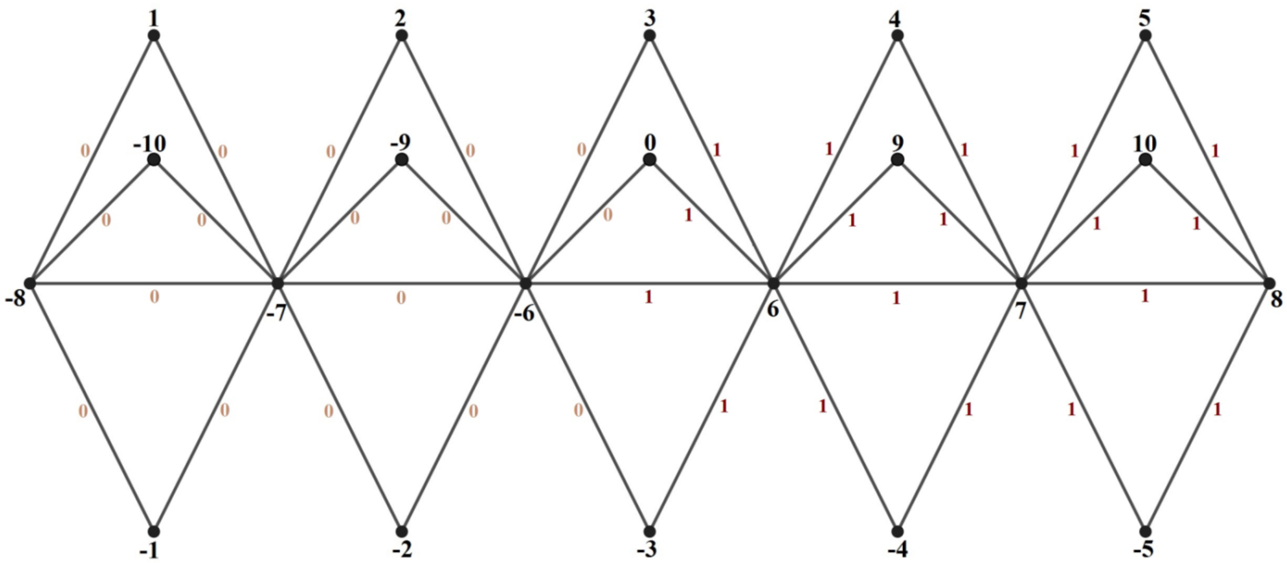

Figure – 3 "triple triangular snake graph with 6 vertices (TT_6)"

Theorem-2.7: The Alternate Triangular snake graph AT_n is integer cordial graph, $n\ge 2$.

Proof: Let $u_1,u_2,....,u_n$ be the n vertices and joining u_i and u_{i+1} alternatively $(i=1,3,5,.....)$ to a new vertex v_i for $1\le i\le n-1$.

There are different four cases related to the value of n and p.

Case-1: If n is even and p is odd.

We define $f:V\to\left[-\left\lfloor\frac{p}{2}\right\rfloor,....,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$$f(u_i)=\begin{cases} i-\dfrac{n+2}{2}; & 1\le i\le \dfrac{n}{2}\\ i-\dfrac{n}{2}; & \dfrac{n}{2}<i\le n\end{cases}$$

$$f(v_i)=\begin{cases} i-\dfrac{3n+2}{4}; & 1\le i<\dfrac{n+2}{4}\\ 0; & i=\dfrac{n+2}{4}\\ i+\dfrac{n-2}{4}; & \dfrac{n+2}{4}<i\le\dfrac{n}{2}\end{cases}$$

Case-2: If n and p are even.

We define $f:V \rightarrow \left[-\frac{p}{2},...,\frac{p}{2}\right]^*$ as follows:

$$f(u_i)=\begin{cases} i-\frac{n+2}{2}; & 1 \le i \le \frac{n}{2} \\ i-\frac{n}{2}; & \frac{n}{2} < i \le n \end{cases}$$

$$f(v_i)=\begin{cases} i-\frac{3n+4}{4}; & 1 \le i \le \frac{n}{4} \\ i+\frac{n}{4}; & \frac{n}{4} < i \le \frac{n}{2} \end{cases}$$

Case-3: If p and n both are odd.

We define $f:V \rightarrow \left[-\left\lfloor\frac{p}{2}\right\rfloor,....,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$$f(u_i)=i-\frac{n+1}{2}; \quad 1 \le i \le n$$

$$f(v_i)=\begin{cases} i-\frac{3n+1}{4}; & 1 \le i \le \frac{n-1}{4} \\ i+\frac{n-1}{4}; & \frac{n-1}{4} < i \le \frac{n-1}{2} \end{cases}$$

Case-4: If p is even and n is odd.

We define $f:V \rightarrow \left[-\frac{p}{2},...,\frac{p}{2}\right]^*$ as follows:

$$f(u_i)=\begin{cases} i-\frac{n+1}{2}; & 1 \le i \le \frac{n-1}{2} \\ i-\frac{n-1}{2}; & \frac{n-1}{2} < i \le n \end{cases}$$

$$f(v_i)=\begin{cases} i-\frac{3(n+1)}{4}; & 1 \le i \le \frac{n+1}{4} \\ i+\frac{n+1}{4}; & \frac{n+1}{4} < i \le n \end{cases}$$

Table – 4 "edge condition for AT_n"

Case No.	Value of n	Value of p	Edge condition
1	n is even	p is odd	$e_f(0)=n-1$ and $e_f(1)=n$
2	n is even	p is even	$e_f(0)=n-1$ and $e_f(1)=n$
3	n is odd	p is odd	$e_f(0)=n-1$ and $e_f(1)=n-1$
4	n is odd	p is even	$e_f(0)=n-1$ and $e_f(1)=n-1$

Thus, in each case we get $\left|e_f(0)-e_f(1)\right|\le 1$.

Hence, Alternate Triangular snake graph AT_n is integer cordial.

Example-2.8: An integer cordial labeling of AT_6 is shown in Figure-4.

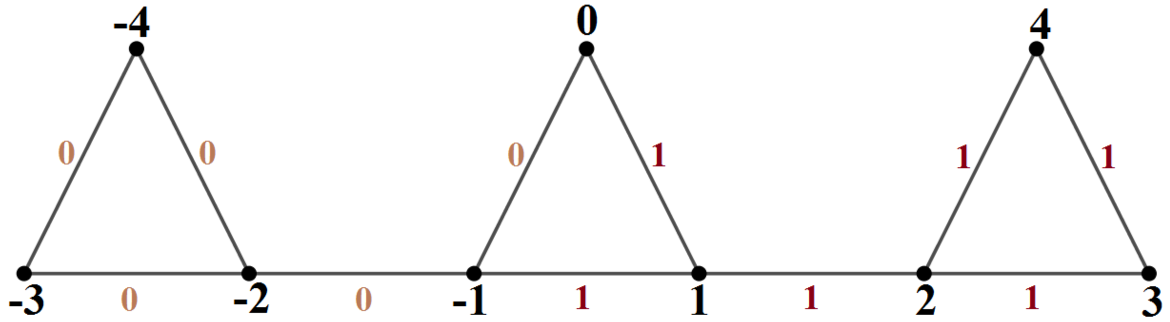

Figure - 4 "alternate triangular snake graph with 6 vertices (AT_6)"

CONCLUSION

In this paper we have proved that triangular snake graph, double triangular snake graph, triple triangular snake graph and alternate triangular snake graph admits integer cordial labeling.

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