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Fourier Series Involving A-Function

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ABSTRACT

The object of the present paper is to establish two Fourier series expansion formulae involving A-function of one variable.

KEYWORDS - Hyper-geometric functions, integration and summation, Fourier sine series, Variable

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1. INTRODUCTION:

The subject of Fourier series for generalized hypergeometric functions occupies a prominent place in the literature of special functions and boundary value problems. Certain double Fourier series of generalized hypergeometric functions play an important role in the development of the theories of special functions and two-dimensional boundary value problems.

The A-function of one variable is defined by Gautam[1] and we will represent here in the following manner:

$$A_{p,q}^{m,n}[x|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}] = \frac{1}{2\pi i}\int_L \theta(s)\, x^s ds \qquad (1)$$

wherei = √(□ 1) and

(i) $$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(a_j+\alpha_j s)\prod_{j=1}^{n}\Gamma(1-b_j-\beta_j s)}{\prod_{j=m+1}^{p}\Gamma(1-a_j-\alpha_j s)\prod_{j=n+1}^{q}\Gamma(b_j+\beta_j s)} \qquad (2)$$

(ii) m, n, p and q are non-negative numbers in which $m \le p$, $n \le q$.

(iii) $x \neq 0$ and parameters a_j, □$_j$, b_k and □$_k$ (j = 1 to p and k = 1 to q) are all complex.

The integral in the right hand side of is convergent if

(i) $x \neq 0$, $k = 0$, $h > 0$, $|\arg(ux)| < \pi h/2$

(ii) $x > 0$, $k = 0 = h$, (ν□σω) <□ 1

where

$$k = \mathrm{Im}\left(\sum_1^p \alpha_j - \sum_1^q \beta_j\right)$$

$$h = \mathrm{Re}\left(\sum_{j=1}^{m}\alpha_j - \sum_{j=m+1}^{p}\alpha_j + \sum_{j=1}^{n}\beta_j - \sum_{j=n+1}^{q}\beta_j\right) \qquad (3)$$

$$u = \prod_1^p \alpha_j^{\alpha_j}\prod_1^q \beta_j^{\beta_j} \qquad (4)$$

$$\nu = \mathrm{Re}\left(\sum_1^p a_j - \sum_1^q b_j\right) - \frac{p-q}{2},$$

$$\omega = \mathrm{Re}\left(\sum_1^q \beta_j - \sum_1^p \alpha_j\right)$$

and s = □ + it is on path L when $|t| \to \infty$.

In our investigation we shall need the following results:

From Macrobert [2, 3]:

$$\frac{\sqrt{\pi}\,\Gamma(2-s)}{2\Gamma(\frac{3}{2}-s)}(\sin\theta)^{1-2s} = \sum_{r=0}^{\infty}\frac{(s)_r}{(2-s)_r}\sin(2r+1)\,\theta, \qquad (5)$$

where $0 < \theta \le \pi$, Re $s \le \frac{1}{2}$.

$$\frac{\sqrt{\pi}\,\Gamma(1-s)}{\Gamma(\frac{1}{2}-s)}\left(\sin\frac{\theta}{2}\right)^{-2s} = 1 + 2\sum_{r=0}^{\infty}\frac{(s)_r}{(1-s)_r}\cos r\theta, \qquad (6)$$

where $0 < \theta \le \pi$.

2. MAIN RESULT:

In this section, we shall establish following Fourier series:

$$\sum_{r=0}^{\infty} A_{p+2,q+2}^{m+1,n+1}\left[z\Big|_{\left(-\frac{1}{2},1\right),\left(b_j,\beta_j\right)_{1,q},(0,1)}^{(r,1),\left(a_j,\alpha_j\right)_{1,p},(-1-r,1)}\right]\sin(2r+1)\theta$$

$$= \frac{\sqrt{\pi}}{2} \sin\theta A_{p,q}^{m,n}\left[\frac{z}{\sin^2\theta}\Big|_{\left(b_j,\beta_j\right)_{1,q}}^{\left(a_j,\alpha_j\right)_{1,p}}\right] \tag{7}$$

provided that |arg(uz)| < ½ □h, where h and u are given in (3) and (4) respectively.

$$A_{p,q+2}^{m,n+1}\left[z\Big|_{\left(\frac{1}{2},1\right),\left(b_j,\beta_j\right)_{1,q},(0,1)}^{\left(a_j,\alpha_j\right)_{1,p}}\right]$$

$$+2A_{p+2,q+2}^{m+1,n+1}\left[z\Big|_{\left(\frac{1}{2},1\right),\left(b_j,\beta_j\right)_{1,q},(0,1)}^{(r,1),\left(a_j,\alpha_j\right)_{1,p},(-r,1)}\right]\cos r\theta$$

$$= \sqrt{\pi}A_{p,q}^{m,n}\left[\frac{z}{\sin^2\frac{\theta}{2}}\Big|_{\left(b_j,\beta_j\right)_{1,q}}^{\left(a_j,\alpha_j\right)_{1,p}}\right]. \tag{8}$$

provided that |arg(uz)| < ½ □h, where h and u are given in (3) and (4) respectively.

Proof of (7):

Using (1), the expression on the left side of (7) can be written as

$$\sum_{r=0}^{\infty} \frac{1}{2\pi i}\int_L \theta(s)\left[\frac{\Gamma\left(\frac{3}{2}-s\right)\Gamma(r+s)}{\Gamma(s)\Gamma(2+r-s)}\sin(2r+1)\,\theta\right] z^s ds$$

On changing the order of integration and summation which is easily seen to be justified, the above expression becomes

$$\frac{1}{2\pi i}\int_L \theta(s)\frac{\Gamma\left(\frac{3}{2}-s\right)}{\Gamma(2-s)}\left[\sum_{r=0}^{\infty}\frac{(s)_r}{(2-s)_r}\sin(2r+1)\,\theta\right] z^s ds.$$

and on using the relation (5), it takes the form

$$\frac{\sqrt{\pi}}{2}\sin\theta.\frac{1}{2\pi i}\int_L \theta(s)\,(z/\sin^2\theta)^s ds.$$

which is just the expression on the right side of (7). (7) is the Fourier sine series for the A-function of one variable.

The Fourier cosine series (8) is proved in an analogous manner by using (1) and (6).

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