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Allegory of Lateral Surface Area of A Cube With A Special Number

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ABSTRACT:

We try to delineate lateral surface area of a cube with a special number using pellian equation. We have also presented some special parabolas and hyperbolas generated from the linear combination between the ranks of the considered number and edge of the cube with some remarkable recurrence relations.

KEYWORDS:

Centered cubic number, Gnomonic number, Binary quadratic equation, Pell equation, Integral solutions.

2010 MATHEMATICS SUBJECT CLASSIFICATION: 11D09

NOTATIONS:

CC_n = Centered cubic number of rank n

Gno_n = Gnomonic number of rank n

$$CG_n = \frac{CC_n}{Gno_n} \quad of\ rank\ n$$

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INTRODUCTION:

Number theory is the investigation of properties of integers. It is an immense and entrancing field of arithmetic. Analytical geometry is agitated about characterizing and depicting geometrical shapes in numerical way and gathering numerical data from shape's definition and portrayals. Various books has been studied for enormous ideas on number theory[1-5].For problem solving techniques on diophantine type of equations, research articles involving diophantine equations has been referred[6-8] .Papers on geometrical figures has been studied for correlation between numbers and geometrical figures[9-19] . Multiple papers have been examined for comparing the geometrical figures with special numbers[20, 21]. Recently Connections between Cylinder, Frustum of a Cone with Truncated Octahedral Number and Other Special Numbers is illustrated[22].

In this paper we delineate the lateral surface area of the cube by a special number CG_n using pellian equation. We have also presented some special parabolas and hyperbolas generated from the linear combination between the ranks of the considered number and edge of the cube with some remarkable recurrence relations.

METHOD OF ANALYSIS:

This section consists of 3 cases. In each case lateral surface area of the cube is equated with CG_n of different ranks.

CASE (1):

Let the lateral surface area of the cube of side 'p' unit is equal to twenty eight times CG_n of rank n. The mathematical statement of our assumption is

$$4p^2 = 28CG_n \qquad (1)$$

which reduces to $y^2 = 7x^2 + 21$ (2)

where $y = 2p$ and $x = 2n - 1$ (3)

The initial solution satisfying (2) is $x_0 = 2$, $y_0 = 7$

Let us now find the general solution of (2).

The pellian equation corresponding to (2) is $y^2 = 7x^2 + 1$ (4)

The initial solution to (4) is $\tilde{x}_0 = 3$, $\tilde{y}_0 = 8$

Therefore the general solution of (4) is

$$\tilde{y}_s = \frac{1}{2} f_s \,,\ \tilde{x}_s = \frac{1}{2\sqrt{7}} g_s$$

where

$$f_s = (8 + 3\sqrt{7})^{s+1} + (8 - 3\sqrt{7})^{s+1}$$

$g_s = (8+3\sqrt{7})^{s+1} - (8-3\sqrt{7})^{s+1}, s = 0,2,4,.....$

Now by applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, we obtain the nontrivial integer solutions to (4).

$$x_{s+1} = f_s + \frac{7}{2\sqrt{7}} g_s \tag{5}$$

$$y_{s+1} = \frac{7}{2} f_s + \frac{7}{\sqrt{7}} g_s \tag{6}$$

Comparing (5) and (6) with (3) we get

$$p_{s+1} = \frac{7}{4} f_s + \frac{7}{2\sqrt{7}} g_s \tag{7}$$

$$n_{s+1} = \frac{1}{2} f_s + \frac{7}{4\sqrt{7}} g_s + \frac{1}{2} \tag{8}$$

From (7) and (8) we obtain the following recurrence relations for the ranks of CG_n and sides of the cube as

$$16n_{s+2} - (n_{s+3} + n_{s+1}) = 7$$

$$16p_{s+2} - 4(p_{s+3} + p_{s+1}) = 0$$

Some numerical examples satisfying our assumptions are given below in table1.

Table 1

S.No	n_{s+1}	p_{s+1}	L.H.S of (1)	R.H.S of (1)
0	19	49	9604	9604
2	4702	12439	618914884	618914884
4	1194163	3159457	$3.992867414 \times 10^{13}$	$3.992867414 \times 10^{13}$

OBSERVATIONS:

From the linear combination between the ranks of CG_n and edge of the cube satisfying (1), we may generate different hyperbolas and parabolas. A few examples are listed below in table2 and table3.

Table 2

S.No	x	y	HYPERBOLA
1.	$37p_{s+1} - 2p_{s+2}$	$2p_{s+2} - 28p_{s+1}$	$4x^2 - 7y^2 = 3969$
2.	$37p_{s+1} - 2p_{s+2}$	$4n_{s+2} - 74n_{s+1} + 35$	$4x^2 - 7y^2 = 3969$
3.	$n_{s+3} - 223n_{s+1} + 111$	$2p_{s+2} - 28p_{s+1}$	$7x^2 - 64y^2 = 36288$
4.	$2n_{s+2} - 28n_{s+1} + 13$	$4n_{s+2} - 74n_{s+1} + 35$	$7x^2 - y^2 = 567$
5.	$37p_{s+1} - 2p_{s+2}$	$n_{s+3} - 295n_{s+1} + 147$	$16x^2 - 7y^2 = 63504$

Table 3			
S.NO	x	y	PARABOLA
1.	$4n_{2s+2}-2p_{s+2}-5$	$n_{s+3}-295n_{s+1}+147$	$y^2=1512x-9072$
2.	$4n_{2s+2}-2p_{s+2}-5$	$4n_{s+2}-74n_{s+1}+35$	$2y^2=189x-1134$
3.	$4n_{2s+2}-2p_{s+2}-5$	$2p_{s+2}-28p_{s+1}$	$2y^2=189x-1134$
4.	$4n_{2s+2}-2p_{s+2}-5$	$p_{s+3}-223p_{s+1}$	$y^2=6048x-36288$

CASE (2):

Let the lateral surface area of the cube of side 'p' unit is equal to twenty four times CC_{n-1} of rank n-1. The mathematical statement of our assumption is

$$4p^2 = 24CG_{n-1} \quad (9)$$

This reduces to

$$y^2 = 6x^2 - 6 \quad (10)$$

where

$$y = 2p, x = 2n - 3 \quad (11)$$

The initial solution of (10) is $x_0 = 5,\ y_0 = 12$

The pellian equation corresponding to (10) is

$$y^2 = 6x^2 + 1 \quad (12)$$

The initial solution to (12) is $\tilde{x}_0 = 2, \tilde{y}_0 = 5$

Therefore the general solution corresponding to (10) is

$$\tilde{y}_s = \frac{1}{2} f_s,\ \tilde{x}_s = \frac{1}{2\sqrt{6}} g_s$$

where

$$f_s = (5+2\sqrt{6})^{s+1} + (5-2\sqrt{6})^{s+1}$$

$$g_s = (5+2\sqrt{6})^{s+1} - (5-2\sqrt{6})^{s+1}, s = -1,0,1,2,.....$$

By applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, the nontrivial integer solutions of (10) are

$$x_{s+1} = \frac{5}{2} f_s + \frac{6}{\sqrt{6}} g_s \quad (13)$$

$$y_{s+1} = 6 f_s + \frac{15}{\sqrt{6}} g_s \quad (14)$$

Comparing (13) and (14) with (11) we have

$$n_{s+1}=\frac{5}{4}f_s+\frac{3}{\sqrt{6}}g_s+\frac{3}{2} \tag{15}$$

$$p_{s+1}=3f_s+\frac{15}{2\sqrt{6}}g_s \tag{16}$$

From (15) and (16), the recurrence relations for the value of rank of $CG_{n-1}$and sides of the cube are given by

$$10n_{s+2}-(n_{s+3}+n_{s+1})=12$$

$$10p_{s+2}-(p_{s+3}+p_{s+1})=0$$

Some numerical examples satisfying our assumption are given below in table4.

Table 4

S.No	n_{s+1}	p_{s+1}	L.H.S of (9)	R.H.S of (9)
0	26	60	14400	14400
1	244	594	1411344	1411344
2	2402	5880	138297600	138297600
3	23764	58206	1.355175374×10^{10}	1.355175374×10^{10}
4	235226	576180	1.32793357×10^{12}	1.32793357×10^{12}
5	2328484	5703594	1.301239381×10^{14}	1.301239381×10^{14}

OBSERVATIONS:

From the linear combinations between the ranks of CG_{n-1} and edge of the cube satisfying (9), different hyperbolas and parabolas may be generated. Some examples are listed below in table5 and table6.

Table 5

S.NO	x	y	HYPERBOLA
1.	$20n_{s+1}-p_{s+1}-30$	$24n_{s+1}-10p_{s+1}-36$	$3x^2-2y^2=12$
2.	$4n_{s+2}-16p_{s+1}-6$	$12n_{s+1}+5p_{s+3}-50p_{s+2}-18$	$3x^2-8y^2=12$
3.	$99n_{s+1}-n_{s+3}-147$	$99p_{s+1}-p_{s+3}$	$6x^2-y^2=150$
4.	$20n_{s+1}-2n_{s+2}-27$	$p_{s+2}-10p_{s+1}$	$3x^2-2y^2=3$
5.	$4n_{s+2}-16p_{s+1}-6$	$99p_{s+1}-p_{s+3}$	$7x^2-2y^2=300$
6.	$20n_{s+1}-2n_{s+2}-27$	$24n_{s+1}-10p_{s+1}-36$	$6x^2-y^2=6$

Table 6			
S.NO	x	y	PARABOLA
1.	$20n_{2s+2}-8p_{2s+2}-28$	$24n_{s+1}-10p_{s+1}-36$	$2y^2=3x-12$
2.	$20n_{2s+2}-8p_{2s+2}-28$	$12n_{s+1}+5p_{s+3}-50p_{s+2}-18$	$8y^2=3x-12$
3.	$20n_{2s+2}-8p_{2s+2}-28$	$99p_{s+1}-p_{s+3}$	$2y^2=75x-300$
4.	$20n_{2s+2}-8p_{2s+2}-28$	$p_{s+2}-10p_{s+1}$	$8y^2=3x-12$
5.	$20n_{2s+2}-8p_{2s+2}-28$	$24n_{s+1}-p_{s+2}-36$	$8y^2=3x-12$

CASE (3):

Let the lateral surface area of the cube of sides ‘p’ unit be equal to twelve times of CG_{n-2} of rank n-2. The mathematical statement of our assumption is

$$4p^2=12CG_{n-2} \qquad (17)$$

which reduces to $y^2=3x^2+9$ (18)

where $y=2p,\ x=2n-5$ (19)

The initial solution of (18) is $x_0=3, y_0=6$

The pellian equation corresponding to (18) is

$$y^2=3x^2+1 \qquad (20)$$

The initial solution to (20) is $\tilde{x}_0=1, \tilde{y}_0=7$

Therefore the general solution to (20) are given by

$$\tilde{y}_s=\frac{1}{2}f_s \quad ; \tilde{x}_s=\frac{1}{2\sqrt{3}}g_s$$

where

$$f_s=(2+\sqrt{3})^{s+1}-(2-\sqrt{3})^{s+1}$$

$$g_s=(2+\sqrt{3})^{s+1}-(2-\sqrt{3})^{s+1}, s=\ -1,1,3,5,\ldots..$$

By applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$ nontrivial integer solutions of (18) are given by,

$$x_{s+1}=\frac{3}{2}f_s+\frac{3}{\sqrt{3}}g_s \qquad (21)$$

$$y_{s+1}=3f_s+\frac{9}{2\sqrt{3}}g_s \qquad (22)$$

Comparing (21) and (22) with (19) we have

$$p_{s+1}=\frac{3}{2}f_s+\frac{9}{4\sqrt{3}}g_s$$

$$n_{s+1} = \frac{3}{4} f_s + \frac{3}{2\sqrt{3}} g_s + \frac{5}{2}$$

Some numerical examples satisfying (17) are listed below in table 7

Table 7

S	n_{s+1}	p_{s+1}	L.H.S Of (1)	R.H.S Of(1)
-1	4	3	36	36
1	25	39	6084	6084
3	316	543	1179396	1179396
5	4369	7568	228795876	228795876

OBSERVATIONS:

From the linear combination between the rank of CG_{n-2} and edge of the cube satisfying (17), different hyperbolas and parabolas may be generated. Some numerical examples are listed below in table8 and table9.

Table 8

S.No	x	y	HYPERBOLA
1.	$60p_{s+1} - 4p_{s+3}$	$8p_{s+2}$ $-28p_{s+1}$	$3x^2 - 16y^2 = 1728$
2.	$p_{s+2} - 4p_{s+1}$	$3p_{s+3} - 39p_{s+1}$	$108x^2 - y^2 = 243$
3.	$4n_{s+2} - 14n_{s+1} + 25$	$4n_{s+2} - 16n_{s+1} + 30$	$4x^2 - 3y^2 = 36$
4.	$31n_{s+1} - n_{s+3} - 75$	$15n_{s+1} - n_{s+3} - 35$	$4x^2 - 3y^2 = 2304$
5.	$4p_{s+3} - 15p_{s+2}$	$21p_{s+3} - 78p_{s+2}$	$432x^2 - 16y^2 = 972$

Table 9

S.No	x	y	PARABOLA
1.	$4p_{2s+2} - 6n_{2s+2} + 18$	$4n_{s+2}$ $-16n_{s+1} + 30$	$y^2 = 2x - 12$
2.	$4p_{2s+2} - 6n_{2s+2} + 18$	$15n_{s+1} - n_{s+3} - 35$	$y^2 = 128x - 768$
3.	$4p_{2s+2} - 6n_{2s+2} + 18$	$21p_{s+3} - 78p_{s+2}$	$16y^2 = 162x - 972$
4.	$4p_{2s+2} - 6n_{2s+2} + 18$	$8p_{s+2} - 28p_{s+1}$	$y^2 = 18x - 108$
5.	$4p_{2s+2} - 6n_{2s+2} + 18$	$3p_{s+3} - 39p_{s+1}$	$4y^2 = 162x - 972$

CONCLUSION:

In this paper we have represented lateral surface area of the cube with a special number of different ranks and presented some notable relations. One can also try the same with any other geometrical figures and special numbers.

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