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ON NANOα^{s*}CLOSED ANDNANOα^{s*}-CONTINUOUS

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ABSTRACT

The purpose of this paper is to define and study a new class of sets called Nano α^{s}*–closed sets in nano topological spaces. Basic properties of nano α^{s}*–closed sets are analyzed. We also used them to introduce the new notions like Nano α^{s}*-closure and Nano α^{s}*-continuous and their relation with already existing well known sets are also investigated.

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KEYWORDS: Nano closed set, Nano α-closed, Nano α^{s}*-closed, Nano α^{s}* -closure, Nano α^{s}*-continuous.

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1 INTRODUCTION:

The notion of α–open sets was introduced by O.Njastad[7] in 1965.In 1970 ,Levine[2] introduced the concept of generalized closed sets as a generalization of closed sets in Topological space. The notion of nano topology was introduced by Lillis Thivagar[3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets ,nano interior and nano closure. He also introduced the weak forms of Nano open sets namely Nano –αopen sets,Nano semi open sets and Nano pre open sets. In this paper we have introduced a new class of sets on nano topological spaces called nano α^{s}*-closed sets and investigate their basic properties. We also discuss their relationship with already existing concepts.

2 PRELIMINARIES:

DEFINITION 2.1[8]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the in discernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X)=\bigcup_{x\in U}\{R(x): R(x) \subseteq U\}$where R(X) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X)=\bigcup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not –X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X)=U_R(X)-L_R(X)$.

PROPERTY 2.2[8]

If (U,R) is an approximation space and X,Y⊆U , then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\emptyset)=U_R(\emptyset)$ and $L_R(U)=U_R(U)=U$

(iii) $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X) \cup L_R(Y) \subseteq L_R(X\cup Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$

(x) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

DEFINITION 2.3[3]

Let U be the universe,R be an equivalence relation on U and τ_R (X) $=\{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property $\tau_R(X)$satisfies the following axioms:

(i) U and $\emptyset$belongs to $\tau_R(X)$.

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X.(U,$\tau_R(X)$ is called the Nano topological space.Elements of the Nano topology are known as Nano open sets in U.Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

REMARK2.4[8]

If $\tau_R(X)$ is a Nano topology on U with respect to X, Then the set B={U,$L_R(X)$,$B_R(X)$) is the basis for $\tau R(X)$.

DEFINITION 2.5[3]

If (U,$\tau_R(X)$is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano interior of the set A is defined as the union of all Nano open sets contained in A and is denoted by NInt(A),NInt(A) is the largest Nano open subsets of A.

(ii) The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by NCl(A),NCl(A) is the smallest Nano closed sets containing A.

DEFINITION 2.6[3]

Let (U,$\tau_R(X)$) be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) Nano semiopen if $A \subseteq NCl[Nint(A)]$.

(ii) Nano semi closed if $NInt[NCl(A)] \subseteq A$.

(iii)Nano pre open if $A \subseteq NInt[NCl(A)]$.

(iv)Nano α- open if $A \subseteq NInt[NCl(NInt(A))]$.

The class of all Nano semi open ,Nano Semi closed, Nano pre open and Nano α –open is defined by NSO(U,X),NSF(U,X),NPO(U,X) and $\tau_R{}^{\alpha}(X)$resp.

DEFINITION 2.7[1]

(i) A subset A of (U, τ_R(X)) is called Nano generalized closed set (briefly Ng-closed) if NCl(A) ⊆V whenever A⊆V and V is Nano open in (U, τ_R(X)).

(ii) Nano generalized open (briefly Ng-open)if U\A is Ng-closed.

The intersection of all Ng-closed sets containing A is called the Nano g-closure of A and denoted by NCl*(A) and the Nano g-interior of A is the union of all Nano g-open sets contained in A and is denoted by NInt*(A).

DEFINITION 2.8[6]

Let (U, τ_R(X)) and (V, τ_R^*(Y)) be two nano topological space . The mapping f:(U, τ_R(X))→ (V, τ_R^*(Y)) is said to Nano α–continuous(Nano continous) if f^{-1}(A) is Nano α open (Nano opem) in U for every Nano open set A in V.

3 NANOα^s* -CLOSED SETS

DEFINITION 3.1

A subset A of a Nano topological space (U, τ_R(X)) is called Nano α^s* - closed if NCl*(NInt(NCl(A)))⊆A.The collection of all Nano α^s*-closed set in X is denoted by Nα^s*C(X).

THEOREM 3.2

Every Nano α-closed set is Nano α^s*-closed.

PROOF:

SupposeAisNanoαclosedthenNCl(NInt(NCl(A)))⊆A.Now,NCl*(NInt(NCl(A)))⊆NCl(NInt(NCl(A)))⊆A, it follows that A is Nano α^s*-closed.

THEOREM 3.3

Every Nano closed set is Nano α^s*-closed.

PROOF:

Since every Nano closed set is Nano α-closed and by Theorem 3.2, the result is clear.

REMARK 3.4

The converse of the above theorem is not true as shown in the following example.

EXAMPLE 3.5

Consider the Nano topological space (U, τ_R(X)) where U={a,b,c,d}U\R={{a},{c},{b,d}} and X={a,b} then L_R(X)={a}, U_R(X)={a,b,d} B_R(X)={b,d}. Therefore the Nano topology

$\tau_R(X)$={∅,U,{a},{a,b,d},{b,d}}. Here {a} is Nano α^{s}*-closed but neither Nano α-closednor Nano closed.

THEOREM 3.6

Arbitrary intersection of Nano α^{s}*-closed set is Nano α^{s}*-closed.

PROOF:

Let $\{F_\alpha\}$ be collection of Nano α^{s}*closed sets. Then NCl*(NInt(NCl(F_α))) ⊆ F_α for each ∝. Now,NCl*(NInt(NCl(∩ F_α))) ⊆NCl*(NInt(∩(NCl(F_α)))) ⊆NCl*(∩ (NInt(NCl(F_α))) ⊆ ∩NCl*(NInt(NCl(F_α))) ⊆∩ F_α.Therefore ∩ F_α is Nanoα^{s}*-closed.

REMARK 3.7

Union of two Nano α*-closed sets need not be Nano α*-closed as shown in the following example.

EXAMPLE 3.8

Consider the Nano topological space (U, $\tau_R(X)$)whereU={a,b,c,d,e}U\R={{a},{c},{b,d},(e)} and X={a,b} then $L_R(X)$={a}, $U_R(X)$={a,b,d} $B_R(X)$={b,d}. Therefore the Nano topology $\tau_R(X)$={∅,U,{a},{a,b,d},{b,d}}. Here {a,c} and {b,d} are Nano α^{s}*-closed but {a,b,c,d} is not Nano α^{s}*-closed.

DEFINITION 3.9

Let A be a subset of (U, $\tau_R(X)$) then Nano α^{s}* - closure of A is defined as the intersection of all Nano α^{s}*-closed set in X containing A.Iit is denoted by Nα^{s}*Cl(A). that is Nα^{s}*Cl(A) =∩ $\{F: A \subseteq F \text{ and } F$ is Nano α^{s}*-closed}

THEOREM 3.10

Let A be a subset of X. then A is Nano α^{s}* -closed iff Nα^{s}* Cl(A)=A.

PROOF:

Suppose A is Nano α^{s}* closed then by definition Nα^{s}*Cl(A)=A. conversely , suppose Nα*Cl(A) =A by using theorem3.6 , A is Nα^{s}*-closed.

THEOREM 3.11

Let A and B be subsets of (U, $\tau_R(X)$), then the following results hold.

(i) Nα^{s}*Cl (∅)= ∅ and Nα^{s}*Cl (X)= X.

(ii) A⊆Nα^{s}*Cl (A)

(iii)If A⊆B then Nα^{s}*Cl ((A)⊆Nα^{s}*Cl (B).

(iv) $A \subseteq N\alpha^{s}*Cl(A) \subseteq N\alpha Cl(A) \subseteq NCl(A)$

(v) $N\alpha^{s}*Cl(N\alpha^{s}*Cl(A)) = N\alpha^{s}*Cl(A)$

PROOF:

(i),(ii) and (iii) follows from Definition. (iv) follows from Theorem 3.2 and Theorem 3.3 and (v) follows from Theorem 3.10 and Definition

THE OREM 3.12

Let A and B be subset of $(U, \tau_R(X))$, then

(i) $N\alpha^{s}*Cl(A) \cup N\alpha^{s}*Cl(B) \subseteq N\alpha^{s}*Cl(A \cup B)$

(ii) $N\alpha^{s}*Cl(A \cap B) \subseteq N\alpha^{s}*Cl(A) \cap N\ \alpha^{s}*Cl(B)$

PROOF:

(i) by (iii) of Pre-theorem , $N\alpha^{s}*Cl(A) \subseteq N\alpha^{s*}Cl(A \cup B)$ and$N\alpha^{s*}Cl(B) \subseteq N\alpha^{s}*Cl(A \cup B)$. This implies that $N\alpha^{s}*Cl(A) \cup N\alpha^{s}*Cl(B) \subseteq N\alpha^{s}*Cl(A \cup B)$

(ii) Again byBy (iii) of Pre-Theorem $N\alpha^{s}*Cl(A \cap B) \subseteq N\alpha^{s}*Cl(A)$ and $N\alpha^{s}*Cl(A \cap B) \subseteq N\alpha^{s}*Cl(B)$.Hence $N\alpha^{s}*Cl(A \cap B) \subseteq N\alpha^{s}*Cl(A) \cap N\alpha^{s}*Cl(B)$

REMARK 3.13

The inclusions in Theorem 3.12 (i) may be strict and equality may also hold.

This can be seen from the following Example.

EXAMPLE 3.14

Consider the Nano topological space $(U, \tau_R(X))$where U={a,b,c,d,e}U\R={{a},{c},{b,d},(e)} and X={a,b} then $L_R(X)$={a}, $U_R(X)$={a,b,d} $B_R(X)$={b,d}. Therefore the Nano topology $\tau_R(X)$={∅,U,{a},{a,b,d},{b,d}}.

Let A={a,c} B={b,d} then $N\alpha^{s}*Cl(A)$={a} $N\alpha^{s}*Cl(B)$={b,d}and $N\alpha^{s}*Cl(A \cup B)$= $N\alpha^{s}*Cl$({a,b,c,d})=U.Therefore $N\alpha^{s}*Cl(A) \cup N\alpha^{s}*Cl(B) \subsetneq N\alpha^{s}*Cl(A \cup B)$

Consider the Nano topological space $(U, \tau_R(X))$ where U={a,b,c,d,e} U\R={{a},{c},{b,d},(e)} and X={a,b} then $L_R(X)$={a}, $U_R(X)$={a,b,d} $B_R(X)$={b,d}. Therefore the Nano topology $\tau_R(X)$={∅,U,{a},{a,b,d},{b,d}}.

Let A={b} B={c} then $N\alpha^{s}*Cl(A)$={b,c,d} $N\alpha^{s}*Cl(B)$={c}and $N\alpha^{s}*Cl(A \cup B)$= $N\alpha^{s}*Cl$({b,c})={b,c,d}.Therefore $N\alpha^{s}*Cl(A) \cup N\alpha^{s}*Cl(B) = N\alpha^{s}*Cl(A \cup B)$

REMARK 3. 15

The inclusions in Theorem 3.12(ii) may be strict and equality may also hold. This can be seen from the following Example.

EXAMPLE 3.16

Consider the Nano topological space $(U, \tau_R(X))$ where U={a,b,c,d,e}U\R={{a},{c},{b,d},(e)} and X={a,b} then $L_R(X)$={a}, $U_R(X)$={a,b,d} $B_R(X)$={b,d}. Therefore the Nano topology $\tau_R(X)$={∅,U,{a},{a,b,d},{b,d}}.

Let A={a,c} B={b,c} then $N\alpha^{s}$*Cl(A)={a,c} $N\alpha^{s}$*Cl(B)={b,c,d} and $N\alpha^{s}$*Cl(A∩B)= $N\alpha^{s}$*Cl({c})={c}.

Therefore $N\alpha^{s}$*Cl(A∩B) = $N\alpha^{s}$*Cl(A) ∩$N\alpha^{s}$*Cl(B)Let C={c} D={d} then $N\alpha^{s}$*Cl(C)={c} $N\alpha^{s}$*Cl(D)={b,c,d} and $N\alpha^{s}$*Cl(C∩D)= $N\alpha^{s}$*Cl(∅)=∅.

There fore $N\alpha^{s}$*Cl(C∩ D) ⊊$N\alpha^{s}$*Cl(C) ∩$N\alpha^{s}$*Cl(D).

4 NANOα^{s}* CONTINUOUS

DEFINITION 4.1

A subset of a space $(U, \tau_R(X))$ is called Nano α^{s*} open set if U \A is Nano α^{s*} -closed. Let $N\alpha^{s*}O(X)$ denote the collection of all Nano α^{s*} open set in $(U, \tau_R(X))$.

THEOREM 4. 2

A subset A of a space X is Nano α^{s*} open iff A⊆NInt*(NCl(NInt(A)))

PROOF:

Let A be a Nanoα^{s*} open set then U\A is Nano α^{s*}closed. That is iffNCl*(NInt(NCl(U\A))⊆U\A. that is iffNInt*(NCl(NInt(A)))⊇A. note that if A⊆ B then U\A⊇U\B and NCl*(U\A)= U\NInt*A and U \Ncl*A=Nint*(U\A).

THEOREM 4.3

Every Nano open(α-open set) is Nano α^{s*} open.

PROOF:

Suppose A is Nano open (Nano α-open) . then U\A is Nano closed(Nano αclosed) . By Theorem 3.2 , U\A is Nano α^{s*} closed. Therefore A is Nano α^{s*}open .

DEFINITION 4.4

Let $(U, \tau_R(X))$ and $(V, \tau_R^{*}(Y))$ be two nano topological space . The mapping $f:(U, \tau_R(X))\rightarrow (V, \tau_R^{*}(Y))$ is said to Nano α^{s*} -continuous if $f^{-1}(A)$ is Nano α^{s*} open in U for every Nano open set A in V.

THEOREM 4.5

Every Nano continuous (Nano α continuous) is Nano α^{s^*} -continuous.

PROOF:

Let f:(U, τ_R(X))→ (V, τ_R^*(Y)) be Nano continuous (Nano α continuous). Let V be an open set in Y . then f^{-1}(V) is Nano open (Nano α open in X).By Theorem 4.3 f^{-1}(V) is Nano α^{s^*} open in X. Hence f is Nano α^{s^*} -continuous.

But the converse is not true in the following Example.

EXAMPLE 4.6

Consider the Nano topological spaces (U, τ_R(X))and(V, τ_R^*(Y)) where U={a,b,c,d} =V U\R={{a},{c},{b,d}} X={a,b} and V\R*={{a},{d},{b,c}} Y={a,d}.If f:(U, τ_R(X))→ (V, τ_R(Y)) is defined by f(a)=b f(b)=a f(c)=d f(d)=a then f is Nano α^{s^*}-continuous but not Nano α continuous and not Nano continuous. Since f^{-1}({a,d})={b,c,d} is Nano α^{s^*}-open but not Nano open (Nano αopen).

2.3 CONCLUSION

Thus we introduced Nano α^{s}* -closed and Nano α^{s}* -open sets in topological spaces and discussed their relationship with already existing closed and open sets. The properties of the functions namely Nano α^{s}*-continuous are studied Further we gave some characterizations for these functions.

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