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## Ranking Interval valued Fuzzy Numbers

## S.Shunmugapriya*and G.Uthra

Department of Mathematics, Pachaiyappa's College for Men, Chennai, Tamilnadu, India.
Email: priya010978@ gmail.com
Department of Mathematics, Pachaiyappa's College for Men, Chennai, Tamilnadu, India.
Email:uthragopalsamy@yahoo.com


#### Abstract

: In this paper,interval valuedfuzzy numbershave been defined and a new ranking formula have been proposed.The membershiparea of the fuzzy numbers aresplitted into plane figures and centroid of the centroids of these plane figures are calculated. The ranking formula is calculated by finding the area of this centroid from the origin. The advantage of this paper is that the ranking IVFN by this approach yields better solution when compared with other ranking methods. This approach is illustrated with numerical examples.


KEY WORDS:Triangular fuzzy number, Interval valued fuzzy number, centroid ranking method.

## *Corresponding author

## S.Shunmugapriya

Assistant Professor,
Department of Mathematics,
Pachaiyappa's College for Men, Chennai, Tamilnadu, India.
Email: priya010978@gmail.com

## INTRODUCTION:

Generally, fuzzy numbers are employed to express uncertainity. Type 1 fuzzy set theory is used to deal with imprecision. But it is not always possible for a membership function of the type to assign one point from[0,1]. For a fuzzy number, the degree of the membership is a crisp number whereas the degree of the membership for an interval valued fuzzy number is an interval.

In May 1975, Sambuc ${ }^{1}$ presented in his doctoral research, the concept of IVFS named as $\phi$ fuzzy set. Interval valued fuzzy sets were suggested by Gorzlczany ${ }^{2}$ andTurksen ${ }^{3}$. Wnag and $\mathrm{Li}^{4}$ defined interval valued fuzzy numbers IVFN and gave their extended operations.Stephen Dinagar and Abirami ${ }^{6}$ proposed an analytical method for finding critical path using IVFNS in fuzzy project network. $\mathrm{Feng}^{7}$ solved job shop scheduling problem with imprecise processing time as $(\lambda, 1)$ interval valued fuzzy numbers.Abirami and Stephen ${ }^{8}$ developed a new approach on ranking L-R type interval valued fuzzy numbers.

In this paper we define triangular, trapezoidal, pentagonal , hexagonal and octagonal interval valued fuzzy numbers and develop new ranking technique on those fuzzy numbers which are more efficient.

## INTERVAL VALUED TRIANGULAR FUZZY NUMBER:

Atriangularfuzzy number A issaidto be an interval valued triangular fuzzy number(IVTFN) in the parameter $b_{1} \leq a_{1} \leq b_{2} \leq a_{2} \leq a_{3} \leq b_{3}$ denoted by $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}\right),\left(b_{1}, b_{2}, b_{3}\right) ; w_{A}^{L}, w_{A}^{U}\right\}$, $0 \leq w_{A}^{L} \leq w_{A}^{U} \leq 1$ if its membership functions are as follows.
$\mu_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x})=\left\{\begin{array}{cl}0 & x<a_{1} \\ \frac{W_{A}^{L}\left(x-a_{1}\right)}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\ W_{A}^{L}-\frac{W_{A}^{L}\left(x-a_{3}\right)}{a_{3}-a_{2}}, & a_{2} \leq x \leq a_{3} \\ 0, & x>a_{3}\end{array}\right\}$
$\mu_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x})=\left\{\begin{array}{cc}0 & x<b_{1} \\ \frac{W_{A}^{U}\left(x-b_{1}\right)}{b_{2}-b_{1}}, & b_{1} \leq x \leq b_{2} \\ W_{A}^{U}-\frac{W_{A}^{U}\left(x-b_{3}\right)}{b_{3}-b_{2}}, & b_{2} \leq x \leq b_{3} \\ 0, & x>b_{3}\end{array}\right\}$


Consider the IVTFN A=\{( $\left.\left.a_{1}, a_{2}, a_{3}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, w_{A}^{U}\right) ;\right\}$, The centroid of a traingle is considered to be the balancing point of the triangle.
The Centroid of the triangle ABC is $=\left(\frac{a_{1}+a_{2}+a_{3}}{3}, \frac{w_{A}^{L}}{3}\right)$
Now we define $\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{a_{1}+a_{2}+a_{3}}{3} * \frac{w_{A}^{L}}{3}\right)$
This is the area between the centroid of the centroid and the original point.
Similarly $\mathrm{S}\left(\mu_{A}^{U}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{b_{1}+b_{2}+b_{3}}{3} * \frac{w_{A}^{U}}{3}\right)$
Using the above definitions, the rank of A is defined as follows:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$

## INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBER

We define a Trapezoidal fuzzy number A to be anterval valued Trapezoidal fuzzy number(IVTrFN) in the parameter $b_{1} \leq a_{1} \leq b_{2} \leq a_{2} \leq a_{3} \leq b_{3} \leq a_{4} \leq b_{4}$ denoted by $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, w_{A}^{U}\right) ;\right\}, 0 \leq w_{A}^{L} \leq w_{A}^{U} \leq 1$, if its membership functions are as follows.

$$
\mu_{A}^{L}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<a_{1} \\
\frac{w_{A}^{L}\left(x-a_{1}\right)}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\
w_{A}^{L}, & a_{2} \leq x \leq a_{3} \\
w_{A}^{L}-\frac{\left(w_{A}^{L}\right)\left(x-a_{3}\right)}{a_{4}-a_{3}}, & a_{3} \leq x \leq a_{4} \\
0, & x>a_{4}
\end{array}\right\}
$$

$$
\mu_{A}^{U}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<b_{1} \\
\frac{w_{A}^{U}\left(x-b_{1}\right)}{b_{2}-b_{1}}, & b_{1} \leq x \leq b_{2} \\
w_{A}^{U}, & b_{2} \leq x \leq b_{3} \\
w_{A}^{U}-\frac{\left(w_{A}^{U}\right)\left(x-b_{3}\right)}{b_{4}-b_{3}}, & b_{3} \leq x \leq b_{4} \\
0, & x>b_{4}
\end{array}\right\}
$$



Consider the IVTrFN $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, w_{A}^{U}\right)\right\}$. The centroid of a Trapezoid is considered to be the balancing point of the Trapezoid. Divide the membership part of trapezoid into three plane figures namely a triangle, a quadrilateral (kite) and a triangle respectively. Let $G_{1}, G_{2}, G_{3}$ be the centroids of these three plane figures.

The Centroid of these centroids $G_{1}, G_{2}, G_{3}$ is considered as the point of reference to define the ranking of generalized Interval valued fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIPFN.

The Centroids of these plane figures are $G_{1}=\left(\frac{a_{1}+2 a_{2}}{3}, \frac{w_{A}^{L}}{3}\right) ; \quad G_{2}=\left(\frac{a_{2}+a_{3}}{2}, \frac{w_{A}^{L}}{2}\right)$ and $\quad G_{3}=\left(\frac{2 a_{3}+a_{4}}{3}, \frac{w_{A}^{L}}{3}\right)$ respectively.
Thus $\mathrm{G}_{1}, \mathrm{G}_{2}$ and $\mathrm{G}_{3}$ are not collinear and they form a triangle. Thus the centroid of these centroids is
$\mathrm{G}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(2 a_{1}+7 a_{2}+7 a_{3}+2 a_{4}\right)}{18}, \frac{7 w_{A}^{L}}{18}\right)$
Now we define $\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\frac{\left(2 a_{1}+7 a_{2}+7 a_{3}+2 a_{4}\right)}{18} \cdot \frac{7 w_{A}^{L}}{18}$
This is the area between the centroid of the centroids and the original point.
Similarly the trapezoid corresponding to the upper membership function is divided into three plane figures. In similar fashion, the centroid of the three plane figures and the centroid of these centroids are evaluated. The centroid of these plane figures are
$G_{1}=\left(\frac{b_{1}+2 b_{2}}{3}, \frac{W_{A}^{U}}{3}\right) ; \quad G_{2}=\left(\frac{b_{2}+b_{3}}{2}, \frac{w_{A}^{U}}{2}\right)$ and $G_{3}=\left(\frac{2 b_{3}+b_{4}}{3}, \frac{w_{A}^{U}}{3}\right)$ respectively.
Thus $\mathrm{G}_{1}, \mathrm{G}_{2}$ and $\mathrm{G}_{3}$ are not collinear and they form a triangle. Thus the centroid of these centroids is
$\mathrm{G}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(2 b_{1}+7 b_{2}+7 b_{3}+2 b_{4}\right)}{18}, \frac{7 w_{A}^{U}}{18}\right)$
Now we define $\mathrm{S}\left(\mu_{A}^{U}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\frac{\left(2 b_{1}+7 b_{2}+7 b_{3}+2 b_{4}\right)}{18} \cdot \frac{7 w_{A}^{U}}{18}$
Using the above definitions, the rank of A is defined as follows:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$

## INTERVAL VALUED PENTAGONAL FUZZY NUMBER

We define apentagonal fuzzy number A to be aInterval valued pentagonal fuzzy number(IVPFN) in the parameter $b_{1} \leq a_{1} \leq b_{2} \leq a_{2} \leq b_{3} \leq a_{3} \leq a_{4} \leq b_{4} \leq a_{5} \leq b_{5}$ denoted by $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, w_{A}^{U}\right) ;\right\}, \quad 0 \leq k_{1} \leq k_{2} \leq w_{A}^{L} \leq w_{A}^{U} \leq 1$, if its membership functions are as follows.

$$
\begin{aligned}
& \mu_{A}^{L}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<a_{1} \\
K_{1}-\frac{K_{1}\left(x-a_{2}\right)}{a_{1}-a_{2}}, & a_{1} \leq x \leq a_{2} \\
w_{A}^{L}+\frac{\left(K_{1}-w_{A}^{L}\right)\left(x-a_{3}\right)}{a_{2}-a_{3}}, & a_{2} \leq x \leq a_{3} \\
w_{A}^{L}+\frac{\left(K_{1}-w_{A}^{L}\right)\left(x-a_{3}\right)}{a_{4}-a_{3}}, & a_{3} \leq x \leq a_{4} \\
K_{1}-\frac{K_{1}\left(x-a_{4}\right)}{a_{5}-a_{4}}, & a_{4} \leq x \leq a_{5} \\
0, & x>a_{5}
\end{array}\right\} \\
& \mu_{A}^{U}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<b_{1} \\
K_{2}-\frac{K_{2}\left(x-b_{2}\right)}{b_{1}-b_{2}}, & b_{1} \leq x \leq b_{2} \\
w_{A}^{L}+\frac{\left(K_{2}-w_{A}^{L}\right)\left(x-b_{3}\right)}{b_{2}-b_{3}}, & b_{2} \leq x \leq b_{3} \\
w_{A}^{L}+\frac{\left(K_{2}-w_{A}^{L}\right)\left(x-b_{3}\right)}{b_{4}-b_{3}}, & b_{3} \leq x \leq b_{4} \\
K_{1}-\frac{K_{2}\left(x-b_{4}\right)}{b_{5}-b_{4}}, & b_{4} \leq x \leq b_{5} \\
0, & x>b_{5}
\end{array}\right\}
\end{aligned}
$$



Consider the IVPFN $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, w_{A}^{U}\right)\right\}$. The centroid of a pentagon is considered to be the balancing point of the pentagon. Divide the lower membership part of pentagon into three plane figures namely a quadrilateral (kite ) and two triangles. Let $G_{1}, G_{2}, G_{3}$ be the centroids of these three plane figures.

The Centroid of these centroids $G_{1}, G_{2}, G_{3}$ is considered as the point of reference to define the ranking of generalized pentagonal Institutionistic fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIPFN.

The Centroids of these plane figures are

$$
G_{1}=\left(\frac{a_{1}+a_{2}+a_{3}}{3}, \frac{K_{1}}{3}\right) ; \quad G_{2}=\left(\frac{a_{2}+a_{3}+a_{4}}{3}, \frac{w_{A}^{L}+K_{1}}{3}\right) \text { and } \quad G_{3}=\left(\frac{a_{3}+a_{4}+a_{5}}{3}, \frac{K_{1}}{3}\right) \text { respectively. }
$$

Thus $G_{1}, G_{2}$ and $G_{3}$ are not collinear and they form a triangle. Thus the centroid of these centroids is

$$
\mathrm{G}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(a_{1}+2 a_{2}+3 a_{3}+2 a_{4}+a_{5}\right)}{9}, \frac{3 K_{1}+w_{A}^{L}}{9}\right)
$$

Now we define $\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(a_{1}+2 a_{2}+3 a_{3}+2 a_{4}+a_{5}\right)}{9}\right) \times \frac{3 K_{1}+w_{A}^{L}}{9}$.
This is the area between the centroid of the centroids and the original point.
Similarly the pentagon corresponding to the upper membership function is divided into three plane figures. In similar fashion, the centroid of the three plane figures and the centroid of these centroids are evaluated. The centroid of these plane figures are

$$
G_{1}=\left(\frac{b_{1}+b_{2}+b_{3}}{3}, \frac{K_{1}}{3}\right) ; G_{2}=\left(\frac{b_{2}+b_{3}+b_{4}}{3}, \frac{w_{A}^{U}+K_{1}}{3}\right) ; G_{3}=\left(\frac{b_{3}+b_{4}+b_{5}}{3}, \frac{K_{1}}{3}\right) .
$$

The centroid of these centroids is

$$
\mathrm{G}^{\prime}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(b_{1}+2 b_{2}+3 b_{3}+2 b_{4}+b_{5}\right)}{9}, \frac{3 K_{1}+w_{A}^{U}}{9}\right) .
$$

Now we define $\mathrm{S}\left(\mu_{A}^{U}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(b_{1}+2 b_{2}+3 b_{3}+2 b_{4}+b_{5}\right)}{9}\right) \times \frac{3 K_{1}+w_{A}^{U}}{9}$
Using the above definitions, the rank of A is defined as follows:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$

## INTERVAL VALUED HEXAGONAL FUZZY NUMBER

We definehexagonal fuzzy number $A$ to be an interval valued hexagonal fuzzy number(IVHFN) in the parameter $b_{1} \leq a_{1} \leq b_{2} \leq a_{2} \leq b_{3} \leq a_{3} \leq a_{4} \leq b_{4} \leq a_{5} \leq b_{5} \leq a_{6} \leq$ $b_{6}$ denoted byA= $\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, b_{6}, w_{A}^{U}\right)\right\}, 0 \leq k_{1} \leq k_{2} \leq w_{A}^{L} \leq$ $w_{A}^{U} \leq 1$, ifits membership functions are as follows.

$$
\begin{aligned}
& \mu_{A}^{L}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<a_{1} \\
\frac{K_{1}\left(x-a_{1}\right)}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\
K_{1}+\frac{\left(w_{A}^{L}-K_{1}\right)\left(x-a_{3}\right)}{a_{3}-a_{2}}, & a_{2} \leq x \leq a_{3} \\
w_{A}^{L} & a_{3} \leq x \leq a_{4} \\
w_{A}^{L}+\frac{\left(K_{1}-w_{A}^{L}\right)\left(x-a_{4}\right)}{a_{4}-a_{3}}, & a_{4} \leq x \leq a_{5} \\
K_{1}-\frac{K_{1}\left(x-a_{5}\right)}{a_{6}-a_{5}}, & a_{5} \leq x \leq a_{6} \\
0, & x>a_{6}
\end{array}\right\} \\
& \mu_{A}^{U}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<b_{1} \\
\frac{K_{2}\left(x-b_{1}\right)}{b_{2}-b_{1}}, & b_{1} \leq x \leq b_{2} \\
K_{2}+\frac{\left(w_{A}^{U}-K_{2}\right)\left(x-b_{3}\right)}{b_{3}-b_{2}}, & b_{2} \leq x \leq b_{3} \\
w_{A}^{U} & b_{3} \leq x \leq b_{4} \\
w_{A}^{U}+\frac{\left(K_{2}-w_{A}^{U}\right)\left(x-b_{4}\right)}{b_{4}-b_{3}}, & b_{4} \leq x \leq b_{5} \\
K_{2}-\frac{K_{2}\left(x-b_{5}\right)}{b_{6}-b_{5}}, & b_{5} \leq x \leq b_{6} \\
0, & x>b_{6}
\end{array}\right\}
\end{aligned}
$$



Consider the IVHFN $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, b_{6}, w_{A}^{U}\right)\right\}$. The centroid of a Hexagon is considered to be the balancing point of the Hexagon. The centroid of Hexagon is the centroid of centroids $G_{1}, G_{2}, G_{3}$ which is considered as the point of reference to define the ranking of generalized hexagonal Intuitionistic fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIHFN.

The Centroids of these plane figures are

$$
G_{1}=\left(\frac{a_{1}+a_{2}+a_{3}}{3}, \frac{k_{1}}{3}\right), G_{3}=\left(\frac{a_{4}+a_{5}+a_{6}}{3}, \frac{k_{1}}{3}\right) \text { and } G_{2}=\left(\frac{2 a_{1}+7 a_{3}+7 a_{4}+2 a_{5}}{18}, \frac{7, w_{A}^{L}+4 k_{1}}{18}\right)^{\text {respectively }} .
$$

From the above figure, the centroid of hexagon is calculated as
$\mathrm{G}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(6 a_{1}+8 a_{2}+13 a_{3}+13 a_{4}+8 a_{5}+6 a_{6}\right)}{54}, \frac{7 w_{A}^{L}+16 k_{1}}{54}\right)$
Now we define $\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(6 a_{1}+8 a_{2}+13 a_{3}+13 a_{4}+8 a_{5}+6 a_{6}\right)}{54}\right) \times \frac{7 w_{A}^{L}+16 k_{1}}{54}$
This is the area between the centroid of the hexagon and the original point Similarly
we define $\mathrm{S}\left(\mu_{A}^{U}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(6 b_{1}+8 b_{2}+13 b_{3}+13 b_{4}+8 b_{5}+6 b_{6}\right)}{54}\right) \times \frac{7 w_{A}^{U}+16 k_{2}}{54}$
Using the above definitions, the rank of A is defined as follows:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$

## INTERVAL VALUED OCTAGONAL FUZZY NUMBER:

An Octagonal fuzzy number $A$ is said to be a interval valued Octagonal fuzzy number(IVOFN) in the parameter $b_{1} \leq a_{1} \leq b_{2} \leq a_{2} \leq b_{3} \leq a_{3} \leq b_{4} \leq a_{4} \leq a_{5} \leq b_{5} \leq a_{6} \leq$ $b_{6} \leq a_{7} \leq b_{7} \leq a_{8} \leq b_{8}$ denotedby
$\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, a_{7}, a_{8}, w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, b_{6}, b_{7}, b_{8}, w_{A}^{U}\right)\right\}, \quad 0 \leq k_{1} \leq k_{2} \leq w_{A}^{L} \leq$ $w_{A}^{U} \leq 1$, if its membership functions are as follows.
$\mu_{A}^{L}(\mathrm{x})=\left\{\begin{array}{cc}0 & x<a_{1} \\ \frac{k_{1}\left(x-a_{1}\right)}{a_{2}-a_{1}}, & a_{1} \leq x \leq a_{2} \\ k_{1}, & a_{2} \leq x \leq a_{3} \\ k_{1}+\frac{\left(w_{A}^{L}-K_{1}\right)\left(x-a_{3}\right)}{a_{4}-a_{3}}, & a_{3} \leq x \leq a_{4} \\ w_{A}^{L} a_{4} \leq x \leq a_{5} \\ w_{A}^{L}+\frac{\left(K_{1}-w_{A}^{L}\right)\left(x-a_{5)}\right.}{a_{6}-a_{5}}, & a_{5} \leq x \leq a_{6} \\ K_{1}, & a_{6} \leq x \leq a_{7} \\ K_{1}-\frac{K_{1}\left(x-a_{7}\right)}{a_{8}-a_{7}}, & a_{7} \leq x \leq a_{8} \\ 0, & x>a_{8}\end{array}\right\}$

$$
\mu_{A}^{U}(\mathrm{x})=\left\{\begin{array}{cc}
0 & x<b_{1} \\
\frac{k_{2}\left(x-b_{1}\right)}{b_{2}-b_{1}}, & b_{1} \leq x \leq b_{2} \\
k_{2}, & b_{2} \leq x \leq b_{3} \\
k_{2}+\frac{\left(w_{A}^{U}-K_{2}\right)\left(x-b_{3}\right)}{b_{4}-b_{3}}, & b_{3} \leq x \leq b_{4} \\
w_{A}^{U} b_{4} \leq x \leq b_{5} \\
w_{A}^{U}+\frac{\left(K_{2}-w_{A}^{U}\right)\left(x-b_{5}\right)}{b_{6}-b_{5}}, & b_{5} \leq x \leq b_{6} \\
K_{2}, & b_{6} \leq x \leq b_{7} \\
K_{2}-\frac{K_{2}\left(x-b_{7}\right)}{b_{8}-b_{7}}, & b_{7} \leq x \leq b_{8} \\
0, & x>b_{8}
\end{array}\right\}
$$



In similar fashion, The centroid of octagon is calculated as
$\mathrm{G}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\frac{\left(2 a_{1}+7 a_{2}+9 a_{3}+9 a_{4}+9 a_{5}+9 a_{6}+7 a_{7}+2 a_{8}\right)}{54}, \frac{7 w_{A}^{L}+18 k_{1}}{54}\right)$
Now we define $\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(2 a_{1}+7 a_{2}+9 a_{3}+9 a_{4}+9 a_{5}+9 a_{6}+7 a_{7}+2 a_{8}\right)}{54}\right) \times \frac{7 w_{A}^{L}+18 k_{1}}{54}$.
This is the area between the centroid of the centroids and the original point.
Similarly,
Now we define $\mathrm{S}\left(\gamma_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\left(\frac{\left(2 b_{1}+7 b_{2}+9 b_{3}+9 b_{4}+9 b_{5}+9 b_{6}+7 b_{7}+2 b_{8}\right)}{54}\right) \times . \frac{7 w_{A}^{U}+18 k_{2}}{54}$
Using the above definitions, the rank of A is defined as follows:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$

## ARITHMETIC OPERATIONS:

If $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4}\right),\left(b_{1}, b_{2}, b_{3}, b_{4}\right) ; w_{A}^{L}, w_{A}^{U}\right\}$, and
$\mathrm{B}=\left\{\left(c_{1}, c_{2}, c_{3}, c_{4}\right),\left(d_{1}, d_{2}, d_{3}, d_{4}\right) ; w_{B}^{L}, W_{B}^{U}\right\}$, are two IVTFN then
$\left.\mathrm{A}+\mathrm{B}=\left\{\left(a_{1}+c_{1}, a_{2}+c_{2}, a_{3}+c_{2}, a_{4}+c_{4}\right), b_{1}+d_{1}, b_{2}+d_{2}, b_{3}+d_{3}, b_{4}+d_{4}\right) ; \mathrm{w}, \mathrm{u}\right\}$ where $\mathrm{w}=$ $\min \left\{w_{A}^{L}, w_{B}^{L}\right\}, \mathrm{u}=\max \left\{w_{A}^{U}, w_{B}^{U}\right\}$
$\mathrm{A}-\mathrm{B}=\left\{\left(a_{1}-c_{4}, a_{2}-c_{3}, a_{3}-c_{2}, a_{4}-c_{1}\right)\left(b_{1}-d_{4}, b_{2}-d_{3}, b_{3}-d_{2}, b_{4}-d_{1}\right) ; w, u\right\}$ where $\mathrm{w}=$ $\min \left\{w_{A}^{L}, w_{B}^{L}\right\}, \mathrm{u}=\max \left\{w_{A}^{U}, w_{B}^{U}\right\}$
$\mathrm{A} * \mathrm{~B}=\left\{\left(a_{1} \cdot R(B), a_{2} \cdot R(B), a_{3} \cdot R(B), a_{4} \cdot R(B)\right)\left(b_{1} \cdot R(B), b_{2} \cdot R(B), b_{3} \cdot R(B), b_{4} \cdot R(B)\right) ;\right.$ $\mathrm{w}, \mathrm{u}\}$ where $\mathrm{w}=\min \left\{w_{A}^{L}, w_{B}^{L}\right\}, \mathrm{u}=\max \left\{w_{A}^{U}, w_{B}^{U}\right\}$ if $\mathrm{R}(\mathrm{B})>0$ $\mathrm{A} / \mathrm{B}=\left\{\left(\frac{a_{1}}{R(B)}, \frac{a_{2}}{\mathrm{R}(\mathrm{B})}, \frac{a_{3}}{\mathrm{R}(\mathrm{B})}, \frac{a_{4}}{\mathrm{R}(\mathrm{B})}\right)\left(\frac{b_{1}}{\mathrm{R}(\mathrm{B})}, \frac{b_{2}}{\mathrm{R}(\mathrm{B})}, \frac{b_{3}}{\mathrm{R}(\mathrm{B})}, \frac{b_{4}}{\mathrm{R}(\mathrm{B})}\right) ; \mathrm{w}, \mathrm{u}\right\}$ where $\mathrm{w}=\min \left\{w_{A}^{L}, w_{B}^{L}\right\}, \mathrm{u}=$ $\max \left\{w_{A}^{U}, w_{B}^{U}\right\}$
In Similar fashion the arithmetic operations can be defined for other fuzzy numbers.

## NUMERICAL ILLUSTRATION.

Consider the Interval valued trapezoidal fuzzy number denoted by $\mathrm{A}=\left\{\left(a_{1}, a_{2}, a_{3}, a_{4} ; w_{A}^{L}\right),\left(b_{1}, b_{2}, b_{3}, b_{4} ; w_{A}^{U}\right)\right\}=\{(2,4,5,7 ; 1)(1,3,6,8 ; 1)$

Rank of A by the proposed method:
$\mathrm{R}(\mathrm{A})=\frac{w_{A}^{L} \mathrm{~S}\left(\mu_{A}^{L}\right)+w_{A}^{U} \mathrm{~S}\left(\mu_{A}^{U}\right)}{w_{A}^{L}+w_{A}^{U}}$ where
$\mathrm{S}\left(\mu_{A}^{L}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\frac{\left(2 a_{1}+7 a_{2}+7 a_{3}+2 a_{4}\right)}{18} \cdot \frac{7 w_{A}^{L}}{18}=1.75$
$\mathrm{S}\left(\mu_{A}^{U}\right)=\mathrm{x}_{0} \cdot \mathrm{y}_{0}=\frac{\left(2 b_{1}+7 b_{2}+7 b_{3}+2 b_{4}\right)}{18} \cdot \frac{7 w_{A}^{U}}{18}=1.75$
$\mathrm{R}(\mathrm{A})=1.75$

Rank of A defined by the existing method in [6]
$\mathrm{R}(\mathrm{A})=\frac{\left(a_{1}+a_{2}+a_{3}+a_{4}+b_{1}+b_{2}+b_{3}+b_{4}\right)}{8}=4.5$

## CONCLUSION:

This paper proposes interval valued triangular, trapezoidal, Pentagonal, hexagonal and Octagonalfuzzy numbers along with a efficient ranking technique. This centroid based ranking method gives more efficient result and is illustrated with a numerical example.

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