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On Complex Numbers by Untraditional Method of Number Building Blocks - 5

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ABSTRACT

In this unattempted system of number theory, a number building block is a ratio of two consecutive numbers and is defined as a function $f(x)$ which is exponential in x. This system based on number building blocks, is proved to be applicable to complex numbers also. On the basis of this system, we have given altogether new series expansions for Pi, natural logarithm of 2, tangent inverse of any real quantity, natural logarithm of any real quantity and have also determined their actual values. In addition, we have explained the concept of expression of a complex number in exponential form. This unconventional method of number building blocks has disclosed a number of interesting results including new algebraic identities and has also opened new doors in number theory where multiplications and divisions of quantities are easier than addition and subtraction.

KEYWORDS: Numbers, Approximation, Complex Number Building Blocks, Exponentiation, Logarithm, Pi, Series Expansion, Tangent Inverse.

AMS SUBJECT CLASSIFICATION: Complex Analysis 97I80, 37K20, 97F50

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INTRODUCTION

Numbers traditionally proceed adding unity to their magnitude and are expressible by simply writing their magnitude. In this paper numbers proceed by multiplication of a function $f(x)$ which is ratio of two consecutive numbers $x/(x-1)$ and is expressed as

$$f(x) \simeq x/(x-1) \qquad (1a)$$

Sign $\simeq$ denotes approximation and is used in above equation in stead of sign of equality $=$ owing to the fact that $f(x)$ is an exponential function and second, this system has inherent characteristics of approximation. When x is appreciably large, approximation is largely exact and when x tends to be infinitely large, approximation tends to exactness. This function $f(x)$ that equals is also called number building block and builds number x from $(x-1)$ and x can be written as $(x-1)\cdot f(x)$.

In this system of number theory, number 1 can not be created from 0 as number building block in this particular case will be $(1/0)$ which is not defined and further multiplication of 0 with $(1/0)$ is also not defined. This is the reason why numbers are built from 2 onwards. Theory of this system is not applicable to numbers less than 2 and more than -2. However, by taking reciprocals, numbers can be generated from $1/2$ to 0 also but 2 to $1/2$, numbers can not be built. With a little manipulation, by enlarging the number by a factor n which is a large quantity, all numbers including 2 to ½ can be generated.

In case of negative numbers, these proceed with decreasing magnitude, therefore, -1 is generated from -2 but 0 can not be generated from -1 on the same grounds as discussed earlier. Therefore, in general, it can be taken as this system is applicable for positive numbers, when x either equals or is more than 2 and for negative numbers, when x equals -1 or is less than -1 unless otherwise mentioned. For simplicity and uniformity, this theory is applicable to modulus x written as $|x|$ which must be equal or more than 2. It is further submitted if only number building block is mentioned, it means it pertains to creation of number x from $(x-1)$. Numbers here do not mean integers only but these can be fractions also satisfying relation $|x|$ is equal or more than 2. Moot question is what $f(x)$ is in exponential form. By mathematics of approximation, f(x) can be derived as

$$f(x) \simeq e^{\frac{2}{2.x-1-1/x^3}} \simeq x/(x-1)\ . \qquad (1b)$$

Or

$$x/(x-1) \simeq \exp[2/(2x-1-1/x^3)]$$

where $\exp[2/(2x-1-1/x^3)]$ means $e^{1/(2x-1-1/x^3)}$. Or

$$\ln\{x/(x-1)\} \simeq 2/(2x-1-1/x^3)$$

Details of this derivation are not given here as it is not the subject matter of this paper. If x is appreciably large, $1/x^3$ can be ignored then number building block can be written as

$$x/(x-1) \simeq e^{\left(\frac{2}{2.x-1}\right)} \quad or\ \ln\{x/(x-1)\} \simeq 1/(2x-1) \qquad (1c)$$

Formula $x/(x-1) \simeq e^{2/(2x-1)}$ yields error when x is in the vicinity of 2 but error gets reduced when x is large. To have better approximation nearby 2 and also above 2, equation (1c) is used.

THEORY AND CONCEPT

Splitting Real Number Building Block Into Complex Number Building Block

As already stated, $x/(x-1) \simeq e^{1/(2x-1-1/x^3)}$ and if x on left hand side (LHS) is replaced by y that is $y = x$, then $y \simeq (x-1) \cdot e^{1/(2x-1-1/x^3)}$ is a universal equation of straight line with unit slope where e is natural number and x is any number as explained earlier but with condition $|x|$ must be equal or more than 2. If x is large, $1/x^3$ can be neglected and equation can be written as $x \simeq (x-1) \cdot e^{2/(2x-1)}$. If $(x-1)$ is transposed to left hand side by cross multiplication, then $x/(x-1)$ on left hand side is a number building block when x is real number as it has been elaborated in introduction. Right hand side that is $e^{2/(2x-1)}$ which equals $x/(x-1)$ is also a number building in exponential form for large x.

In z alphabet, $z/(z-1)$ can also be written as

$$\frac{z}{z-1} = \frac{\{(z-1)+1\}}{z-1} = 1 + \frac{1}{z-1} \tag{1d}$$

Let y be a number such that $y^2 = z-1$, then

$$\frac{z}{z-1} = \frac{y^2+1}{y^2} = \left(\frac{i\cdot y+1}{i\cdot y}\right).\left(\frac{-i\cdot y+1}{-i\cdot y}\right) = A \cdot A^* \tag{1e}$$

where A is a complex number and A^* is conjugate given by

$$A = \left(\frac{i\cdot y+1}{i\cdot y}\right), \tag{1f}$$

$$A^* = \left(\frac{-i\cdot y+1}{-i\cdot y}\right) \tag{1g}$$

and i is pure imaginary number equal to $\sqrt{-1}$. That is $i = \sqrt{-1}$. Therefore, a number building block can be split up as product of two complex number building blocks (hereinafter abbreviated as CNBB) in the manner as

$$\frac{y^2+1}{y^2} = \left(\frac{i\cdot y+1}{i\cdot y}\right) \cdot \left(\frac{-i\cdot y+1}{-i\cdot y}\right) = A \cdot A^* \tag{1h}$$

Lemma 1

A real number building block can be the product of two complex number building blocks which are conjugate of each other. Conversely, if there is a complex number building block, there will always exist an another complex number building block which is conjugate of the former and product of complex number building block with its conjugate is always a real number building block. That means imaginary number building blocks and its conjugate gives birth to a real number building block. Applying equation (1) which is applicable to real number, to these complex number building blocks A and A^*,

$$i \cdot y + 1 \simeq (i \cdot y) \cdot e^{2/(2 \cdot i \cdot y + 2 - 1)} \simeq (i \cdot y) \cdot e^{2/(2 \cdot i \cdot y + 1)} \quad (2a)$$

$$-i \cdot y + 1 \simeq (-i \cdot y) \cdot e^{2/(-2 \cdot i \cdot y + 2 - 1)} \simeq (-i \cdot y) \cdot e^{2/(-2 \cdot i \cdot y + 1)} \quad (2b)$$

It is clear from equations (2a) and (2b) that multiplier in right hand sides$(i \cdot y)$ and $(-i \cdot y)$ are one less than the complex numbers $(i \cdot y + 1)$ and $(-i \cdot y + 1)$ respectively on left hand sides. In real number building block, $|z|$ equals or is more than 2, applying the analogy since magnitude of complex number in the above case is always real, $|\sqrt{1 + y^2}|$ should be equal or more than 2. On dividing both left hand side LHS and right hand side RHS of equations (2a) and (2b) by i, equations can be written

$$y - i \simeq (y) \cdot e^{2/(2 \cdot i \cdot y + 1)} \quad (2c)$$

$$-y - i \simeq (-y) \cdot e^{2/(-2 \cdot i \cdot y + 1)} \quad (2d)$$

Coming to equation (1d)

$\frac{z}{z-1} = 1 + \frac{1}{z-1}$, on putting $z - 1 = \frac{1+y^2}{3}$ then

$$\frac{z}{z-1} = \frac{1 + \frac{1+y^2}{3}}{\frac{1+y^2}{3}} = \frac{4+y^2}{1+y^2} = \left(\frac{2+i \cdot y}{1+i \cdot y}\right)\left(\frac{2-i \cdot y}{1-i \cdot y}\right) \quad (2e)$$

On putting $z - 1 = \frac{4+y^2}{5}$ then

$$\frac{z}{z-1} = \frac{9+y^2}{4+y^2} = \left(\frac{3+i \cdot y}{2+i \cdot y}\right)\left(\frac{3-i \cdot y}{2-i \cdot y}\right).$$

On putting $z - 1 = \frac{9+y^2}{7}$ then

$$\frac{z}{z-1} = \frac{16+y^2}{9+y^2} = \left(\frac{4+i \cdot y}{3+i \cdot y}\right)\left(\frac{4-i \cdot y}{3-i \cdot y}\right)$$

… ..

By mathematical induction, on putting

$$z - 1 = \frac{(x-1)^2 + y^2}{2.x - 1} \quad (2f)$$

$$\frac{z}{z-1} = \frac{x^2+y^2}{(x-1)^2+y^2} = \left(\frac{x+i \cdot y}{x-1+i \cdot y}\right)\left(\frac{x-i \cdot y}{x-1-i \cdot y}\right) \quad (2g)$$

From above equations, it is explicit that by putting $z - 1 = y^2$, number building block $\frac{z}{z-1}$ equals product of two complex number building blocks $\left(\frac{1+i \cdot y}{i \cdot y}\right)\left(\frac{1-i \cdot y}{-i \cdot y}\right)$.

By putting $z - 1 = \frac{1+y^2}{3}$, number building block $\frac{z}{z-1}$ equals product of two complex number building blocks $\left(\frac{2+i \cdot y}{1+i \cdot y}\right)\left(\frac{2-i \cdot y}{1-i \cdot y}\right)$,

By putting $z - 1 = \frac{4+y^2}{5}$, number building block $\frac{z}{z-1}$ equals product of two complex number building blocks $\left(\frac{3+i \cdot y}{2+i \cdot y}\right)\left(\frac{3-i \cdot y}{2-i \cdot y}\right)$ and finally

By putting $z-1=\frac{(x-1)^2+y^2}{2x-1}$, number building block $\frac{z}{z-1}$ equals product of two complex number building blocks $\left(\frac{x+i.y}{x-1+i.y}\right)\left(\frac{x-i.y}{x-1-i.y}\right)$.

Lemma 2

In general, a real number building block $\frac{z}{z-1}$ is product of two complex number building blocks $\left(\frac{x+i\cdot y}{x-1+i\cdot y}\right)\left(\frac{x-i\cdot y}{x-1-i\cdot y}\right)$, where $z=1+\frac{(x-1)^2+y^2}{2x-1}$, and $|z|$ or $|1+\frac{(x-1)^2+y^2}{2x-1}|$ equals or is more than 2.

From equation (2g), $\frac{z}{z-1}=\frac{x^2+y^2}{(x-1)^2+y^2}=\left(\frac{x+i\cdot y}{x-1+i\cdot y}\right)\left(\frac{x-i\cdot y}{x-1-i\cdot y}\right),=\frac{x^2+y^2}{(x-1)^2+y^2}=1+\frac{1}{\left\{\frac{(x-1)^2+y^2}{2x-1}\right\}}$

and from equation (2f), $z=1+\frac{(x-1)^2+y^2}{2.x-1}$, therefore, from equation (1a) i.e. $z\simeq(z-1)e^{\frac{2}{2z-1}}$,

$$z=\frac{x^2+y^2}{2\cdot x-1}\simeq\left(\frac{x^2+y^2}{2\cdot x-1}-1\right)\cdot e^{\frac{2(2x-1)}{2x^2+2y^2-2x+1}}$$

On simplification,

$$\frac{x^2+y^2}{x^2+y^2-2x+1}\simeq e^{\frac{2(2x-1)}{2x^2+2y^2-2x+1}}\simeq e^{\frac{2x-1}{\left(x-\frac{1}{2}\right)^2+y^2+\frac{1}{4}}} \qquad \text{(2h)}$$

Taking log

$$\ln\left(\frac{x^2+y^2}{x^2+y^2-2x+1}\right)\simeq\frac{2(2x-1)}{x^2+y^2-2x+1}\simeq\frac{2x-1}{\left(x-\frac{1}{2}\right)^2+y^2+\frac{1}{4}}$$

Assuming x or y or both x so large that $1/4$ can be neglected then $\left(x-\frac{1}{2}\right)^2+y^2+\frac{1}{4}$ can be approximated as $\left(x-\frac{1}{2}\right)^2+y^2$. Further, referring to equation (1c), negative term that is $(-1/z^3)$ has already been neglected, therefore, its effects were also to reduce error due to positive ¼ and if x and y were large, effect of $(1/4-1/z^3)$ can be neglected. On ignoring effect of $(1/4-1/z^3)$ as discussed,

$$\ln\left(\frac{x^2+y^2}{x^2+y^2-2x+1}\right)\simeq\frac{2x-1}{\left(x-\frac{1}{2}\right)^2+y^2}$$

Taking log

$$\ln\left(\frac{x+i\cdot y}{x-1+i\cdot y}\right)+\ln\left(\frac{x-i\cdot y}{x-1-i\cdot y}\right)\simeq\frac{2}{2x+2i\cdot y-1}+\frac{2}{2x-2i\cdot y-1}$$

From symmetry, above equation can be split into two equations,

$$\ln\left(\frac{x+i\cdot y}{x-1+i\cdot y}\right)\simeq\frac{2}{2x+2i\cdot y-1}$$

and

$$\ln\left(\frac{x - i \cdot y}{x - 1 - i \cdot y}\right) \simeq \frac{2}{2x - 2i \cdot y - 1}.$$

Taking antilog,

$$\frac{x + i \cdot y}{x - 1 + i \cdot y} \simeq e^{\frac{2}{2x+2i \cdot y-1}},$$

$$\frac{x - i \cdot y}{x - 1 - i \cdot y} \simeq e^{\frac{2}{2x-2i \cdot y-1}}$$

Above are equations of complex number building blocks and these are similar to equations of real number building blocks given by equation (1). These equations are best approximated when effect of $(1/4 - 1/z^3)$ can be neglected in view of large value x and y. Therefore, when x and y are small, these may yield error. In order to eliminate error, x and y are multiplied with large positive integer number n so as to increase their value by factor n. Effective x as $n.x$ and y as $n.y$ will thus reduce error appreciably. Data in Table 1 proves correctness of these equations. Inspection of the figures given in the table reveals that there is zero or very very small error for complex numbers $(1 + i)$, $(2 + 3i)$, $(3 + i)$ where removal of error by multiplication with n has been adopted. Removal of error by multiplication factor will further be explained at appropriate paragraph. For all intent and purposes, effect of $(1/4 - 1/z^3)$ in denominator is neglected in view of multiplication factor n which is large positive integer as compared to $(1/4 - 1/z^3)$. Complex number building block $\frac{x+i \cdot y}{x-1+i \cdot y}$ which is analogous to $\frac{z}{z-1}$ if z is considered as $(x + i.y)$ as proved above can be written as

$$\frac{x+i \cdot y}{x-1+i \cdot y} \simeq e^{\frac{2}{2x+2i \cdot y-1}} \tag{3}$$

Or

$$(x + i \cdot y) \simeq (x + i \cdot y - 1).e^{\frac{2}{2x+2i \cdot y-1}}$$

and its conjugate complex number building block as

$$\frac{x - i \cdot y}{x - 1 - i \cdot y} \simeq e^{\frac{2}{2x-2i \cdot y-1}}$$

Or

$$(x - i \cdot y) \simeq (x - i \cdot y - 1) \cdot e^{\frac{2}{2x-2i \cdot y-1}}$$

On dividing numerator and denominator of LHS of equation (3) by i, it transforms to

$$\frac{y - i \cdot x}{y - i \cdot x + i} \simeq e^{\frac{2}{2x+2i \cdot y-1}} \simeq e^{\frac{-i2}{2y-2ix+i}}$$

$$y - i \cdot x + i \simeq (y - i \cdot x)e^{\frac{2i}{2y-2ix+i}}$$

If complex number is $y + i.x$ then

$$y + i \cdot x - i \simeq (y + i \cdot x)e^{\frac{-2i}{2y+2ix-i}} \tag{3*}$$

However, in stead equation (3*), equation (3) will be used extensively in this paper.

Conversion Of A Complex Number To Product Of Complex Numbers In Exponential Form And Other Complex Number, That Is From Complex Number $a + i \cdot b$ To Product $(c + i \cdot d) \cdot e^{f+i \cdot g}$

It has already been proved in equation (3) that $\frac{x+i \cdot y}{x-1+i \cdot y} = 1 + \frac{1}{x+i \cdot y-1} \simeq e^{\frac{2}{2x+2i \cdot y-1}}$. On rationalising exponent $\frac{2}{2x-1+2i \cdot y}$ into real and imaginary parts, equation (3) can be written as

$$(x + i \cdot y) \simeq (x + i \cdot y - 1)e^{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}} \cdot e^{\frac{-i \cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}} \tag{3a}$$

By transposition and rationalisation,

$$\frac{x^2 - x + y^2}{(x-1)^2 + y^2} + i.\frac{y(2x-1)}{(x-1)^2 + y^2} \simeq e^{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}} \cdot e^{\frac{-i.y}{\left(x-\frac{1}{2}\right)^2+y^2}}$$

On taking logarithm of both sides of equation (3a),

$$\ln\left(\frac{x+iy}{x+iy-1}\right) \simeq \frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2} - i\frac{y}{\left(x-\frac{1}{2}\right)^2+y^2} \tag{3b}$$

These equations are universally true and give better approximation when $|(x^2 + y^2)^{\frac{1}{2}}|$ equals or is more than 2. When these are less than 2, equations start giving errors. Values of x or y or both are enlarged by multiplication factor n so as to reduce error as discussed earlier.

Approximation When $|(x^2 + y^2)^{\frac{1}{2}}|$ Is In The Proximity Of Two Or Less Than Two – Removal Of Error

As complex number $(x + i \cdot y)$ has magnitude $\sqrt{x^2 + y^2}$ and if this magnitude is less or in proximity of 2, it will yield error as denominator of exponent $\frac{2}{2x-1+2i \cdot y}$ will not be large enough to ignore $(1/4 - 1/z^3)$.Therefore, it is desirable to increase x and y. Let there be any large real positive integer "n" so that when it is multiplied with $(x + i \cdot y)$, it transforms the complex number into $n \cdot x + i \cdot n \cdot y$. It is n times larger than the original complex number. This will have CNBB as

$$\frac{n(x+i \cdot y)}{(n \cdot x+i \cdot n \cdot y-1)} \simeq e^{\frac{2}{2n \cdot x+2i \cdot n \cdot y-1}} \tag{4a}$$

Denominator of exponent has increased approximately n times. On cancelling n in numerator and denominator, numerator in left hand side is restored to its original value $(x + i \cdot y)$, therefore,

$$(x + i \cdot y) \simeq \left(x + i \cdot y - \frac{1}{n}\right) e^{\frac{2}{2n \cdot x+2i \cdot n \cdot y-1}} \tag{4b}$$

This transformed CNBB $\frac{n\cdot(x+i\cdot y)}{(n\cdot x+i\cdot n\cdot y-1)}$ which can also be written $\frac{(x+i\cdot y)}{\left(x+i\cdot y-\frac{1}{n}\right)}$ will yield better approximation. Larger the value of n, better will be the approximation. For example, if a complex number is $1+i$ or $1-i$ or its magnitude is less than 2 or any other complex number whose magnitude is even less, it will yield error. Also if a complex number is $1/m+i/m$ where m is any number more than 1, then magnitude of this complex number will be less than 2, here also multiplying the complex number by n and adopting the same procedure, modified CNBB can be obtained and that will give better approximation. Therefore, in such cases, complex number is multiplied by n which is a large integer and the error is eliminated.

Complex Number Building Block Transfers Another Complex Number To Exponent

It is already explained how a real number building block comprises of two complex number building block and a CNBB can be written as $(x+i\cdot y)\simeq(x+i\cdot y-1)\cdot e^{\frac{2}{2x+2i\cdot y-1}}$. It is evident from above that a complex number yields two factors on RHS as $(x+i\cdot y-1)$ and $e^{\frac{2}{2x+2i\cdot y-1}}$. Latter factor has exponent $2/(2x+2i\cdot y-1)$ and is a complex number. It leads to the conclusion that a complex number by method of CNBB transposes another complex number to the exponent of natural number 'e'.

Thus $(x+i\cdot y)\simeq(x+i\cdot y-1)\cdot e^{\frac{2}{2x+2i\cdot y-1}}$. On rationalising $2/(2x+2i\cdot y-1)$ into real and imaginary parts, $\frac{2}{2x+2i\cdot y-1}=\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}-i\cdot\frac{y}{\left(x-\frac{1}{2}\right)^2+y^2}$, then

$$(x+i\cdot y)\simeq(x+i\cdot y-1)e^{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}}\cdot e^{\frac{-i.y}{\left(x-\frac{1}{2}\right)^2+y^2}} \tag{5a}$$

Or

$$\ln(x+i\cdot y)\simeq\ln(x+i\cdot y-1)+\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}-i.\frac{y}{\left(x-\frac{1}{2}\right)^2+y^2}$$

Similarly,

$$(x-i\cdot y)\simeq(x-i\cdot y-1)\cdot e^{\left\{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}\cdot e^{\left\{\frac{i.y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}} \tag{5b}$$

Data in the table given below compares magnitude of LHS $(x^2+y^2)^{\frac{1}{2}}$ with $\{(x-1)^2+y^2\}^{\frac{1}{2}}\cdot e^{\frac{x-\frac{1}{2}}{\left\{\left(x-\frac{1}{2}\right)^2+y^2\right\}}}$ of equation (3b) in the given Table.

Table 1 Showing Magnitude Of Complex Number And That Calculated With CNBB Method

$x+i\cdot y$	Magnitude $(x^2+y^2)^{\frac{1}{2}}$	$\{(x-1)^2+y^2\}^{\frac{1}{2}}\cdot e^{\left\{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}$	Percentage Difference
$1+i$	1.41421	1.41424*	.0000266
$2+3i$	3.60555	3.60555*	.000000
$3+i$	3.16227	3.16227*	.000000
$3+5i$	5.83095	5.83367	.0468087
$7+i$	7.07106	7.06920	−.0263404
$8+10i$	12.80624	12.80675	.0039976
9+7i	11.40175	11.40209	.0030150
10+11i	14.86606	14.86640	.0023500
12+13i	17.69180	17.692038	.0013446
$11+14i$	17.80449	17.80475	.001476

In the table, data marked * pertains to calculations done adopting method of removal by multiplication with a large number n and approximation is as good as exactness. From above Table, it is clear that magnitude of a complex number matches with that value calculated from complex number building block with minor error. This error goes on decreasing as magnitude goes on increasing. The above Table proves that in equation (3a) magnitude of $e^{\left\{\frac{-i.y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}$ is unity.

Dividing LHS $(x+i\cdot y)$ by its magnitude $(x^2+y^2)^{\frac{1}{2}}$

and RHS $(x+i\cdot y-1)\cdot e^{\left\{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}\cdot e^{\left\{\frac{-i.y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}$ by its magnitude $\{(x-1)^2+y^2\}^{\frac{1}{2}}\cdot e^{\left\{\frac{x-\frac{1}{2}}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}$, resultant equation is

$$\frac{x}{(x^2+y^2)^{\frac{1}{2}}}+i\cdot\frac{y}{(x^2+y^2)^{\frac{1}{2}}}\simeq e^{\left\{\frac{-i.y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}\cdot\left[\frac{x-1}{\{(x-1)^2+y^2\}^{\frac{1}{2}}}+i\cdot\frac{y}{\{(x-1)^2+y^2\}^{\frac{1}{2}}}\right].$$

Since complex number on LHS and RHS are divided by respective magnitude, therefore, both have unit magnitude. On transposing and rationalising,

$$\frac{x^2+y^2-x-i\cdot y}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}}\simeq e^{\left\{\frac{-iy}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}} \tag{6a}$$

And this has a unit magnitude. Its conjugate

$$\frac{x^2+y^2-x+i\cdot y}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}}\simeq e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}} \tag{6b}$$

also has a unit magnitude. In complex plane, $+i$ is taken on + Y axis and $-i$ on -Y axis and real part is taken on +X axis and minus real part on – X axis. Since complex number has unit magnitude, real part of LHS of equation (6a) is $\frac{x^2+y^2-x}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}}$ and imaginary part of LHS is

$\frac{-i\cdot y}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}}$. If a unit magnitude complex number building block CNBB makes an angle A with real axis then

$$\cos(A) \simeq \frac{x^2+y^2-x}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}} \tag{7a}$$

$$\sin(A) \simeq \frac{y}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}} \tag{7b}$$

Therefore, from equation (6a),

$$\cos(A) - i\cdot\sin(A) \simeq e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}} \tag{8a}$$

From these equations, it is proved that if natural number *e* has its power which is imaginary, then it has real and imaginary parts. Real part equals cosine of an angle and imaginary sine of an angle. To which quantity that angle corresponds will be proved in appropriate paragraph. Similarly, LHS of equation (6b) can written as

$$\cos(A) + i\cdot\sin(A) \simeq e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}} \tag{8b}$$

Also on adding equations (8) and (8/1),

$$\mathrm{Cos}(A) \simeq \frac{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}+e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}{2} \tag{9a}$$

$$\sin(A) \simeq \frac{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}-e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}{2i} \tag{9b}$$

From equations (7) and (9),

$$\cos(A) \simeq \frac{x^2+y^2-x}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}} \simeq \frac{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}+e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}{2} \tag{9c}$$

Similarly, from equations (7b) from (9b) ,

$$\sin(A) \simeq \frac{y}{(x^2+y^2)^{\frac{1}{2}}\cdot\{(x-1)^2+y^2\}^{\frac{1}{2}}} \simeq \frac{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}-e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}{2i} \tag{9d}$$

From above

$$\tan \mathrm{A} = \frac{y}{x^2+y^2-x} \simeq \frac{1}{i}\cdot\left[\frac{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}-e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}{e^{\left\{\frac{i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}+e^{\left\{\frac{-i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}}\right] \tag{10a}$$

Or

$$\tan A \simeq \frac{1}{i}\cdot\left[\frac{e^{\left\{\frac{2i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}-1}{e^{\left\{\frac{2i\cdot y}{\left(x-\frac{1}{2}\right)^2+y^2}\right\}}+1}\right] \tag{10b}$$

where A is the angle that CNBB makes with X axis.

Converting Complex Number Into Pure Exponential Form, That Is From $a + i.b$ to $e^{c+i.d}$.

In equations (2a) to (2d), it was proved, if a complex number has unit real part or unit imaginary part or complex number is reducible to $y + i$ or $y - i$ or $1 + i \cdot y$ or $1 - i \cdot y$ form then complex number transfers another complex number as exponent to base e. For convenience, these equations are reproduced below

$$y - i \simeq (y) \cdot e^{\frac{2}{2i \cdot y+1}} \simeq (y) \cdot e^{2\left(\frac{1-2i \cdot y}{1+4y^2}\right)}$$

$$y + i \simeq (y) \cdot e^{\frac{2}{-2i \cdot y+1}} \simeq (y) \cdot e^{2\left(\frac{1+2i \cdot y}{1+4y^2}\right)}$$

$$i \cdot y + 1 \simeq (i \cdot y) \cdot e^{\frac{2}{2i \cdot y+1}} \simeq (i \cdot y) \cdot e^{2\left(\frac{1-2i \cdot y}{1+4y^2}\right)}$$

$$-i \cdot y + 1 \simeq (-i \cdot y) \cdot e^{\frac{2}{-2i \cdot y+1}} \simeq (-i \cdot y) \cdot e^{2\left(\frac{1-2i \cdot y}{1+4y^2}\right)}$$

Inspection of these equations reveals that these still have y or $i \cdot y$ as a factors of exponential terms in RHS. In this section, method will be explained as to how factors of y or $i \cdot y$ can be got ridden of and a complex number $a + i \cdot b$ can be written in the manner $e^{c+i \cdot d}$ using complex number building blocks. Let the complex number be $x + i \cdot y$. From equation (3), it can be written in complex number building block system as

$i \cdot y + x \simeq (i \cdot y + x - 1) \cdot e^{\frac{2}{2x+2i \cdot y-1}}$ and$(i \cdot y + x - 1)$ can be written as

$(i \cdot y + x - 1) \simeq (i \cdot y + x - 2) \cdot e^{\frac{2}{2x+2i \cdot y-3}}$ and $(i \cdot y + x - 2)$ can be written as

$(i \cdot y + x - 2) \simeq (i \cdot y + x - 3) \cdot e^{\frac{2}{2x+2i \cdot y-5}}$,

… … … … … … … … … … … … … … …. so on

… … … … … … … … … … … … … … … …. and $i \cdot y + 3$ can be written as

$(i \cdot y + 3) \simeq (i \cdot y + 2) \cdot e^{\frac{2}{5+2i \cdot y}}$ and $i \cdot y + 2$ can be written as

$(i \cdot y + 2) \simeq (i \cdot y + 1) \cdot e^{\frac{2}{3+2i \cdot y}}$ and $i \cdot y + 1$ can be written as

$(i \cdot y + 1) \simeq (i \cdot y) \cdot e^{\frac{2}{1+2i \cdot y}}$

On multiplying all these terms,

$$i \cdot y + x \simeq (i \cdot y) \cdot e^{\frac{2}{2x+2i \cdot y-1}} \cdot e^{\frac{2}{2x+2i \cdot y-3}} \cdot e^{\frac{2}{2x+2i \cdot y-5}} \cdot \ldots \cdot e^{\frac{2}{5+2i \cdot y}} \cdot e^{\frac{2}{3+2i \cdot y}} \cdot e^{\frac{2}{1+2i \cdot y}} \tag{14a}$$

On dividing RHS and LHS by i,

$$y - i \cdot x \simeq (y) \cdot e^{\frac{2}{2x+2i \cdot y-1}} \cdot e^{\frac{2}{2x+2i \cdot y-3}} \cdot e^{\frac{2}{2x+2i \cdot y-5}} \cdot \ldots \cdot e^{\frac{2}{5+2i \cdot y}} \cdot e^{\frac{2}{3+2i \cdot y}} \cdot e^{\frac{2}{1+2i \cdot y}} \tag{14b}$$

To write these equations easily, logarithm of both sides is taken,

$$\ln(y-i\cdot x) \simeq \ln(y) + \frac{2}{2x-1+2i\cdot y} + \frac{2}{2x-3+2i\cdot y} + \frac{2}{2x-5+2i\cdot y} + \cdots + \frac{2}{5+2i\cdot y} + \frac{2}{3+2i\cdot y} + \frac{2}{1+2i\cdot y}.$$

On rationalising term $\frac{2}{2x-1+2i\cdot y}$ by multiplying and dividing by $(2x-1-2i\cdot y)$, it transforms to $\frac{2(2x-1)}{(2x-1)^2+4y^2} - i\cdot\frac{4y}{(2x-1)^2+4y^2}$. Similarly, all the remaining terms are rationalised, added and the equation transforms to

$$\ln(y-i\cdot x) \simeq \ln(y) + \frac{2(2x-1)}{(2x-1)^2+4y^2} - i\cdot\frac{4y}{(2x-1)^2+4y^2} + \frac{2(2x-3)}{(2x-3)^2+4y^2} - i\cdot\frac{4y}{(2x-3)^2+4y^2} + \frac{2(2x-5)}{(2x-5)^2+4y^2} - i\cdot\frac{4y}{(2x-5)^2+4y^2} + \cdots + \frac{2}{1+4y^2} - i\cdot\frac{4y}{1+4y^2}.$$

On rearranging by writing the last term first and proceeding to first and also separating real and imaginary parts,

$$\ln(y-i\cdot x) \simeq \left[\ln(y) + 2\left\{\frac{1}{1+4y^2} + \frac{3}{3^2+4y^2} + \frac{5}{5^2+4y^2} + \cdots + \frac{2x-1}{(2x-1)^2+4y^2}\right\}\right] - 4i\cdot y\left[\left\{\frac{1}{1+4y^2} + \frac{1}{3^2+4y^2} + \frac{1}{5^2+4y^2} + \cdots + \frac{1}{(2x-1)^2+4y^2}\right\}\right] \quad (15a)$$

Real part of equation (15) is

$$\ln(y-i\cdot x) \simeq \left[\ln(y) + 2\left\{\frac{1}{1+4y^2} + \frac{3}{3^2+4y^2} + \frac{5}{5^2+4y^2} + \cdots \frac{2x-1}{(2x-1)^2+4y^2}\right\}\right] \quad (15b)$$

Imaginary part of equation (15) is

$$4y\left[\left\{\frac{1}{1+4y^2} + \frac{1}{3^2+4y^2} + \frac{1}{5^2+4y^2} + \cdots + \frac{1}{(2x-1)^2+4y^2}\right\}\right] \quad (15c)$$

Therefore,

$$\ln(y-i\cdot x) \simeq \ln(y) + 2\sum_{n=1}^{x}\left\{\frac{2n-1}{(2n-1)^2+4y^2}\right\} - 4i\cdot y\sum_{n=1}^{x}\left\{\frac{1}{(2n-1)^2+4y^2}\right\} \quad (16)$$

where $\sum_{x=1}^{n}$ denotes summation of terms when x varies from 1 to n. Also in earlier paper, it was proved that

$$\ln(n) \simeq 2\sum_{x=2}^{n}\left(\frac{1}{2x-1}\right) + 2\sum_{x=2}^{n}\left\{\frac{1}{x^3.(2x-1)^2}\right\} \quad (17)$$

and that paper is under consideration and therefore, it is not being proved here. Equation (16) can be written as

$$\ln(y-i\cdot x) \simeq 2.\sum_{n=2}^{y}\left(\frac{1}{2n-1}\right) + 2.\sum_{n=2}^{y}\left\{\frac{1}{n^3.(2n-1)^2}\right\} + 2.\sum_{n=1}^{x}\left\{\frac{2n-1}{(2n-1)^2+4y^2}\right\} - 4i\cdot y\sum_{n=1}^{x}\left\{\frac{1}{(2n-1)^2+4y^2}\right\} \quad (18a)$$

Let

$$p = 2\sum_{n=2}^{y}\left(\frac{1}{2n-1}\right) = 2\left\{\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2y-1}\right\} \quad (18b)$$

$$q = 2\sum_{n=2}^{y}\left\{\frac{1}{n^3(2n-1)^2}\right\} = 2\left\{\frac{1}{2^3.3^2} + \frac{1}{3^3.5^2} + \frac{1}{4^3.7^2} + \cdots + \frac{1}{y^3(2y-1)}\right\} \quad (18c)$$

$$r = 2\sum_{n=1}^{x}\left\{\frac{2n-1}{(2n-1)^2+4y^2}\right\} = 2\left\{\frac{1}{1+4y^2} + \frac{3}{3^2+4y^2} + \frac{5}{5^2+4y^2} + \cdots + \frac{2x-1}{(2x-1)^2+4y^2}\right\} \quad (18d)$$

$$s = 4y\sum_{n=1}^{x}\left\{\frac{1}{(2n-1)^2+4y^2}\right\} = 2\left\{\frac{1}{1+4y^2} + \frac{1}{3^2+4y^2} + \frac{1}{5^2+4y^2} + \cdots + \frac{1}{(2x-1)^2+4y^2}\right\} \quad (18e)$$

Equation (18a) can be written as

$$\ln(y - i \cdot x) \simeq p + q + r - i \cdot s \tag{19a}$$

On taking antilog, it can be written as

$$(y - i \cdot x) \simeq e^{p+q+r-i \cdot s} \tag{19b}$$

Similarly, $(y + i \cdot x)$ which is conjugate complex number can be written by changing sign of i as

$(y + i \cdot x) \simeq e^{p+q+r+i.s}$

(19c)

where p, q, r and s have values as stated above. In this way, a complex number $(y + i \cdot x)$ or $(y - i \cdot x)$ can be written purely in exponential form.

When Complex Number Is Of $x + i \cdot y$ Form.

We have already derived the formula for $\ln(y - i \cdot x)$ as

$$\ln(y - i \cdot x) \simeq \ln(y) + 2\sum_{n=1}^{x}\left\{\frac{2n-1}{(2n-1)^2+4y^2}\right\} - 4i \cdot y\sum_{n=1}^{x}\left\{\frac{1}{(2n-1)^2+4y^2}\right\} \tag{20}$$

Above equation pertains to $(y - i \cdot x)$ for $(x + i \cdot y)$, y will replace x along with sign of i.

From above, complex number $(x + i \cdot y)$ is given by equation

$$\ln(x + i \cdot y) \simeq \ln(x) + 2\sum_{n=1}^{y}\left\{\frac{2n-1}{(2n-1)^2+4x^2}\right\} + 4i \cdot x\sum_{n=1}^{y}\left\{\frac{1}{(2n-1)^2+4x^2}\right\} \tag{21}$$

$$\ln(x + i \cdot y) \simeq$$
$$2\sum_{n=2}^{x}\left(\frac{1}{2n-1}\right) + 2\sum_{n=2}^{x}\left\{\frac{1}{n^3 \cdot (2n-1)^2}\right\} + 2\sum_{n=1}^{y}\left\{\frac{2n-1}{(2n-1)^2+4x^2}\right\} + 4i \cdot x\sum_{n=1}^{x}\left\{\frac{1}{(2n-1)^2+4x^2}\right\} \tag{22a}$$

Or $\ln(x + i \cdot y) \simeq p' + q' + r' + i.s'$.

$$\ln(x + i \cdot y) \simeq \ln(x) + 2\sum_{n=1}^{y}\left\{\frac{2n-1}{(2n-1)^2+4x^2}\right\} + 4i \cdot x\sum_{n=1}^{y}\left\{\frac{1}{(2n-1)^2+4x^2}\right\} \tag{22b}$$

On taking antilog, it can be written as $x + i \cdot y \simeq e^{p'+q'+r'+i.s'}$. Or

$$x + i \cdot y \simeq x \cdot e^{r'+i \cdot s'} \tag{22c}$$

And its conjugate

$$x - i \cdot y \simeq x \cdot e^{r'-i \cdot s'} \tag{22d}$$

where

$$p' = 2\sum_{n=2}^{x}\left(\frac{1}{2n-1}\right) = 2\left\{\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2n-1}\right\}$$

$$q' = 2\sum_{n=2}^{x}\left\{\frac{1}{n^3(2n-1)}\right\} = 2\left\{\frac{1}{2^3 \cdot 3^2} + \frac{1}{3^3 \cdot 5^2} + \frac{1}{4^3 \cdot 7^2} + \cdots + \frac{1}{n^3(2n-1)^2}\right\}$$

$$r' = 2\sum_{n=1}^{y}\left\{\frac{2n-1}{(2n-1)^2 + 4x^2}\right\} = 2\left\{\frac{1}{1+4x^2} + \frac{3}{3^2+4x^2} + \frac{5}{5^2+4x^2} + \cdots + \frac{2y-1}{(2y-1)^2+4x^2}\right\}$$

$$s' = 4x\sum_{n=1}^{y}\left\{\frac{1}{(2n-1)^2 + 4x^2}\right\} = 4x\left\{\frac{1}{1+4x^2} + \frac{1}{3^2+4x^2} + \frac{1}{5^2+4x^2} + \cdots + \frac{1}{(2y-1)^2+4x^2}\right\}$$

Examples

In these examples, main stress is upon converting imaginary numbers into exponential form as exercises on converting real number to exponential forms have already been done in my earlier paper.

a. Using complex number building blocks, convert $\frac{4}{100}+\frac{i}{10}$ to exponential form.

This can be written as $\frac{1}{100}.(4+10.i)$. Therefore,

$$\begin{aligned}\ln(4+10\cdot \mathrm{i}) \simeq &\ln(4)\\ &+2\left\{\frac{1}{1+4(4^2)}+\frac{3}{3^2+4(4^2)}+\frac{5}{5^2+4(4^2)}+\frac{7}{7^2+4(4^2)}+\frac{9}{9^2+4(4^2)}\right.\\ &\left.+\frac{11}{11^2+4(4^2)}+\frac{13}{13^2+4(4^2)}+\frac{15}{15^2+4(4^2)}+\frac{17}{17^2+4(4^2)}+\frac{19}{19^2+4(4^2)}\right\}\\ &+\mathrm{i}.16\left\{\frac{1}{1+4(4^2)}+\frac{1}{3^2+4(4^2)}+\frac{1}{5^2+4(4^2)}+\frac{1}{7^2+4(4^2)}+\frac{1}{9^2+4(4^2)}\right.\\ &\left.+\frac{1}{11^2+4(4^2)}+\frac{1}{13^2+4(4^2)}+\frac{1}{15^2+4(4^2)}+\frac{1}{17^2+4(4^2)}+\frac{1}{19^2+4(4^2)}\right\}\\ &Or\ \ln(4+i\cdot 10)\ \simeq \ln(4)+.99339447378154+i(1.190537132278)\end{aligned}$$

$$4+i\cdot 10\simeq 4\cdot e^{.99339+i(1.19054)}\ and\ \frac{4}{100}+\frac{i}{10}\simeq\left(\frac{4}{100}\right).e^{.99339+i(1.19054)}$$

b. Using complex number building blocks, convert $i-\frac{4}{10}$ to exponential form.

Here $x=4/10$ and $y=1$. This can also be written as $-\left(\frac{1}{10}\right)(4-10i)$

Therefore,

$$\begin{aligned}\ln(4-10\cdot \mathrm{i}) \simeq &\ln(4)\\ &+2\left\{\frac{1}{1+4(4^2)}+\frac{3}{3^2+4(4^2)}+\frac{5}{5^2+4(4^2)}+\frac{7}{7^2+4(4^2)}+\frac{9}{9^2+4(4^2)}+\frac{11}{11^2+4(4^2)}\right.\\ &\left.+\frac{13}{13^2+4(4^2)}+\frac{15}{15^2+4(4^2)}+\frac{17}{17^2+4(4^2)}+\frac{19}{19^2+4(4^2)}\right\}\\ &-\mathrm{i}.16\left\{\frac{1}{1+4(4^2)}+\frac{1}{3^2+4(4^2)}+\frac{1}{5^2+4(4^2)}+\frac{1}{7^2+4(4^2)}+\frac{1}{9^2+4(4^2)}\right.\\ &\left.+\frac{1}{11^2+4(4^2)}+\frac{1}{13^2+4(4^2)}+\frac{1}{15^2+4(4^2)}+\frac{1}{17^2+4(4^2)}+\frac{1}{19^2+4(4^2)}\right\}\\ &Or\ \ln(4-i\cdot 10)\ \simeq \ln(4)+.99339447378154-i(1.190537132278)\end{aligned}$$

$$4-i\cdot 10\simeq 4.e^{.99339-i(1.19054)},\qquad \text{therefore,}\qquad i-\frac{4}{10}\simeq-\left(\frac{1}{10}\right)e^{.99339-i(1.19054)}$$

c. Using complex number building blocks, convert $-10-i\cdot 15$ to exponential form.

Here x is -10 and y is -15. This can also be written as $-(10+i\cdot 15)$. Therefore,

$$\ln(10 + i \cdot 15) \simeq \ln 10 \; + 2\left\{\frac{1}{1 + 4 \cdot 10^2} + \frac{3}{3^2 + 4 \cdot 10^2} + \frac{5}{5^2 + 4 \cdot 10^2} + \cdots + \frac{29}{29^2 + 4 \cdot 10^2}\right\} + i \cdot 40\left\{\frac{1}{1 + 4 \cdot 10^2} + \frac{1}{3^2 + 4 \cdot 10^2} + \frac{1}{5^2 + 4 \cdot 10^2} + \cdots + \frac{1}{29^2 + 4 \cdot 10^2}\right\}$$

Or

$$\ln(10 + i \cdot 15) \; \simeq \ln 10) + .589794256 + i(.9829120174).$$

Or

$$-(10 + i15) \simeq -10e^{.589794256 + i(.9829120174)}$$

***Pure imaginary number iota i in exponential form and its relation with* π.**

Equations (19/1) and (19/2) are reproduced below

$$(\mathrm{y} - \mathrm{i} \cdot \mathrm{x}) \simeq e^{p+q+r-i \cdot s}$$
$$(\mathrm{y} + \mathrm{i} \cdot \mathrm{x}) \simeq e^{p+q+r+i \cdot s}$$

Let $y = x = n$ where n large positive integer then

$$(\mathrm{n} - \mathrm{i} \cdot \mathrm{n}) \simeq e^{p+q+r-i \cdot s}$$
$$(\mathrm{n} + \mathrm{i} \cdot \mathrm{n}) \simeq e^{p+q+r+i \cdot s}$$

where

$$p + q = ln(n)$$

$$r = 2\sum_{x=1}^{n} \frac{2x - 1}{(2x - 1)^2 + 4n^2},$$

$$s = 4n\sum_{x=1}^{n} \frac{1}{(2x - 1)^2 + 4n^2}.$$

Therefore,

$$n - i \cdot n \simeq ne^{r - i \cdot s}$$
$$n + i \cdot n \simeq ne^{r + i \cdot s}$$

Or

$$1 - i \simeq e^{r - i \cdot s} \quad (23)$$
$$1 + i \simeq e^{r + i \cdot s} \quad (24)$$

Dividing equation (24) by (23),

$$\tfrac{1+i}{1-i} \simeq e^{2i \cdot s} \quad (25)$$

On taking square root of both sides,

$$\sqrt{\frac{1 + i}{1 - i}} \simeq e^{i \cdot s}$$

On rationalising,

$$e^{i \cdot s} \simeq \tfrac{1}{\sqrt{2}} + i\tfrac{1}{\sqrt{2}}. \quad (26)$$

On squaring (26),

$$e^{2i \cdot s} \simeq i \quad (27)$$

Multiplying equations (26) and (27),

$$e^{3i \cdot s} \simeq -\tfrac{1}{\sqrt{2}} + i\tfrac{1}{\sqrt{2}}. \quad (28)$$

Squaring equation (27),

$$e^{4i \cdot s} \simeq -1 \quad (29)$$

Multiplying equations (29) and (26),

$$e^{5i \cdot s} \simeq -\tfrac{1}{\sqrt{2}} - i\tfrac{1}{\sqrt{2}}. \quad (30)$$

Squaring equation (28),

$$e^{6i\cdot s} \simeq -i \tag{31}$$

Multiplying equations (31) and (26),

$$e^{7i\cdot s} \simeq \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}. \tag{32}$$

Squaring equation (29),

$$e^{8i\cdot s} \simeq 1 \tag{33}$$

Multiplying equations (33) with (26),

$$e^{9i\cdot s} \simeq \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}. \tag{34}$$

Inspection of equations (25) to (34) reveals that right hand sides of these equations has unity as their magnitude. In other words, $e^{i\cdot s}$, $e^{2i\cdot s}$, $e^{3i\cdot s}$, $e^{4i\cdot s}$, $e^{5i\cdot s}$, $e^{6i\cdot s}$, $e^{7i\cdot s}$, $e^{8i\cdot s}$ and $e^{9i\cdot s}$ all have unit magnitude. Equations (25) to (34) can also be written as

$$e^{i\cdot s} \simeq \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right) + i\cdot\sin\left(\frac{\pi}{4}\right)$$

$$e^{2i\cdot s} \simeq i = \cos\left(\frac{2\pi}{4}\right) + i\cdot\sin\left(\frac{2\pi}{4}\right)$$

$$e^{3i\cdot s} \simeq -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos\left(\frac{3\pi}{4}\right) + i\cdot\sin\left(\frac{3\pi}{4}\right)$$

$$e^{4i\cdot s} \simeq -1 = \cos\left(\frac{4\pi}{4}\right) + i\cdot\sin\left(\frac{4\pi}{4}\right)$$

$$e^{5i\cdot s} \simeq -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos\left(\frac{5\pi}{4}\right) + i\cdot\sin\left(\frac{5\pi}{4}\right),$$

$$e^{6i\cdot s} \simeq -i = \cos\left(\frac{6\pi}{4}\right) + i\cdot\sin\left(\frac{6\pi}{4}\right),$$

$$e^{7i\cdot s} \simeq \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}} = \cos\left(\frac{7\pi}{4}\right) + i\cdot\sin\left(\frac{7\pi}{4}\right),$$

$$e^{8i\cdot s} \simeq 1 = \cos\left(\frac{8\pi}{4}\right) + i\cdot\sin\left(\frac{8\pi}{4}\right),$$

$$e^{9i\cdot s} \simeq \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos\left(\frac{9\pi}{4}\right) + i\cdot\sin\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) + i\cdot\sin\left(\frac{\pi}{4}\right)$$

It is clear from above that when exponential term is $e^{i\cdot s}$, it is expressed as $\cos\left(\frac{\pi}{4}\right) + i\cdot\sin\left(\frac{\pi}{4}\right)$, when the term is $e^{2i\cdot s}$, it is expressed as $\cos\left(\frac{2\pi}{4}\right) + i\cdot\sin\left(\frac{2\pi}{4}\right)$, similarly, when the term is $e^{3i\cdot s}$, it is expressed as $\cos\left(\frac{3\pi}{4}\right) + i\cdot\sin\left(\frac{3\pi}{4}\right)$so on. By mathematical induction, term $e^{k\cdot i\cdot s}$ is expressible as $\cos\left(\frac{k\pi}{4}\right) + i\cdot\sin\left(\frac{k\pi}{4}\right)$.

It is also clear from equations (26) to (34) that $e^{i\cdot s}$ when multiplies with itself adds an angle $\left(\frac{\pi}{4}\right)$

to its original angle of $\left(\frac{\pi}{4}\right)$ and makes angle $\left(\frac{2\pi}{4}\right)$. If is multiplied thrice, it further converts its angle to $\left(\frac{3\pi}{4}\right)$. If $e^{i\cdot s}$ multiplies with any other complex number k times, it makes the angle $\left(\frac{k\pi}{4}\right)$. By mathematical induction, if $e^{i\cdot s}$ divides a complex number k times, it results in subtracting an angle of $\left(\frac{k\pi}{4}\right)$ from the original angle that the complex number makes. $e^{i\cdot s}$ is therefore, an operator of $\left(\frac{\pi}{4}\right)$ radian that rotates a complex number by an angle of $\left(\frac{\pi}{4}\right)$ radians and since magnitude of $e^{i\cdot s}$ is unity, it does not effect the magnitude of the complex number with which it multiplies or divides. It is also observed that at $e^{8i\cdot s}$, operator completes an angle of $\left(\frac{8\pi}{4}\right)$ radians and further multiplication with operator $e^{i\cdot s}$ further rotates it by $\left(\frac{\pi}{4}\right)$ thus making $e^{9i\cdot s} = e^{i(2\pi+s)} = \cos(2\pi + \pi/4) + i\cdot\sin(2\pi+\pi/4) = \cos(\pi/4) + i\cdot\sin(\pi/4)$

If $k = t/s$ then $e^{i\cdot k\cdot s} = \cos(s\cdot t/s) + i\cdot\sin(s\cdot t/s) = \cos(t) + i\cdot\sin(t)$ or $e^{i\cdot\frac{t}{s}\cdot s} = e^{i\cdot t} = \cos(t) + i\cdot\sin(t)$. Conversely, a complex number if expressed as $\cos(t) + i\cdot\sin(t)$ equals $e^{i\cdot t}$. It is also proved above s in the term, $e^{i\cdot s} = e^{8i\cdot s + i\cdot s}$ and $8s$ equals $\left(\frac{8\pi}{4}\right)$ and s equals $\left(\frac{\pi}{4}\right)$

And $8s + s$ equals $\left(\frac{8\pi}{4}\right) + \left(\frac{\pi}{4}\right)$ and $(8k\cdot s) + s = (2k\pi + \pi/4)$. Therefore,

$$e^{i\cdot s} \simeq e^{i\cdot\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right) + i\cdot\sin\left(\frac{\pi}{4}\right) \tag{35}$$

and $s = \left(\frac{\pi}{4}\right)$ or $\left(2k\pi + \frac{\pi}{4}\right)$ where k is a number $1, 2, 3, ..$ so on. Taking the principal value s which has already been defined by equation

$$s = 4n\left\{\frac{1}{1+4\cdot n^2} + \frac{1}{3^2+4\cdot n^2} + \frac{1}{5^2+4\cdot n^2} + \cdots + \frac{1}{(2x-1)^2+4\cdot n^2}\right\} \tag{36}$$

where x varies from 1 to n and n is a large positive number.This equation can also be written as

$$s = 4n\sum_{x=1}^{n}\frac{1}{(2x-1)^2 + 4n^2}.$$

Since s is proved equal to $\pi/4$, therefore,

$$\frac{\pi}{4} \simeq 4n\sum_{x=1}^{n}\frac{1}{(2x-1)^2 + 4n^2}.$$

$$\frac{\pi}{4} \simeq 4n\left\{\frac{1}{1+4\cdot n^2} + \frac{1}{3^2+4\cdot n^2} + \frac{1}{5^2+4\cdot n^2} + \cdots + \frac{1}{(2n-1)^2+4\cdot n^2}\right\} \tag{37a}$$

Or

$$\pi \simeq 16n\sum_{x=1}^{n}\frac{1}{(2x-1)^2+4n^2} \tag{37b}$$

where x varies from 1 to n and n is any number preferably very large for better result. Sign of approximation $\simeq$ has been used in equations (37a) and (37b) meaning thereby that π will approximate in accordance with above equations. It has also been discussed when n or x as it had appeared herein before, must be greater than 1 and when n is in proximity of 1, it gives error and when n is large, error is appreciably reduced. Therefore, when n is larger, approximation of π will be better. If n tends to infinity, then approximation will tend to exactness. Therefore,

$$\lim_{n\to\infty} 16n\sum_{x=1}^{n}\frac{1}{(2x-1)^2+4n^2} \tag{37c}$$

Table 2 Showing Value Of π With Varying n

n	Calculated Value Of π From Equation (37/1)	n	Calculated Value Of πFrom Equation (37/1)
1	3.2	5	3.14492
2	3.16235	10	3.14242
3	3.15085	15	3.14196
4	3.14680	20	3.14180

Data in the table above show that for a single term when n is one, π approximates as 3.2. This figure of 3.2 is with in error of less than 3 percents. There is no such series for evaluating π that gives so close approximation as is given by equation (37b) which is derived on the basis of complex number building block. Upto 10 terms when n is ten, approximation is so close that it tends to exact value π. In view of its fast approximation to π, this method of calculating value of π can be considered one of the best method.

When n tends to infinity, sum of the series as given by equation (37b) tends to π. At this limit, approximation and exactness overlaps.

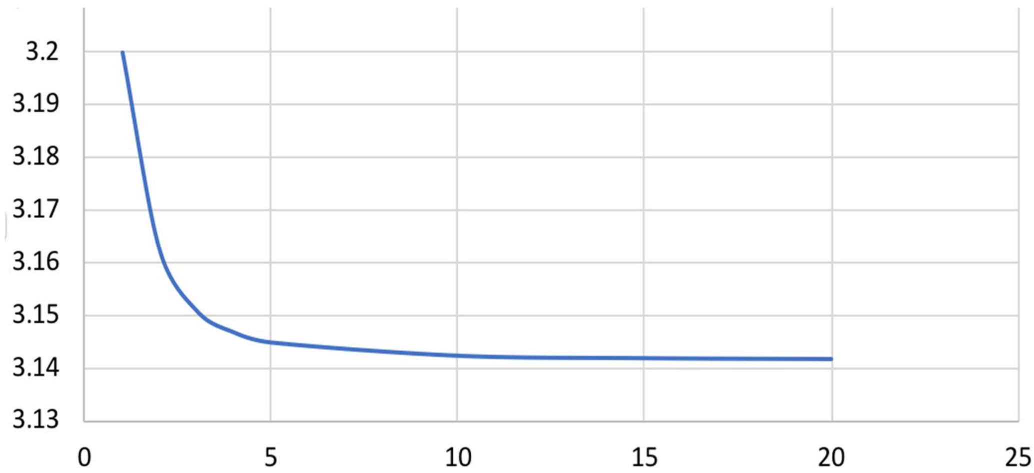

Figure 1 Displaying Calculated Value Of π With Increasing n

A graph is plotted between n on x axis and calculated value of π as

$$16n\sum_{x=1}^{n}\frac{1}{(2x-1)^2+4n^2}$$

on Y axis from the data given in the Table 2.To display variation in calculated value of π from actual π in better and clear way, a graph is plotted on enhanced scale with value 3.13 at origin and 3.2 at $x = 2$. However, calculated value of π at $n = 1, 2, 3, 4, 5, 10, 15$ and 20 are plotted. Inspection of the curve reveals that at $n = 20$, curve is almost touching the actual value of π.

It is also explicit from the Graph that as $n$increases, the gap between calculated and actual value of π decreases. Strictly speaking this gap tends to zero when n will tend to infinity.

Geometrical Representation Of Equation That Determines π

Equation

$$\frac{\pi}{4} \simeq 4n \sum_{x=1}^{n} \frac{1}{(2x-1)^2 + 4n^2}$$

can be represented geometrically by drawing a straight line OAB which equals $4n$ units where n is any positive integer and A is the mid point of OAB. At point B, a perpendicular is drawn and points C, C′, C″, C''' and C'''' are marked on this perpendicular in such a way

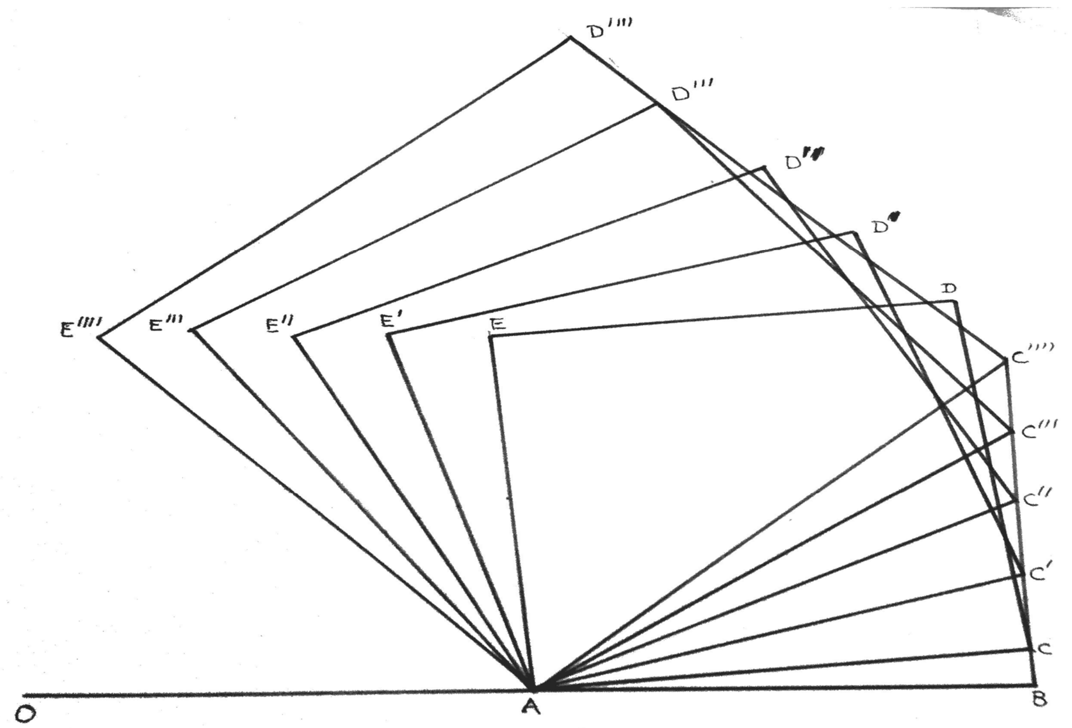

Figure 2 Geometrical Representation Of Formula For π/4 And Ln 2

that $BC = 1$, $BC' = 3$, $BC'' = 5$, $BC''' = 7$, $BC'''' = 9$, ... Squares are then constructed on sides AC, AC', AC'', AC''' and AC''''. Figure 2 shown above pertains to n as 5 cms and that makes, $BC = 1$ cm, $BC' = 3$ cms, $BC'' = 5$ cms, $BC''' = 7$ cms, $BC'''' = 9$ cms, $OAB = 4n = 20$ cms and $AB = 10$ cms. Points C, C', C'', C''' and C'''' are joined with point A. Squares are then constructed on sides AC, AC', AC'', AC''' and AC''''.

Therefore, area of square $ACDE = 1 + 4n^2 = 1 + 100 = 101$,

area of square $AC'D'E' = 3^2 + 4n^2 = 9 + 100 = 109$,

area of square $AC''D''E'' = 5^2 + 4n^2 = 25 + 100 = 125$,

area of square $AC'''D'''E''' = 7^2 + 4n^2 = 49 + 100 = 149$, and area of square $AC''''D''''E''''$ = $9^2 + 4n^2 = 81 + 100 = 181$.

π/4 is then approximated as

$$\frac{\pi}{4} \simeq \frac{2AB}{\text{square } ACDE} + \frac{2AB}{\text{square } AC'D'E'} + \frac{2AB}{\text{square } AC''D''E''} + \frac{2AB}{\text{square } AC'''D'''E'''} + \frac{2AB}{\text{square } AC''''D''''E''''}$$

Let line AC makes an angle α_1, AC' makes α_2, AC'' makes α_3 so on with OAB then $\alpha_1 = cos$ inverse (AB/AC), $\alpha_2 = cos$ inverse (AB/AC′), $\alpha_3 = cos$ inverse $\left(\frac{\text{AB}}{\text{AC}''}\right)$, … so on. Therefore,

$$\frac{\pi}{4} \simeq \frac{2}{AB}\left(\frac{AB^2}{AC^2} + \frac{AB^2}{AC'^2} + \frac{AB^2}{AC''^2} + \frac{AB^2}{AC'''^2} + \cdots\right)$$

Since $AB = 2n$, therefore,

$$\frac{\pi}{4} \simeq \frac{1}{n}\{\cos^2(\alpha_1) + \cos^2(\alpha_2) + \cos^2(\alpha_3) + \cdots + \cos^2(\alpha_n)\}$$

Or

$$\frac{\pi}{4} \simeq \left(\frac{1}{n}\right)\sum_{x=1}^{n} \cos^2(\alpha_x)$$

where $\alpha_x = cos$ inverse $\frac{2n}{\sqrt{(2x-1)^2+4n^2}}$ and higher the value of n better will be the approximation. In Figure 2 above, n is taken as 5, therefore,

$$\frac{\pi}{4} \simeq \frac{1}{n}\{\cos^2(\alpha_1) + \cos^2(\alpha_2) + \cos^2(\alpha_3) + \cdots + \cos^2(\alpha_5)\}$$

Or

$$\frac{\pi}{4} \simeq \frac{1}{5}\left\{\frac{100}{101} + \frac{100}{109} + \frac{100}{125} + \frac{100}{149} + \frac{100}{181}\right\}$$

when n tends to infinity, value of π will reach its exact value. Hence

$$\pi \simeq \left(\frac{4}{n}\right)\sum_{x=1}^{n} \cos^2(\alpha_x)$$

Determination Of $\ln(2)$

Referring to equations (23) and (24) which are reproduced below

$$1 - i \simeq e^{r-i\cdot s}$$
$$1 + i \simeq e^{r+i\cdot s}$$

if these are multiplied with each other then $1 - i^2 \simeq e^{2r}$. Or

$$\ln(2) \simeq 2r \tag{38a}$$

where

$$r \simeq 2\sum_{x=1}^{n} \frac{2x-1}{(2x-1)^2 + 4n^2}$$

Or

$$r = 2\left\{\frac{1}{1^2 + 4n^2} + \frac{3}{3^2 + 4n^2} + \frac{5}{5^2 + 4n^2} + \cdots + \frac{2n-1}{(2n-1)^2 + 4n^2}\right\}$$

$$\ln\,(2) \simeq 2r \simeq 4\sum_{x=1}^{n}\frac{2x-1}{(2x-1)^2+4n^2},$$

$$\ln(2) \simeq 4\left\{\frac{1}{1^2+4n^2}+\frac{3}{3^2+4n^2}+\frac{5}{5^2+4n^2}+\cdots+\frac{2n-1}{(2n-1)^2+4n^2}\right\} \quad (38b)$$

Table 3 Showing Value Of $\ln(2)$ With Varying n

n	Calculated Value Of $\ln(2)$ From Equation (38/1)	n	Calculated Value Of $ln(2)$ From Equation (38/1)
1	.80000	5	.69651
2	.71529	10	.69398
3	.70264	15	.69351
4	.69842	20	.69335

It is clear from the data given in the table that as n increases, value of ln (2) as calculated from equation (38b) tends to exact value of ln(2)

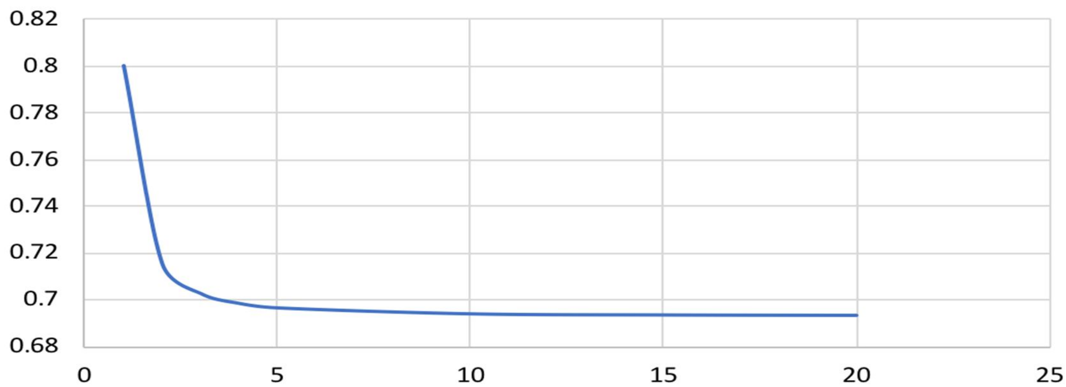

Figure 2 Displaying Calculated Value Of $ln(2)$ With increasing n.

A graph is plotted between n on X axis and calculated value of $\ln(2)$ as $4\sum_{x=1}^{n}\frac{2x-1}{(2x-1)^2+4n^2}$ on Y axis from the data given in Table 3.To display variation in calculated value of $\ln(2)$ from actual $\ln(2)$ in better and clear way, a graph is plotted on enhanced scale with value 0.68 at origin and 0.80 at $x = 2$. However, calculated value of $\ln(2)$ at $n = 1, 2, 3, 4, 5, 10, 15$ and 20 are plotted. Inspection of the curve reveals that at $n = 20$, curve is almost touching the actual value of $\ln(2)$. It is also explicit from the graph that as n increases, the gap between calculated and actual value of $\ln(2)$ decreases. Strictly speaking this gap tends to zero when n to infinity. Therefore,

$$\ln(2) \simeq \lim_{n\to\infty} 4\sum_{x=1}^{n}\frac{2x-1}{(2x-1)^2+4n^2}$$

$$\lim_{n\to\infty}\ln(2) \simeq 4\left\{\frac{1}{1^2+4n^2}+\frac{3}{3^2+4n^2}+\frac{5}{5^2+4n^2}+\cdots+\frac{2n-1}{(2n-1)^2+4n^2}\right\}$$

Geometrical Representation Of $ln(2)$

Referring to figure 2 and equation (38b),

$$\ln(2) \simeq 4\left(\frac{BC}{AC^2} + \frac{BC}{AC'^2} + \frac{BC}{AC''^2} + \frac{BC}{AC'''^2} + \cdots\right)$$

$$\ln(2) \simeq \frac{4BC}{\text{square } ACDE} + \frac{4BC'}{\text{square } AC'D'E'} + \frac{4BC''}{\text{square } AC''D''E''} + \frac{4BC'''}{\text{square } AC'''D'''E'''} + \frac{4BC''''}{\text{square } AC''''D''''E''''}$$

Above equation can also be written in trigonometric ratios as

$$\ln(2) \simeq 4\left[\frac{1}{BC}\left(\frac{BC^2}{AC^2}\right) + \frac{1}{BC'}\left(\frac{BC'^2}{AC'^2}\right) + \frac{1}{BC''}\left(\frac{BC''}{AC''^2}\right) + \cdots\right],$$

$$\ln(2) \simeq 4\left[\frac{\sin^2\alpha_1}{BC} + \frac{\sin^2\alpha_2}{BC'} + \frac{\sin^2\alpha_3}{BC''} + \cdots\right] \simeq 4\left[\frac{\sin^2\alpha_1}{1} + \frac{\sin^2\alpha_2}{3} + \frac{\sin^2\alpha_3}{5} + \frac{\sin^2\alpha_4}{7} + \frac{\sin^2\alpha_5}{9}\right]$$

In general, $\ln(2)$ can be written as

$$\ln(2) \simeq \lim_{x\to\infty} \quad 4\left[\frac{\sin^2\alpha_1}{1} + \frac{\sin^2\alpha_2}{3} + \frac{\sin^2\alpha_3}{5} + \cdots + \frac{\sin^2\alpha_x}{2x-1}\right] \simeq \lim_{n\to\infty} 4\sum_{x=1}^{n}\frac{\sin^2(\alpha_x)}{2x-1}$$

and

$$\alpha_x = sin\ inverse \frac{2x-1}{\sqrt{(2x-1)^2 + (2n)^2}}$$

In the Figure 2 above, $n = 5$, therefore,

$$\ln(2) \simeq 4\left[\frac{1}{101} + \frac{9}{3\cdot 109} + \frac{25}{5\cdot 125} + \frac{49}{7\cdot 149} + \frac{81}{9\cdot 181}\right] \simeq .69651$$

When n tends to infinity, calculated value of $\ln(2)$ will reach exact value of $\ln(2)$.

Determination Of $Tan\ Inverse\ y/x$

In complex plane, complex number $x + i\cdot y$ makes an angle say A which is given by relation $\tan A = y/x$ or $A = \tan\ \text{inverse}\, y/x$. Angle A is calculated by series expansion of $\tan\ \text{inverse}\, y/x$ or otherwise. In this section, independent method is devised to find angle A or $\tan\ \text{inverse}\, y/x$.

From equations (22c) and (22d),

$$x + i\cdot y \simeq x\cdot e^{r'+i\cdot s'},$$

$$x - i\cdot y \simeq x\cdot e^{r'-i\cdot s'}$$

On dividing

$$\frac{x + i\cdot y}{x - i\cdot y} \simeq \frac{x\cdot e^{r'+i\cdot s'}}{x\cdot e^{r'-i\cdot s'}}$$

On rationalising,

$$\frac{x^2 - y^2}{x^2 + y^2} + 2i\frac{xy}{x^2 + y^2} \simeq e^{2i\cdot s'},$$

It has already been proved that $e^{i\cdot k} = \cos k + i\cdot \sin k$. Therefore,

$$e^{2i\cdot s'} = \cos 2s' + i\cdot \sin 2s'.$$

Equating real and imaginary parts,

$$\cos 2s' \simeq \frac{x^2-y^2}{x^2+y^2}, \tag{39}$$

$$\sin 2s' \simeq 2\frac{xy}{x^2+y^2} \tag{40}$$

By componendo and dividendo, equation (39) can be transformed as

$$\frac{1+\cos 2s'}{1-\cos 2s'} = \frac{\cos^2 s'}{\sin^2 s'} \simeq \frac{x^2}{y^2}$$

Or

$$\tan s' \simeq \frac{y}{x}, \tag{41a}$$

$$\text{tan inverse}\frac{\mathrm{y}}{\mathrm{x}} \simeq \quad s'$$

where

$$\text{tan inverse } \frac{\mathrm{y}}{\mathrm{x}} \simeq \quad s' \simeq 4x\sum_{x=1}^{y}\frac{1}{(2x-1)^2+4x^2},$$

$$\text{tan inverse } \frac{\mathrm{y}}{\mathrm{x}} \simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\} \tag{41b}$$

This identity has the limitation that it is not applicable when either x or y equals zero. Also from equation (41a),

$$\cos s' \simeq \frac{1}{\sqrt{1+\frac{y^2}{x^2}}}$$

Or

$$\text{cos inverse } \frac{1}{\sqrt{1+\frac{y^2}{x^2}}} \simeq \quad s' \simeq 4x\sum_{x=1}^{y}\frac{1}{(2x-1)^2+4x^2} \simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\} \tag{42}$$

Also from equation (41),

$$\sin s' \simeq \frac{1}{\sqrt{1+\frac{x^2}{y^2}}}$$

Or

$$\text{sin inverse } \frac{1}{\sqrt{1+\frac{x^2}{y^2}}} \simeq \quad s' \simeq 4x\sum_{x=1}^{y}\frac{1}{(2x-1)^2+4x^2} \simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\} \tag{43}$$

On multiplying, equations (22c) with (22d),

$$x^2+y^2 \simeq x^2\cdot e^{2r'}$$

Or

$$2r' \simeq \ln\left(1+\frac{y^2}{x^2}\right) \tag{44a}$$

Or

$$2r' \simeq 4\sum_{n=1}^{y}\frac{2n-1}{(2n-1)^2+4x^2} \simeq 4\left\{\frac{1}{1+4\cdot x^2}+\frac{3}{3^2+4\cdot x^2}+\frac{5}{5^2+4\cdot x^2}+\cdots+\frac{2y-1}{(2y-1)^2+4\cdot x^2}\right\} \tag{44b}$$

Since y/x or $(1 + y^2/x^2)$ can have any value between 1 to infinity, therefore equations, (44a) and (44b) are useful for approximating logarithm of any quantity say m by equating it with $(1 + y^2/x^2)$. If $(1 + y^2/x^2)$ is less than 1 but more than 0, than value of $-\ln(1 + y^2/x^2)$ can be found. At the same time logarithm of a quantity say m, can also be expanded in finite series. Also from (44a), $\ln(sec\, s') = r'$ or $\ln(cos\, s') + r' = 0$.

Another Way To Determine Value Of π

From equation (41), $s' \simeq \text{tan inverse } \frac{y}{x} \simeq 4x\left\{\frac{1}{1+4\cdot x^2} + \frac{1}{3^2+4\cdot x^2} + \frac{1}{5^2+4\cdot x^2} + \cdots + \frac{1}{(2y-1)^2+4\cdot x^2}\right\}$,

similarly, say $s'' \simeq \text{tan inverse } \frac{x}{y} \simeq 4y\left\{\frac{1}{1+4\cdot y^2} + \frac{1}{3^2+4\cdot y^2} + \frac{1}{5^2+4\cdot y^2} + \cdots + \frac{1}{(2x-1)^2+4\cdot y^2}\right\}$

And $\tan(s' + s'') = \frac{\tan(s') + \tan(s'')}{1 - \tan(s')\cdot\tan(s'')}$ or

$$\tan(s' + s'') = \frac{\left(\frac{y}{x}\right)+\left(\frac{x}{y}\right)}{1-\left(\frac{y}{x}\right)\left(\frac{x}{y}\right)} = \infty = \tan\left\{\frac{(2k+1)\pi}{2}\right\} \quad (45a)$$

where k is any integer 1, 2, 3,…….. Therefore,

$$(s' + s'') \simeq \pi/2 \quad (45b)$$

On putting value of s' and s'',

$$\frac{\pi}{2} \simeq 4x\left\{\frac{1}{1+4\cdot x^2} + \frac{1}{3^2+4\cdot x^2} + \frac{1}{5^2+4\cdot x^2} + \cdots + \frac{1}{(2y-1)^2+4\cdot x^2}\right\} + 4y\left\{\frac{1}{1+4\cdot y^2} + \frac{1}{3^2+4\cdot y^2} + \frac{1}{5^2+4\cdot y^2} + \cdots + \frac{1}{(2x-1)^2+4\cdot y^2}\right\} \quad (46a)$$

where x and y are large quantity and if these are not large, these can be made large by multiplication with large number n or

$$\frac{\pi}{2} \simeq 4x\left\{\frac{1}{1+4\cdot x^2} + \frac{1}{3^2+4\cdot x^2} + \frac{1}{5^2+4\cdot x^2} + \cdots + \frac{1}{(2y-1)^2+4\cdot x^2}\right\} + 4y\left\{\frac{1}{1+4\cdot y^2} + \frac{1}{3^2+4\cdot y^2} + \frac{1}{5^2+4\cdot y^2} + \cdots + \frac{1}{(2x-1)^2+4\cdot y^2}\right\} \quad (46b)$$

If $\text{tan inverse } \frac{y}{x}$ is known, $\text{tan inverse } \frac{x}{y}$ can be found from equation (45/1), as

$$\text{tan inverse } \frac{x}{y} = \frac{\pi}{2} - \text{tan inverse } \frac{y}{x} \quad (46c)$$

Determination Of Logarithm Of Any Quantity

Let the quantity is $5/4$. From equation (44b),

$$\ln\left(1 + \frac{y^2}{x^2}\right) \simeq 4\sum_{n=1}^{y} \frac{2n-1}{(2n-1)^2 + 4x^2}$$

$$\simeq 4\left\{\frac{1}{1 + 4\cdot x^2} + \frac{3}{3^2 + 4\cdot x^2} + \frac{5}{5^2 + 4\cdot x^2} + \cdots + \frac{2y-1}{(2y-1)^2 + 4\cdot x^2}\right\}$$

$\ln\left(1 + \frac{y^2}{x^2}\right) = \ln\left(\frac{5}{4}\right)$ ThAt means $\frac{y}{x} = \frac{1}{2}$. On multiplying and dividing by a number say $n = 10$,

$\frac{y}{x} = 10/20$. Then $\ln\left(\frac{5}{4}\right) \simeq 4\left\{\frac{1}{1+1600} + \frac{3}{3^2+1600} + \frac{5}{5^2+1600} + \cdots + \frac{19}{19^2+1600}\right\}$. On computation, $\ln\left(\frac{5}{4}\right) \simeq .22325$. More the value of n, more exact will be the result.

Examples

1 Expand tan inverse $\frac{18}{10}$ in finite series using number building block and also find its value.

Here $\frac{y}{x} = \frac{18}{10} = \frac{9}{5}$. According to equation (41b)

$$\text{tan inverse } \frac{9}{5} \simeq 20\left[\frac{1}{1^2 + 100} + \frac{1}{3^2 + 100} + \frac{1}{5^2 + 100} + \cdots + \frac{1}{17^2 + 100}\right] \simeq 1.06493$$

Actual value of $\tan^{-1}(9/5)$ is 1.06369.

2 Expand tan inverse $\frac{13}{50}$ in finite series using number building block and also find its value.

Here $\frac{y}{x} = \frac{13}{50}$.. According to equation (41b),

$$\text{tan inverse } \frac{13}{50} \simeq 4(50)\left[\frac{1}{1^2 + 10000} + \frac{1}{3^2 + 1000} + \frac{1}{5^2 + 1000} + \cdots + \frac{1}{25^2 + 1000}\right].25437$$

Actual value of $\tan^{-1}(9/5)$ is 1.06369. Actual value of $\tan^{-1}(13/50)$ is .25437.

3 Expand sin inverse $\frac{1}{5}$ in finite series using number building block and also find its value.

According to equation (43),

$$\text{sin inverse } \frac{1}{\sqrt{1+\frac{x^2}{y^2}}} \simeq s' \simeq 4x\sum_{x=1}^{y}\frac{1}{(2x-1)^2 + 4x^2}$$

$$\simeq 4x\left\{\frac{1}{1 + 4\cdot x^2} + \frac{1}{3^2 + 4\cdot x^2} + \frac{1}{5^2 + 4\cdot x^2} + \cdots + \frac{1}{(2y-1)^2 + 4\cdot x^2}\right\}$$

therefore, $\frac{1}{\sqrt{1+\frac{x^2}{y^2}}} = \frac{1}{5}$ or $\frac{y}{x} = \frac{1}{\sqrt{24}} = .20412$. On changing decimals to fraction, $.29412 = \frac{1}{4.89897} \simeq \frac{10}{49}$. In this way, $y = 10, x = 49$ and

$$\text{sin inverse } \frac{1}{5} \simeq 4(49)\left\{\frac{1}{1 + 4\cdot 49^2} + \frac{1}{3^2 + 4\cdot 49^2} + \frac{1}{5^2 + 4\cdot 49^2} + \cdots + \frac{1}{19^2 + 4\cdot 49^2}\right\}$$

$$\simeq .20132$$

Actual value is .20135.

4 Expand cos inverse $\frac{1}{\sqrt{2}}$ in finite series using number building block and also find its value.

According to equation (42),

$$\text{cos inverse}\ \frac{1}{\sqrt{1+\frac{y^2}{x^2}}} \simeq s' \simeq 4x\sum_{x=1}^{y}\frac{1}{(2x-1)^2+4x^2}$$

$$\simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\}$$

therefore, $\frac{1}{\sqrt{1+\frac{y^2}{y^2}}} = \frac{1}{\sqrt{2}}$ or $\frac{y}{x} = 1 = \frac{n}{n}$. More the value of n, better will be approximation. Let $n = 20$ then $\frac{y}{x} = \frac{20}{20}$.

$$\text{Sin inverse}\ \frac{1}{\sqrt{2}} \simeq 4(20)\left\{\frac{1}{1+4\cdot 20^2}+\frac{1}{3^2+4\cdot 20^2}+\frac{1}{5^2+4\cdot 20^2}+\cdots+\frac{1}{39^2+4\cdot 20^2}\right\}$$

$$\simeq .78545.$$

Actual value is .78539

5 A complex number is $17-7i$, find out the angle it makes with X axis.

Here $y = -7$ and $x = 17$. Negative sign of y will be ignored for time being and tan inverse $\frac{y}{x} \simeq s' \simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\}$, therefore,

$$\text{tan inverse}\ \frac{7}{17} \simeq 4(17)\left\{\frac{1}{1+4\cdot 17^2}+\frac{1}{3^2+4\cdot 17^2}+\frac{1}{5^2+4\cdot 17^2}+\cdots+\frac{1}{(14-1)^2+4\cdot 17^2}\right\}$$

$$\simeq .39069$$

Examination of complex number reveals that y and x has opposite signs therefore, and is in 2nd or 4th quadrant. Therefore, tan inverse $\frac{7}{17} \simeq (\pi-.39069)$ or $(2\pi-.39069)$

6 Expand $\ln(26)$ in finite series using number building block and also find its value.

Let $1+\frac{y^2}{x^2} = 26$ then $\frac{y^2}{x^2} = 5$. For better approximation, 5 can be written as $\frac{15}{3}$. Therefore $y = 15$ and $x = 3$.

$$Ln(26) \simeq 4\left\{\frac{1}{1^2+4(3^2)}+\frac{1}{3^2+4(3^2)}+\frac{1}{5^2+4(3^2)}+\cdots+\frac{1}{(30-1)^2+4(3^2)}\right\} \simeq 3.267.$$

RESULTS AND CONCLUSIONS

Real numbers can be generated from number building block $x/(x-1) \simeq e^{\frac{2}{2.x-1-\frac{1}{x^3}}}$, When x is appreciably large such that $1/x^3$ can be neglected, number building block can be written as $x/(x-1) \simeq e^{2/(2x-1)}$. From these real number building blocks, complex numbers building blocks are generated as a real number building block is product of two complex conjugate number building blocks. Complex number $(\mathrm{x}+\mathrm{i}\cdot\mathrm{y})$ and $(\mathrm{x}-\mathrm{i}\cdot\mathrm{y})$ are generated by the equations of complex number building blocks,

$$\frac{x+iy}{x-1+iy} \simeq e^{\frac{2}{2x+2i.y-1}}$$

$$\frac{x-iy}{x+1+iy} \simeq e^{\frac{2}{2x-2i.y-1}}$$

Provide x and y are appreciably large so that $-\frac{1}{z^3}+1/4$ can be ignored where $z=1+\frac{(x-1)^2+y^2}{2.x-1}$. Otherwise also, x and y can always be made large by multiplication of real positive number n and transformed complex number building blocks are given by relations

$$\frac{n(x+iy)}{(n\cdot x+i\cdot n\cdot y-1)} \simeq e^{\frac{2}{2n\cdot x+2i\cdot n\cdot y-1}},$$

$$(x+i\cdot y) \simeq \left(x+i\cdot y-\frac{1}{n}\right) e^{\frac{2}{2n\cdot x+2i\cdot n\cdot y-1}}$$

and its conjugate

$$\frac{n(x-iy)}{(n\cdot x-i\cdot n\cdot y-1)} \simeq e^{\frac{2}{2n\cdot x-2i\cdot n\cdot y-1}},$$

$$(x-i\cdot y) \simeq \left(x-i\cdot y-\frac{1}{n}\right) e^{\frac{2}{2n\cdot x-2i\cdot n\cdot y-1}}$$

Multiplication by large real positive number n reduces the error appreciably. If n tends to infinity, error itself is eliminated and approximation tends to exactness.

Method of complex number building block proves, a complex number transposes another complex number to the exponent of base of natural number e. If there is a complex number $(\mathrm{x}+\mathrm{i}\cdot\mathrm{y})$, it will transpose another complex number $1/(\mathrm{x}-\mathrm{i}\cdot\mathrm{y}-1/2)$ to the exponent of natural number e. In mathematical form,

$$(\mathrm{x}+\mathrm{i}\cdot\mathrm{y}) \simeq (\mathrm{x}+\mathrm{i}\cdot\mathrm{y}-1)\, e^{\frac{1}{\mathrm{x}-\mathrm{i}\cdot\mathrm{y}-\frac{1}{2}}}.$$

A complex number building block for $(\mathrm{x}+\mathrm{i}\cdot\mathrm{y})$ gives the identity,

$$\ln\left(\frac{x^2+y^2}{x^2+y^2-2x+1}\right) \simeq \frac{2x-1}{\left(x-\frac{1}{2}\right)^2+y^2+\frac{1}{4}}$$

If x and y are large, $1/4$ appearing in RHS of above equation can be ignored and the equation gets modified to

$$\ln\left(\frac{x^2+y^2}{x^2+y^2-2x+1}\right) \simeq \frac{2x-1}{\left(x-\frac{1}{2}\right)^2+y^2}$$

By the method of number building blocks, a complex quantity can be transformed to pure exponential form. That is to say $(x + i \cdot y)$ can be transformed to exponential form as $e^{p'+q'+r'+i.s'}$ where

$$p' = 2\sum_{n=2}^{x}\left(\frac{1}{2n-1}\right) = 2\left\{\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2n-1}\right\}$$

$$q' = 2\sum_{n=2}^{x}\left\{\frac{1}{n^3(2n-1)}\right\} = 2\left\{\frac{1}{2^3 \cdot 3^2} + \frac{1}{3^3 \cdot 5^2} + \frac{1}{4^3 \cdot 7^2} + \cdots + \frac{1}{n^3(2n-1)^2}\right\}$$

$$r' = 2\sum_{n=1}^{y}\left\{\frac{2n-1}{(2n-1)^2 + 4x^2}\right\} = 2\left\{\frac{1}{1 + 4x^2} + \frac{3}{3^2 + 4x^2} + \frac{5}{5^2 + 4x^2} + \cdots + \frac{2y-1}{(2y-1)^2 + 4x^2}\right\}$$

$$s' = 4x\sum_{n=1}^{y}\left\{\frac{1}{(2n-1)^2 + 4x^2}\right\}$$

$$= 4x\left\{\frac{1}{1 + 4x^2} + \frac{1}{3^2 + 4x^2} + \frac{1}{5^2 + 4x^2} + \cdots + \frac{1}{(2y-1)^2 + 4x^2}\right\}$$

Method of number building blocks proves that for complex number $(x + i \cdot y)$ or $e^{p'+q'+r'+i.s'}$, the part $e^{p'+q'+r'}$ is magnitude and equals $\sqrt{x^2 + y^2}$ and $e^{i.s'}$ equals $e^{i \cdot s'} = \cos(s') + i \cdot \sin(s')$ where s' is the angle that complex number makes with real axis and equals tan inverse $\frac{y}{x}$. Conversely, any quantity $e^{c+i.d}$ can be expressed as $(a + i \cdot b)$. Method of complex number building blocks proves if $x = y = n$ and limit n tends to infinity,

$$\frac{\pi}{4} \simeq 4n\sum_{x=1}^{n}\frac{1}{(2x-1)^2 + 4n^2}.$$

Or when x varies from 1 to n and n tends to infinity,

$$\frac{\pi}{4} \simeq 4n\left\{\frac{1}{1+4 \cdot n^2} + \frac{1}{3^2+4 \cdot n^2} + \frac{1}{5^2+4 \cdot n^2} + \cdots + \frac{1}{(2x-1)^2+4 \cdot n^2}\right\}$$

On the basis of method of number building blocks, pure imaginary number i is expressible in exponential form $i = e^{i \cdot 2s} = e^{i \cdot \frac{\pi}{2}}$ and

$$s \simeq \frac{\pi}{4} \simeq 4n\left\{\frac{1}{1 + 4 \cdot n^2} + \frac{1}{3^2 + 4 \cdot n^2} + \frac{1}{5^2 + 4 \cdot n^2} + \cdots + \frac{1}{(2n-1)^2 + 4 \cdot n^2}\right\}$$

where n tends to infinity.

Also $s =$ tan inverse (y/x) and this equation gives the value of angle that the complex number $(x + i \cdot y)$ makes with real axis in complex plane. But this equation has limitation when either x or y is zero. Since tan inverse (y/x) + tan inverse $(x/y) = \pi/2$, therefore,

$$\frac{\pi}{2} \simeq 4x\left\{\frac{1}{1+4\cdot x^2}+\frac{1}{3^2+4\cdot x^2}+\frac{1}{5^2+4\cdot x^2}+\cdots+\frac{1}{(2y-1)^2+4\cdot x^2}\right\} + 4y\left\{\frac{1}{1+4\cdot y^2}+\frac{1}{3^2+4\cdot y^2}+\frac{1}{5^2+4\cdot y^2}+\cdots+\frac{1}{(2x-1)^2+4\cdot y^2}\right\}$$

where x and y are large positive integers. π can also be determined by the equation

$$\frac{\pi}{4} \simeq \left(\frac{1}{n}\right)\sum_{x=1}^{n} \cos^2(\alpha_x)$$

where $\alpha_x = cos \text{ inverse} \frac{2n}{\sqrt{(2x-1)^2+4n^2}}$ and x varies from 1 to n and n tends to infinity. Method of complex building blocks proves that logarithm of a number n where $n = (1 + y^2/x^2)$ is given by relation

$$\ln n \simeq 2r' \simeq \ln\left(1+\frac{y^2}{x^2}\right) \simeq 4\sum_{n=1}^{y} \frac{2n-1}{(2n-1)^2+4x^2} \simeq 4\left\{\frac{1}{1+4\cdot x^2}+\frac{3}{3^2+4\cdot x^2}+\frac{5}{5^2+4\cdot x^2}+\cdots+\frac{2y-1}{(2y-1)^2+4\cdot x^2}\right\}$$

If $y = x$ and x varies from 1 to n and n is a large real integer tending to infinity then

$$\ln 2 \simeq 2r' \simeq 4\sum_{x=1}^{n} \frac{2x-1}{(2x-1)^2+4n^2} \simeq 4\left\{\frac{1}{1+4\cdot n^2}+\frac{3}{3^2+4\cdot n^2}+\frac{5}{5^2+4\cdot n^2}+\cdots+\frac{2n-1}{(2n-1)^2+4\cdot n^2}\right\}$$

Ln(2) can also be written in trigonometric equation as

$$\ln(2) \simeq \lim_{n\to\infty} 4\sum_{x=1}^{n} \frac{\sin^2(\alpha_x)}{2x-1}$$

and

$$\alpha_x = sin\ inverse \frac{2x-1}{\sqrt{(2x-1)^2+(2n)^2}}$$

and x varies from 1 to n and n tends to infinity.

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REFERENCES

1. Method of number building blocks as has been described and is used in this paper to arrive at conclusions given above, is not attempted earlier and, therefore, there is no material available

on number building blocks. In the absence of material related to number buildings which are exhaustively dealt in the paper, nothing could be referred.