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On The Higher Degree Equation with Six Unknowns $x^6 - y^6 - 2z^3 = 10^{2n}T^{2m}(w^2 - p^2)$

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ABSTRACT

We presents non-zero solutions of the (2m+2)th measure non-homogeneous Diophantine equation in six unknowns represented by $x^6 - y^6 - 2z^3 = 10^{2n}T^{2m}(w^2 - p^2)$ in which $m, n \in z^+$. In exacting, unlike patterns of non-zero essential solutions of the on top of equation by the side of with a small number of fascinating properties along with the solutions are exhibited.

KEYWORDS: Higher degree equation with six unknowns, Integral solutions.

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INTRODUCTION

Diophantine equations, homogeneous and non-homogeneous have aroused the interest of various mathematicians since ancient times as can be seen from [1,2].The problem of decision all integer solutions of a Diophantine equation with three or more variables and extent at least three, in universal presents a good transaction of difficulties. There is huge universal hypothesis of homogeneous quadratic equations with three variables [3-7].Cubic equations in two variables drop into the hypothesis of elliptic curves which is a very urbanized assumption but motionless an vital issue of recent delve into [8-11].A bunch is identified regarding equation in two variables in higher degrees. For equations with extra three variables and scale at least three very little is known. It is value to message that undesirability appears in equations, even possibly at degree four with rather small co-efficient. It seems that greatly work has not been done in solving higher order Diophantine equations. In [12-20] a few higher order equations are considered for integral solutions. In this message a $(2m+2)^{th}$ degree non-homogeneous equation with six variables represented by $x^6 - y^6 - 2z^3 = 10^{2n}T^{2m}(w^2 - p^2)$ is considered and in particular a few interesting relations among the solutions are presented.

NOTATIONS USED

- $T_{m,n}$ - Polygonal number of rank n with size m.
- P_m^n - Pyramidal number of rank n with size m.
- Pr_n - Pronic number of rank n
- g_{na} – Gnomonic number of rank n.
- SO_n - Stella Octangular number of rank n.

METHOD OF ANALYSIS

The Diophantine equation representing the higher degree equation with six unknowns under consideration is

$$x^6 - y^6 - 2z^3 = 10^{2n}T^{2m}(w^2 - p^2) \tag{1}$$

Introduction of the transformation

$$x = u + v, y = u - v, z = 2uv, w = uv + 3, p = uv - 3 \tag{2}$$

In (1) leads to $u^2 + v^2 = 10^n T^m$ (3)

Now, we solve (3) through different methods and thus obtain different patterns of solutions to (1).

PATTERN-I

Assume $T = T(a,b) = a^2 + b^2$ (4)

Where a and b are non-zero distinct integers.

Write 10 as $10 = (3+i)\,(3-i)$ (5)

Using (4) & (5) in (3) and applying the method of factorization, define

$u + iv = (3+i)^n\,(a+ib)^m = (\alpha_1 + i\beta_1)(\gamma + i\delta)$, say

Equating the real and imaginary parts, we have

$$u = \alpha_1\gamma - \beta_1\delta$$
$$v = \alpha_1\delta + \beta_1\gamma$$

Hence in view of (2), the corresponding solutions of (1) are given by

$$x = \alpha_1\gamma - \beta_1\delta + \alpha_1\delta + \beta_1\gamma$$
$$y = \alpha_1\gamma - \beta_1\delta - \alpha_1\delta - \beta_1\gamma$$
$$z = 2(\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma)$$
$$w = (\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma) + 3$$
$$p = (\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma) - 3$$

Where

$$\alpha_1 = \frac{1}{2}[(3+i)^n + (3-i)^n]$$

$$\beta_1 = \frac{1}{2i}[(3+i)^n + (3-i)^n]$$

ILLUSTRATION-I

Let n=2, m=3

Thus the corresponding non-zero distinct integral solutions of (1) are

$$x = x(a,b) = 14a^3 + 6a^2b - 42ab^2 - 2b^3$$
$$y = y(a,b) = 2a^3 - 42a^2b - 6ab^2 + 14b^3$$
$$z = z(a,b) = 2(8a^3 - 18a^2b - 24ab^2 + 6b^3)(6a^3 + 24a^2b - 18ab^2 - 8b^3)$$
$$w = w(a,b) = (8a^3 - 18a^2b - 24ab^2 + 6b^3)(6a^3 + 24a^2b - 18ab^2 - 8b^3) + 3$$
$$p = p(a,b) = (8a^3 - 18a^2b - 24ab^2 + 6b^3)(6a^3 + 24a^2b - 18ab^2 - 8b^3) - 3$$
$$T = T(a,b) = a^2 + b^2$$

A few interesting properties observed are as follows:

1) $\left[\frac{z}{2}(a,b), w(a,b)-3, p(a,b)+3\right]$ *forms a pythagorean triple*

2) $x(1,b(b+1))-7y(1,b(b+1))+50SO_{b(b+1)}=350\,\mathrm{Pr}_b$

3) $x^2(a,b)-y^2(a,b)=2z(a,b)+w(a,b)-p(a,b)+6$

4) $x(a,1)-7y(a,1)+T(a,1)=301t_{4,a}-99$

5) $10[7x(a,1)+y(a,1)+150(gn_a-1)]$ *is a cubical integer*

PATTERN-II

Instead of (5), write 10 as 10= (1+3i) (1-3i) (6)

Following the procedure similar to pattern-I and performing a few calculations, the corresponding non-zero distinct integral solution s of (1) are found be

$$x=\alpha_2\gamma-\beta_2\delta+\alpha_2\delta+\beta_2\gamma$$
$$y=\alpha_2\gamma-\beta_2\delta-\alpha_2\delta-\beta_2\gamma$$
$$z=2(\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma)$$
$$w=(\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma)+3$$
$$p=(\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma)-3$$

Where

$$\alpha_2=\frac{1}{2}[(1+3i)^n+(1-3i)^n]$$
$$\beta_2=\frac{1}{2i}[(1+3i)^n+(1-3i)^n]$$

ILLUSTRATION-II

Let n=2, m=5

The corresponding non-zero distinct integral solutions of (1) are

$$x=x(a,b)=-2a^5+20a^3b^2-10ab^4-70a^4b+140a^2b^3-14b^5$$
$$x=x(a,b)=-14a^5+140a^3b^2-70ab^4+10a^4b-20a^2b^3-2b^5$$
$$z=z(a,b)=2(-8a^5+80a^3b^2-40ab^4-30a^4b+60a^2b^3-6b^5)(6a^5-60a^3b^2+30ab^4-40a^4b+80a^2b^3-8b^5)$$
$$w=w(a,b)=(-8a^5+80a^3b^2-40ab^4-30a^4b+60a^2b^3-6b^5)(6a^5-60a^3b^2+30ab^4-40a^4b+80a^2b^3-8b^5)+3$$
$$p=p(a,b)=(-8a^5+80a^3b^2-40ab^4-30a^4b+60a^2b^3-6b^5)(6a^5-60a^3b^2+30ab^4-40a^4b+80a^2b^3-8b^5)-3$$
$$T=T(a,b)=a^2+b^2$$

PROPERTIES:

1) $w(a,b)+p(a,b)-2z(a,b)=0$
2) $14x(1,b)-2y(1,b)=1000SO_b-200b^5$
3) $2y(a,1)-14x(a,1)=1000t_{4,a^2}-80t_{4,5a}$

PATTERN-III

Substituting m=0 in (3), we have

$$u^2+v^2=10^n \tag{7}$$

Applying the method of factorization, the corresponding non-zero distinct integral solutions of (7) are given by

$$\left.\begin{aligned} u_0&=\frac{1}{2}[(3+i)^n+(3-i)^n] \\ v_0&=\frac{1}{2i}[(3+i)^n-(3-i)^n] \end{aligned}\right\} \tag{8}$$

Taking m=1 in (3), we have

$$u^2+v^2=10^n T \tag{9}$$

Considering $T=T(a,b)=a^2+b^2$ and employing the method of factorization, the corresponding non-zero distinct integral solutions of (9) are given by

$$u_1=au_0-bv_0$$
$$v_1=au_0+bv_0$$

The repetition of the above process leads to the solutions of (3) represented by

$$u_m=\frac{1}{2i}(iAu_0+Bv_0)$$
$$v_m=\frac{1}{2i}(Bu_0+iAv_0)$$

Where $A=(a+ib)^m+(a-ib)^m$
$B=(a+ib)^m-(a-ib)^m$

Hence the corresponding non-zero distinct integral solutions of (1) are given by

$$x=\frac{1}{2i}\{(iAu_0+Bv_0)+(Bu_0+iAv_0)\}$$
$$y=\frac{1}{2i}\{(iAu_0+Bv_0)-(Bu_0+iAv_0)\}$$
$$z=-\frac{1}{2}\{(iAu_0+Bv_0)(Bu_0+iAv_0)\}$$
$$w=-\frac{1}{4}\{(iAu_0+Bv_0)(Bu_0+iAv_0)\}+3$$
$$p=-\frac{1}{4}\{(iAu_0+Bv_0)(Bu_0+iAv_0)\}-3$$
$$T=a^2+b^2$$

CONCLUSION

In linear renovation (2), the variables w and p may also be represented by

$w = 3u + v, p = 3u - v$.

Applying the process parallel to that of patterns I to III, other choices of essential solutions to (1) are obtained. To terminate, as sexticequations are rich in multiplicity, one may regard as other forms of sexticequations and rummage around for analogous properties.

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