Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Study of Chaoticity In Hadron-nucleus Interaction

Pal Sitaram

Kanchrapara College, North 24 Pgs, 743145, West Bengal, India
E-mail: sitaram_ju@yahoo.co.in, palsitaram2012@gmail.com

ABSTRACT:

In this paper a study on chaotic (erratic) behavior of produced pions in high-energy interactions induced by protons at 400 GeV have been performed with the help of the parameter- "entropy index" μ_q. The analysis reveals chaotic (erratic) behavior of the produced pions signifying chaotic multiparticle production in high-energy hadron-nucleus interaction. It has been further observed that multipion production process becomes less chaotic with increasing average multiplicity of the final states.

KEY WORDS: Erraticity, chaos, entropy index, hadron-nucleus interaction

***Corresponding author**

Sitaram Pal

Kanchrapara College, North 24 Pgs, 743145,

West Bengal, India

E-mail: sitaram_ju@yahoo.co.in, palsitaram2012@gmail.com

1. INTRODUCTION:

In physics chaos theory describes the behavior of certain complex nonlinear dynamical systems that under specific conditions exhibit dynamics that are sensitive to initial conditions. The behavior of chaotic systems appears to be random, because of an exponential growth of errors in the initial conditions. This happens even though these systems are deterministic in the sense that their future dynamics are well defined by their initial conditions, and there are no random elements involved. This behavior is known as chaos. .Chaotic behavior has been observed in the nature in a variety of natural systems.

The notion of chaos in the description of particle production processes in high energy physics is not well established and the measurement of chaoticity in multiparticle production is a complicated problem. The possibility of chaotic behaviour of multiparticle production in branching processes has been explored by Cao and Hwa[1] with emphasis on the search for appropriate measures of chaoticity. They considered two branching processes in particle production: one is pure gluon theory in perturbative QCD (quantum chromodynamics) that was later extended to include quarks also[2] and the other is an Abelion cascade model (known as the χ model). Characteristics of particle production were investigated by generating events according to the perturbative QCD and the cascade model. Because of the non-classical nature of the system, search for new measures and observables are required.The spatial behavior was studied in terms of fluctuations of the normalized event factorial moment F_q^e and the entropy index μ_q. It was suggested that QCD branching is chaotic, while the χ model is not. Out of the different measures considered to describe the degree of chaoticity in the branching process, we do not consider those measures which are not accessible to experiment. However, the normalized event factorial moment F_q^e, the moment of moments $C_{p,q}$, and the entropy index μ_q describe the characteristics of the final state, and can be determined experimentally in most high-energy collisions. The entropy index μ_q is regarded as an appropriate parameter for measuring the chaotic behaviour of particle production. It describes the degree of fluctuation of the scaled factorial moments in event space as well as the spatial pattern of the particles in the final states, and it also characterizes the degree of fluctuation of the parton multiplicity that initiates branching. A small μ_q implies no chaotic behaviour while a large μ_q implies chaotic behaviour.

Chaoticity analysis was applied to different high-energy interactions multiparticle production in simulated data of hadronic collisions whose parameters are tuned to that of NA22 data[3], in 400 *GeV/c p-p* collision data from NA 27[4], $\pi^+ p$ and $k^+ p$ collisions data at 250 *GeV/c*[5], But so far no attempt has been made to study the chaotic behaviour of pions produced in high-energy hadron-nucleus interactions, which offer unique opportunity to learn about the space-time structure of a strongly interacting process. In view of this we here present a detailed study on p – AgBr interactions at 400 GeV/c to see whether the multipion production process in high energy hadron-nucleus collisions is chaotic (erratic).

2. EXPERIMENTAL DETAILS:

Stacks of G5 nuclear emulsion plates were horizontally exposed to a proton beam of 400 GeV incident energy at Fermilab. The emulsion plates were area scanned with a Leitz Metalloplan Microscope fitted with a semiautomatic scanning device, having a resolution along the X and Y axes of 1 μm while that along the Z axis is 0.5 μm . A sample of 380 events of π^--*AgBr* at 350 GeV/c was chosen, following the usual emulsion methodology for selection criteria of the events.

According to the emulsion terminology[6], the particles emitted from interactions are classified as:

a. Black particles: -They are target fragments with ionization greater or equal to $10I_0$, I_0being the minimum ionization of a singly charged particle. The range of them is less than 3 mm, the velocity less than 0.3c and the energy less than 30 MeV, where c is the velocity of light in vacuum.
b. Grey particles: -They are mainly fast target recoil protons with energy up to 400 MeV. They have ionization $1.4\ I_0 \leq I < 10\ I_0$. Their ranges are greater than 3 mm and they have velocities(v), $0.7c \geq v \geq 0.3c$.
c. Shower particles: -The relativistic shower tracks with ionization I less than or equal to $1.4I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle.

3. METHOD OF ANALYSIS:

Cao and Hwa[1,2] proposed to measure the phase-space pattern of a multiparticle system by factorial moments. In contrast to the sample factorial moments, they defined event factorial moments for studying spatial patterns of a multiparticle system as

$$F_q^{(e)}(M) = \left(\frac{1}{M}\sum_{i=1}^{M} n_i(n_i-1)\ldots\ldots\ldots(n_i-q+1)\right) \times \left(\frac{1}{M}\sum_{i=1}^{M} n_i\right)^{-q} \quad \ldots\ldots\ldots\ldots\ (1)$$

where M is the partition number in phase space, n_i is the number of particles in the i^{th} bin for e^{th} event and q = 2,3,4,……. is the order of the moment. Since $F_q^{(e)}(M)$ fluctuates from event to event, one obtains a distribution $P(F_q^e)$ for the whole event sample. Let the average of $F_q^e(M)$ determined from $P(F_q^e)$ be denoted by $\langle F_q^e(M)\rangle$.

In order to quantify the degree of the fluctuation, a new normalized moment is defined as

$$C_{p,q}(M) = \frac{\langle F_q^p(M)\rangle}{\langle F_q(M)\rangle^p} \qquad \text{.........................(2)}$$

where $\langle F_q^p \rangle = \frac{1}{N}\sum_{e=1}^{N}(F_q^e)^p$, N being the total number of events. The order p is a positive real number. For $p\rangle 1$, $C_{p,q}(M)$ reflects the large F_q^e behaviour of $P(F_q^e)$, which is sensitive to the spikes in phase space. For $p\langle 1$, $C_{p,q}(M)$ probes the low F_q^e behaviour of $P(F_q^e)$, which is influenced mainly by bins with low multiplicities, including empty bins. Thus knowing $C_{p,q}(M)$ for $0\langle p \langle 2$, say, reveals a great deal about the properties of $P(F_q^e)$, all of which are not probed by intermittency analysis.If $C_{p,q}(M)$ has a power law behavior as the division number M goes to infinity

$$C_{p,q}(M) \propto M^{\psi_q(p)},\ M \to \infty, \qquad \text{...................(3)}$$

then this corresponds to chaoticity in a dynamic system where the time sequence can be generated. The power-law behavior of Eq. (3) is referred to as erraticity of a multiparticle system and $\psi_q(p)$ as erraticity exponent. Since $C_{p,q}(M)$ are the moments of $P(F_q^e)$, they describe the deviation of F_q^e from the mean $\langle F_q^e(M)\rangle$. Consequently, $C_{p,q}(M)$ is sensitive to the erratic fluctuations F_q^e from event to event. Those fluctuations depend on the bin size because F_q^e itself is a description of the spatial pattern that varies according to resolution. Thus if those fluctuations scale with bin size, then the erraticity exponent $\psi_q(p)$ is an economical way of characterizing an aspect of the self-similar dynamics that has some order in its erratic fluctuations. Erraticity is characterized by the slope μ_q of $\psi_q(p)$ at $p=1$ which is called entropy index defined by

$$\mu_q = \frac{d}{dp}\psi_q(p)\Big|_{p=1} \qquad \text{................................(4)}$$

and it describes the width of the fluctuation. A positive value of μ_q ($\mu_q > 0$) would corresponds to a broad $P(F_q^e)$ distribution which in turn would mean unpredictable large fluctuations of the spatial patterns from event-to-events. By applying this method to known classical chaotic systems, it has been shown[1,2] that μ_q can be used as a measure of chaoticity in systems where only the spatial patterns could be observed and a positive value of μ_q would signal the presence of chaos in the system. The entropy index μ_q is related to the entropy in event space as[1,2]

$S'_q = \ln(NM^{-\mu_q})$ ……………………… (5)

Evidently, a small μ_q corresponds to large entropy, which means no chaotic behaviour of particle production in branching processes. In order to decrease the entropy, the μ_q must be large, and so a large μ_q means chaotic behaviour. From Eq. (5) we can say that as μ_q increases, i.e. the event-to-event fluctuation of the factorial moment increases, S'_q decreases.

At large M , only large spikes in small bins contribute to F_q^e , especially when q is large. Events with large spikes are rare. Consequently, the fluctuation in F_q^e from event-to-event becomes more pronounced with increasing q . That behavior is now quantified by μ_q . We may therefore use μ_q to characterize the 'spatial' properties of the chaotic behavior of multiparticle production processes.

4. RESULTS AND DISCUSSION:

We have performed a study on p- AgBr interactions data at 400 GeV in pseudorapidity space using the above procedure in search for signs of chaos in pionisation of hadron-nucleus collisions. To do a rigorous study we have done four multiplicity cuts on our data set and got four overlapping sub-samples of events having different average multiplicities. The details of which are given in table 1. In order to eliminate the effect of non-flat average distribution, the pseudorapidity phase space variable η is transformed into the corresponding cumulant form[7] $X(\eta)$ as usual. After the transformation, the phase space region $X(\eta)$ becomes [0,1].

Table 1: Parameters for data-subsamples

Interaction	Sub-sample	Event Multiplicity (Nev)	No. of Events	Average Multiplicity
p-AgBr at 400 GeV	Sub-sample I	Nev $\leq$ 12	258	7.0 ± 0.1
	Sub-sample II	Nev $\geq$ 2	380	10.1 ± 0.2
	Sub-sample III	Nev $\geq$ 6	276	12.5 ± 0.2
	Sub-sample IV	Nev $\geq$ 8	235	13.6 ± 0.2

We have divided the $X(\eta)$ region for each sub-sample into M bins and have calculated the 2nd order factorial moments $F_2^e(M)$ for each event using Eq. (1) with M=3,4,5,…………,16 and then perform average of $F_q^e(M)$ for all the events. Fig. 1 represents the log-log plot of event-averaged factorial moment, $< F_q^e(M) >$ against phase-space partition number, M respectively for the whole data set. All the plots show excellent linear rise with decreasing bin size signifying intermittency.

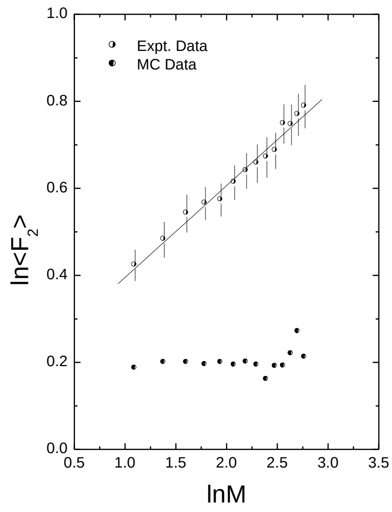

Fig.1: ln<$F_2(M)$> vs ln$M$ graph for experimental data as well as for IEH data sets

To check whether the observation is non-statistical in nature, Monte-Carlo simulated events are generated according to the independent emission hypothesis (IEH), which is based on the following assumptions:

(a) Pions are emitted independently of each other.

(b) The multiplicity distribution and rapidity distribution of Monte Carlo simulated events reproduces those of the real ensemble.

The log-log plots of event-averaged factorial moment for IEH data against M are also included in Fig. 1 with the corresponding experimental data, which signifies non-intermittent behavior. Thus we can conclude from here that our data is dynamically important, so we can use this data for studying the erraticity (chaoticity) analysis.

The factorial moment, $F_q^e(M)$ describes the pattern of the distribution of produced pions of the e^{th} event. As the pattern changes from event-to-event, $F_q^e(M)$ also changes. Large fluctuation in F_2^e is clearly observed for fixed M[1,2]. This large fluctuation in F_2^e is what we want to capture and would be lost if F_2^e is averaged over all events. To probe this event-to-event fluctuation of $F_2^e(M)$,we have calculated $C_{p,2}(M)$, the moment of factorial moments, using Eq.(2). Here p is the order for event-to-event fluctuation. We have calculated the values of $C_{p,2}(M)$ for p=0.7, 0.9, 1.0, 1.1, 1.3 and 1.5. The variation of ln$C_{p,2}(M)$ with ln$M$ including all p values has been depicted in Fig. 2(a)-(d) for sub-sample

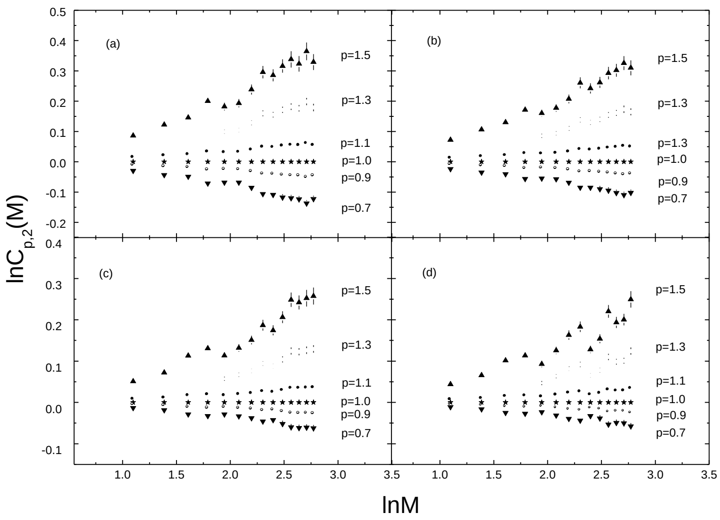

Fig. 2: ln $C_{p,2}(M)$ vs ln*M* for *p*=0.7,0.9,1.0,1.1,1.3 and 1.5 for (a) Sub-sample I, (b)Sub-sample II, (c) Sub-sample III and (d) Sub-sample IV

I-IV for p-AgBr interactions at 400 GeV. $C_{p,2}(M)$ shows power law behavior with *M* in the neighborhood of *p*=1 for the entire range of *M*. For all the sub-samples the linear best fits to the plots corresponding to *p*=0.9 and 1.1 have been performed. The confidence levels for the best fits never fall below 90%. According to Eq.(3) the slopes of plots give $\Psi_2(p)$. The slopes are given in Table 2 for p-AgBr interactions at 400 GeV. To quantify the degree of fluctuation of $F_2^e(M)$ from event-to-event the values of entropy index, μ_2 has been calculated using these slopes following the definition given in Eq.(4). The values of the entropy index μ_2, are also included in Table 2 correspondingly which signify chaos in p-AgBr interactions at 400 GeV.

Table 2: Entropy indices and relative parameters forexperimental and VENUS generated data of p-AgBr interactions at 400 GeV.

Sub-sample	Average Multiplicity	p	$\Psi_2(p)$	μ_2
Sub-sample I	7.0 ± 0.1	0.9	-0.0247 ± 0.0017	0.262 ± 0.018
		1.1	0.0277 ± 0.0019	
Sub-sample II	10.1 ± 0.2	0.9	-0.0206 ± 0.0012	0.223 ± 0.013
		1.1	0.0240 ± 0.0014	
Sub-sample III	12.5 ± 0.2	0.9	-0.0136 ± 0.0011	0.155 ± 0.013
		1.1	0.0173 ± 0.0014	
Sub-sample IV	13.6 ± 0.2	0.9	-0.0115 ± 0.0012	0.131 ± 0.014
		1.1	0.0146 ± 0.0016	
VENUS Data	10.9	0.9	0.082 ± 0.001	0.11 ± 0.02
		1.1	0.104 ± 0.002	

In order to find whether the results from our experimental data could be reproduced by the standard generators of particle production in heavy ion collisions, we simulated 10000 p- AgBr collisions at 400 GeV using the VENUS generator. The $C_{p,2}(M)$ moments for VENUS generated data for p-AgBr interactions at 400 GeV have been estimated and using that the values of the entropy index μ_2, are also calculated and included in Table 2 correspondingly. Thus it is transparent from the Table 2 that the experimental data yields significantly higher value for entropy index compare to VENUS generated data for both the interactions. This suggests that VENUS event generator do not reproduce the event-to-event fluctuations of spatial patterns of final states.

Fig. 3 exhibits the dependence of chaoticity (erraticity) on average multiplicity for p-AgBr interactions at 400 GeV and p-p collision at 400 GeV/c by Wang et al. [4] respectively. It transpires from the above plots that multiparticle production process becomes less chaotic with the increase of average multiplicity for our data sets of hadron-nucleus interaction.

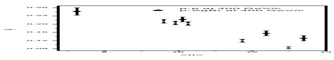

Fig. 3: The dependence of μ_2 on average multiplicity of data sub-samples for p-AgBr and p-p collision at 400 GeV

Fig. 3 reflects that the parameter, entropy index, is sensitive to beam and its energy. Both the plots of Fig.3 are for identical projectile with two different targets: one is nucleus and another is hadron but the values of μ_2 differ from each other significantly. This may be due to fact that nucleus is composed of many nucleons and a hadron-nucleus collision at a particular impact parameter involves a number of participants. When we are changing the multiplicity-cut, we are changing the impact parameter so that different number of participants are involved. The multiplicity distribution is therefore a result of smearing the multiplicities produced by different participants. On the other hand, in case of p-p collision there is only one source. This is the fundamental difference between hadron - nucleus data and hadron-hadron data.

5. CONCLUSIONS:

In this paper, he chaoticity (erraticity) behaviour of pions produced in p - AgBr interactions at 400 *GeV/c* have been analyzed systematically. From the analysis the following conclusions can be drawn:

1. The values of the entropy indices for different event samples are positive and quite large. Therefore, it can be concluded that multiparticle production in p-AgBr interactions at 400 *GeV/c* exhibits chaotic behaviour.
2. Erraticity behaviour depends strongly on multiplicity. The values of the slope $\psi_2(p)$ are different for various multiplicity samples.
3. The entropy index μ_2 is increased with decreasing average multiplicities.

6. REFERENCES

1. Cao Z., Hwa R. C. In search for signs of chaos in branching processes. *Phys. Rev. Lett.* 1995;75 :1268-1271
2. Cao Z. and Hwa R.C. Chaotic behavior of particle production in branching processes. *Phys. Rev. D.* 1996; 53 (11) : 6608-6618
3. Cao Z. and Hwa R. C. Erraticity analysis of multiparticle production. *Phys. Rev. D.* 2000 ; 61 : 074011-074016
4. Wang S. et al. The measurement of entropy indices in pp collisions at 400 geV/c. *Phys. Lett. B.*1998 ; 416 : 216-219
5. Atayan M.R. et al. Erraticity analysis of multiparticle production in π^+ - p and K^+ - p collisions at 250 GeV/c. *Phys. Lett. B.* 2003; 558 : 22-28
6. Powell C.F., Fowler P.H., Perkins D.H. *The study of elementary particles by photographic method.* Oxford, Pergamon; 1959; 450-464.
7. Ochs. W. Multidimensional intermittency analysis. *Z. Phys. C.* 1991; 50 : 339-344