Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Fuzzy Critical Path With Centroidmeasure

BalasowandariP.*[1] and AnusuyaV.[2]

*[1]PG and Research Department of Mathematics,SeethalakshmiRamaswami College, Tiruchirapalli-2

[2]PG and Research Department of Mathematics,SeethalakshmiRamaswami College, Tiruchirapalli-2

Email:balasowandari@gmail.comanusrctry@gmail.com

Mobile no. 9488604293,9443836302

ABSTRACT

Network diagram plays a vital role to determine project completion time. Network analysis is a technique which determines the various sequences of activities concerning a project and the project completion time. The popular method of this technique is widely used as the critical path method. In this paper, we find the fuzzy critical path in a acyclic project network using centroid measure to identify the fuzzy critical path from type-2 discrete fuzzy numbers and its complement. An illustrative example is also included to demonstrate our proposed approach.

KEYWORDS: Fuzzy critical path, type-2 discrete fuzzy number, Acyclic project network, centroid measure.

***Corresponding author**

P.Balasowandari

Research Scholar,

PG and Research Department of Mathematics,

SeethalakshmiRamaswamicollegeTiruchirapalli-2

Email:balasowandari@gmail.com

INTRODUCTION

Time management play more significant role in project management compares to scheduling control, resource and cost management. Critical path method(CPM) is a powerful tool that is most useful in practice and is applied in the planning and control of complicated projects in real world applications. The main purpose of critical path method is to evaluating project performance and to identifying the critical activities on the critical path so that the available resources could be utilized on these activities in the project network in order to reduce project completion time. With the help of the critical path, the decision maker can adopt a better strategy of optimizing the time and the available resources to ensure the earlier completion and the quality of the project.

This paper analyze the critical path in a general project network with fuzzy activity times. The fuzzy measures were introduced by sugeno[1]. We propose centroid measure[2] for fuzzy numbers to find a critical path method in a fuzzy project network, where the duration time of each activity in a fuzzy project network is represented by type – 2 discrete fuzzy number.

The organization of the paper is as follows: In section 2 some basic concepts are discussed. section 3 gives some properties of total slack fuzzy time. Section 4 gives the network terminology. An algorithm to find the critical path combined with type-2 discrete fuzzy number using centroid measure is given in Section 5.The proposed algorithm is illustrated through a numerical example in section 6.

2. BASIC CONCEPTS

2.1. Type-2 discrete fuzzy number

Let X be a non-empty finite set, which is referred as the universal set. A type-2 fuzzy setA, is characterized by a type-2 membership function $\mu_A(x,u) : X \times I \to I$ where $x \in X$, $I = [0,1]$ and $u \in J_x \subseteq I$ that is $A = \{((x,u); \mu_A(x,u) / x \in X, u \in J_x \subseteq I)\}$, where $0 \le \mu_A(x,u) \le 1$ A can also be expressed as $A = \int_{x \in X} \int_{u \in J_x} \frac{\mu_A(x,u)}{(x,u)} = \int_{x \in U} \frac{p_x(u)/u}{x}, J_x \subseteq I$, where $\iint$ denotes union all admissible x and u. For, discrete $\int$ universe of discourse is replaced by $\sum$.

2.2.Complement of type-2 discrete fuzzy number

The complement of the type-2 fuzzy set houses a fuzzy membership function given in the following formula:

$$\tilde{\tilde{A}} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u) \Big/ (1-u) \right] \Big/ x$$

2.3 Centroid measure

If $\tilde{A}$ is a type-2 fuzzy set in the discrete case, the centroid of $\tilde{A}$ can be defined as follows:

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \cdot \int_{\theta_2 \in J_{x_2}} \cdot \cdots \int_{\theta_R \in J_{x_R}} \cdot \left[f_{x_1}(\theta_1) . f_{x_2}(\theta_1) .... f_{x_R}(\theta_R) \right] \Big/ \frac{\sum_{j=1}^{R} x_j \mu_A(x_j)}{\sum_{j=1}^{R} \mu_A(x_j)}$$

Where $\tilde{A} = \sum_{j=1}^{R} \left[\sum_{u \in J_{x_j}} f_{x_j}(u) \Big/ u \right] \Big/ x_j$

2.4.Notations:

t_{ij}= The activity between node i and j.

ESF_j =The earliest starting fuzzy time of node j.

LFF_i = The latest finishing fuzzy time of node i.

TSF_{ij} = The total slack fuzzy time of t_{ij}.

p_n= the n^{th} fuzzy path.

P = the set of all fuzzy paths in a project network

$F(p_n)$= The total slack fuzzy time of path p_n in a project network.

3.PROPERTIES:

Property :3.1 (Forward pass calculation)

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie) $ESF_1 = (0.0,0.0,0.0,0.0)$ $ESF_j = \max_i \{ESF_j + TSF_{ij}\},\ j \neq i,\ j \in N$, i=number of preceding nodes.(ESF_j =The earliest starting fuzzy time of node j).

Ranking value is utilized to identify the maximum value.

Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

Property 3.2. *(Backward pass calculation)*

To calculate the latest finishing time in the project network set $LFF_n = ESFn$. $LFF_j = \min_j \{LFF_j (-) SET_{ij}\}, i \neq n, i \in N$, j = number of succeeding nodes. Ranking value is utilized to identify the minimum value.

Latest starting fuzzy time= Latest finishing Fuzzy time (-) Fuzzy activity time.

Property 3.3.

For the activity t_{ij}, i<j

Total fuzzy slack:

$SFT_{ij} = LFF_j (-)(ESF_i (+) SFT_{ij})$ (or) $(LFF_j (-) SFT_{ij})(-) ESF_i, 1 \leq i \leq j \leq n; i, j \in N$,

Property 3.4.

$F(p_n) = \sum_{\substack{1 \leq i \leq j \leq n \\ i, j \in p_k}} SFT_{ij}$, $p_k \in P$, p_n denotes the number of possible paths in a network from source node to the destination node, k=1 to m.

4.NETWORK TERMINOLOGY

A directed acyclic project network consisting of six nodes and eight edges are considered. Each edge in this network is assigned by type-2 discrete fuzzy numbers. The set of all possible paths are denoted by P. The fuzzy critical path is identified from P.

5. ALGORITHM (FOR FINDING CRITICAL PATH)

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $ESF_1 = (0.0, 0.0, 0.0, 0.0)$ and calculate ESF_j , j =2,3,…. n by using property 1.

Step 3: Let $LFF_n = ESFn$ and calculate LFF_i ,i=n-1, n-2,….. 2,1. By using property 2.

Step 4: Calculate SFT_{ij} with respect to each activity in a project network by using property 3.

Step 5 : Calculate centroid measure for each activity using definition 2.3.

Step 6 : If centroid measure = 0, the corresponding path is a fuzzy critical path.

6.NUMERICAL EXAMPLE

The problem is to find the fuzzy critical path in acyclicproject network whose edges are assigned with type-2 discrete fuzzy numbers.

Solution :

The edge lengths are

$\tilde{P} = (0.3/0.8 + 0.2/0.7)/2 + (0.3/0.9)/4$

$\tilde{Q} = (0.5/0.8 + 0.3/0.6)/3$

$\tilde{R} = (0.7/0.6)/1 + (0.5/0.7)/2$

$\tilde{S} = (0.4/0.4 + 0.5/0.5)/2 + (0.2/0.9)/3$

$\tilde{T} = (0.5/0.7)/2 + (0.7/0.4)/3$

$\tilde{U} = (0.2/0.6)/1 + (0.3/0.5 + 0.4/0.4)/3$

$\tilde{V} = (0.6/0.6)/2 + (0.7/0.5 + 0.4/0.4)/3$

$\tilde{W} = (0.6/0.8)/1 + (0.4/0.5)/3$

Complement of edge lengths are

$\overline{\tilde{P}} = (0.3/0.2 + 0.2/0.3)/2 + (0.3/0.1)/4$

$\overline{\tilde{Q}} = (0.5/0.2 + 0.3/0.4)/3$

$\overline{\tilde{R}} = (0.7/0.4)/1 + (0.5/0.3)/2$

$\overline{\tilde{S}} = (0.4/0.6 + 0.5/0.5)/2 + (0.2/0.1)/3$

$\overline{\tilde{T}} = (0.5/0.3)/2 + (0.7/0.6)/3$

$\overline{\tilde{U}} = (0.2/0.4)/1 + (0.3/0.5 + 0.4/0.6)/3$

$\overline{\tilde{V}} = (0.6/0.4)/2 + (0.7/0.5 + 0.4/0.6)/3$

$\overline{\tilde{W}} = (0.6/0.2)/1 + (0.4/0.5)/3$

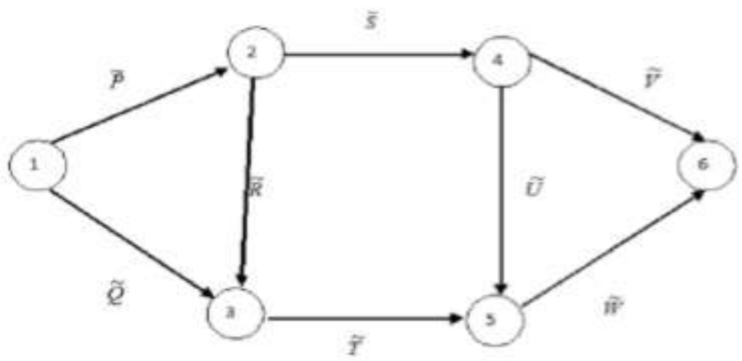

Fig1. Fuzzy acyclic project network

Activities, fuzzy durations and total slack time for each activity for type-2 discrete fuzzy number and its complement are given in the table 6.1 and table 6.2.

Table:6.1

Activity (i-j)i<j	**Fuzzy activity time**	**Defuzzified activity time**	**Total float**
1-2	$(0.3/0.8 + 0.2/0.7)/2 + (0.3/0.9)/4$	0.47	0
1-3	$(0.5/0.8 + 0.3/0.6)/3$	0.15	1.82
2-3	$(0.7/0.6)/1 + (0.5/0.7)/2$	0.54	0.96
2-4	$(0.4/0.4 + 0.5/0.5)/2 + (0.2/0.9)/3$	0.48	0
3-5	$(0.5/0.7)/2 + (0.7/0.4)/3$	0.35	0.96
4-5	$(0.2/0.6)/1 + (0.3/0.5 + 0.4/0.4)/3$	0.2586	1.11
4-6	$(0.6/0.6)/2 + (0.7/0.5 + 0.4/0.4)/3$	1.61	0
5-6	$(0.6/0.8)/1 + (0.4/0.5)/3$	0.24	0.96

Table:6.2

Activity (i-j)i<j	Fuzzy activity time	Defuzzified activity time	Total float
1-2	$(0.3/0.2 + 0.2/0.3)/2 + (0.3/0.1)/4$	0.393	0
1-3	$(0.5/0.2 + 0.3/0.4)/3$	0.04	1.643
2-3	$(0.7/0.4)/1 + (0.5/0.3)/2$	0.35	0.943
2-4	$(0.4/0.6 + 0.5/0.5)/2 + (0.2/0.1)/3$	0.39	0
3-5	$(0.5/0.3)/2 + (0.7/0.6)/3$	0.35	0.94
4-5	$(0.2/0.4)/1 + (0.3/0.5 + 0.4/0.6)/3$	0.3	0.3
4-6	$(0.6/0.4)/2 + (0.7/0.5 + 0.4/0.6)/3$	1.493	0
5-6	$(0.6/0.2)/1 + (0.4/0.5)/3$	0.24	0.94

All the possible paths P = {(1-2-4-6),(1-2-3-5-6),(1-3-5-6),(1-2-4-5-6)} are found in a given network using properties of network. The path 1-2-4-6 is identified as a fuzzy critical path by defuzzification using centroid measure.

CONCLUSION:

In this paper an attempt is made to find fuzzy critical path in a acyclic project network using type-2discrete fuzzy numbers and its complement with the help of centroid measure. The same path is also identified as a fuzzy critical path using complement of type-2 discrete fuzzy number.

REFERENCES

1. Sugeno,M. "*Fuzzy measures and fuzzy integrals-a survey*", In Gupta, saridis and Gaines, 1977; 89-102.
2. Yao S andLinFT "*Fuzzy critical path method based on signed distance ranking of fuzzy members"*, IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans, 2000; 30(1):76-82.
3. Chanas S and Zielinski P, "*Critical path analysis in the network with fuzzy activity times"*, *Fuzzy Sets and Systems*, 2001; 122: 195-204.
4. NasutionS.H, "*Fuzzy critical path method"*, *IEEE Trans. System Man Cybernetics*, 1994; 48-57.
5. O'Brien J.J, "*CPM in Construction Management"*, Mc-Graw-Hill, New York,1993.
6. ElizabethS andSujathaL, "*Fuzzy critical path problem for project network*", International journal of pure and applied mathematics ,2013; 85: 223-240.
7. ZadehL.A, "*Fuzzy sets*", *Information and Control*,1965; 8:138-353.
8. ZadehL.A, "*The concepts of a linguistic variable and its application to approximate reasoning part 1, 2, 3*", *Information Sciences*, 1975; 8:199-249,1976; 9 :43-58.
9. Anusuya Vand.SathyaR,*"Fuzzy shortest path by Type reduction Method"*, International Journal of FuzzyMathematical Archive,2014; 4:11-18.
10. Anusuya V and Balasowandari P, Fuzzy critical path with type-2 trapezoidal fuzzy numbers, Jamal Academic Research Journal, 2016; 171-176.