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Fractional Order Generalized Thermoelastic Problem with Initial Stress, Rotation and Temperature Dependent Properties

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ABSTRACT

In this manuscript, thermo elastic interactions in a homogeneous isotropic generalized medium are analyzed through estimating the effects of rotation, initial stress, fractional order parameter and temperature dependent properties on different fields inside the medium. Formulation of the problem is employed in the context of fractional order theory of generalized thermo elasticity. Governing equations of the general one-dimensional problem are shaped into matrix form using Laplace transform and state-space approach. Solutions of the problem in the physical domain are obtained by using a numerical method of the Laplace inverse transform based on the Fourier expansion technique, and the expressions for the displacement, temperature, and stress inside the medium are obtained. Numerical computations are carried out for a particular material for illustrating the results. Results obtained for the field variables are displayed graphically.

KEYWORDS: Fractional order thermo elasticity, initial stress, rotation, temperature dependent properties.

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INTRODUCTION

During recent years fractional calculus has developed rapidly. This process is still going on, but we can already recognize, that within the framework of fractional calculus new concepts and strategies emerge, which make it possible, to obtain new challenging insights and surprising correlations between different branches of science and technology. Povstenko[1] has constructed a quasi-static uncoupled thermo elasticity model based on the heat conduction equation with fractional order time derivatives. Sherief *et al.*[2] and Youssef[3] presented theories of fractional order thermo elastic models under different considerations. A new model of fractional heat conduction equation using new taylor series expansion for time-fractional order was established by Ezzat[4]. Kumar and Gupta[5] analyzed the reflection and transmission phenomena at an interface between elastic solid half space and a micro polar thermo elastic solid half-space with fractional order derivative.

The development of initial stresses in the medium is due to many reasons, for example, resulting from variation of temperature, process of quenching, overburden layer and slow process of creep, gravitation, weight, largeness, and so forth. Many authors have contributed their efforts to study the wave propagation in solids under initial stresses for various models. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay *et al.*[6]. Montanaro[7] constructed the model of isotropic linear thermo elasticity with hydrostatic initial stress. Singh[8] explored the effect of hydrostatic initial stress on waves in a thermo elastic half-space. Deswal *et al.*[9] studied the dynamical interactions of the thermal, elastic and diffusion fields under the fractional order generalized thermo elasticity theory with diffusion, two-temperature and initial stress.

The elastic modulus is an important physical property of materials reflecting the elastic deformation capacity of the material when subjected to an applied external load. It is well known that the physical properties of engineering materials vary with temperature. Ezzat *et al.*[10] solved a problem of generalized thermo elasticity with two relaxation times in an isotropic elastic medium with temperature-dependent mechanical properties. A general model of the equations of generalized thermomicrostretch for a homogeneous isotropic half-space with temperature-dependent properties was presented by Othman *et al.*[11]. Othman and Said[12] investigated the influence of magnetic field and temperature dependent properties on the plane waves in a fiber reinforced thermo elastic medium in the context of three-phase lag theory and Green–Naghdi theory without energy dissipation. Recently, Yadav *et al.*[13] studied propagation of waves in an initially stressed generalized electromicrostretch thermo elastic medium with temperature-dependent properties under the effect of rotation.

In the present paper, we study the effects of temperature dependent properties, rotation and initial stress under fractional order theory of thermo elasticity proposed by Sherief *et al.*[2]. We employ a state space approach developed by Bahar and Hetnarski[14] on the formulation. Resulting formulation is then applied to a specific problem of an elastic half-space whose boundary is subjected to an instantaneous thermal load. Inversions of the Laplace transforms are computed numerically by using the method introduced by Honig and Hirdes[15]. Finally, numerical estimates of the field variables have been displayed graphically in order to illustrate the problem in greater detail and the effects of rotation, initial stress, fractional parameter and temperature dependent properties on the distribution of different fields are analyzed.

PROBLEM FORMULATION

Following Sherief *et al.*[2] and Montanaro[7], The generalized field equations and constitutive relations for a rotating initially stressed thermo elastic continuum with temperature dependent properties are:

The constitutive relation:

$$\sigma_{ij} = 2\bar{\mu}e_{ij} + \bar{\lambda}e\,\delta_{ij} - \beta\,\theta\delta_{ij} - p(\delta_{ij} + \omega_{ij}), \tag{1}$$

with
$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{2}$$

Equation of motion:

$$\sigma_{ji,j} = (\rho\ddot{\vec{u}} + \vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2\times(\vec{\Omega}\times\dot{\vec{u}}))_i, \tag{3}$$

Heat conduction equation with fractional derivative heat transfer:

$$K\theta_{,ii} = \left(1 + \tau_0\frac{\partial^m}{\partial t^m}\right)\left(\rho c_E\frac{\partial\theta}{\partial t} + \beta\,T_0\frac{\partial u_{i,i}}{\partial t}\right), \tag{4}$$

where $\omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j})$, σ_{ij} and e_{ij} denote the components of stress tensor and strain tensor respectively, $\vec{u}$ is displacement vector, e is cubical dilatation and $\beta = \left(3\bar{\lambda} + 2\bar{\mu}\right)\alpha_t$ is the material constants, α_t is coefficient of linear thermal expansion, $\bar{\lambda}$ and $\bar{\mu}$ are generalized Lame's constants, satisfying the relations $\bar{\lambda} = \frac{E\upsilon}{\eta(1+\upsilon)(1-2\upsilon)}$ and $\frac{E}{\bar{\mu}} = 2\eta(1+\upsilon)$. Here E, υ and η are Young's modulus, Poisson's ratio and initial stress parameter respectively. τ_0 is thermal relaxation time, $\theta = T - T_0$, T is absolute temperature, T_0 is temperature of medium in natural state, ρ is the density of medium, K is the thermal conductivity, p is initial stress, δ_{ij} is the Kronecker delta function and m is the fractional order parameter such that $0 \le m < 1$. The medium is rotating with uniform angular velocity $\vec{\Omega} = \Omega \hat{n}$, where $\hat{n}$ is the unit vector in the direction of the axis of rotation.

Here, a comma followed by a suffix denotes material derivative, a superimposed dot denotes derivative with respect to time and the tensor convention of summing over repeated indices is used.

Our aim is to examine the effect of the temperature dependent nature of the material. So we assume that (Following Othman and Said[12])

$$(\bar{\lambda}_1, \bar{\mu}_1, \beta_1) = \frac{1}{1-\alpha^* T_0}(\bar{\lambda}, \bar{\mu}, \beta), \tag{5}$$

where α^* is an empirical elastic constant.

We consider a one-dimensional problem in which the field variables depend on a single space co-ordinate x (say) and time t. Hence we take $u = u(x, t), \theta = \theta(x, t)$ with $\vec{u} = (u, 0, 0)$, $\vec{\Omega} = (0, 0, \Omega)$ and $e = \frac{\partial u}{\partial x}$.

Proceeding with the analysis, we state the non-dimensional quantities

$$x' = \frac{\omega^*}{c_1}x,\ u' = \frac{\rho\omega^* c_1}{\beta_1 T_0}u,\ \theta' = \frac{\theta}{T_0},\ (t', \tau_0') = \omega^*(t, \tau_0),\ \Omega' = \frac{1}{\omega^*}\Omega,\ (\sigma_{ij}', P') = \frac{(\sigma_{ij}, P)}{\beta_1 T_0},$$

where

$$\omega^* = \frac{\rho c_E c_1^2}{K}, \quad c_1^2 = \frac{(\bar{\lambda}_1 + 2\bar{\mu}_1)}{\rho}.$$

Using the above values, equations (1), (3) and (4) can be reduced to non-dimensional forms as (omitting the primes for simplicity)

$$\sigma = a_0(e - \theta) - p, \tag{6}$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial^2 u}{\partial t^2} - \Omega^2 u, \tag{7}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left(1 + \varepsilon_2 \frac{\partial^m}{\partial t^m}\right)\left(\frac{\partial \theta}{\partial t} + \varepsilon_1 \frac{\partial e}{\partial t}\right), \tag{8}$$

where $\varepsilon_1 = \frac{a_0 \beta_1^2 T_0}{\rho K \omega^*}, \varepsilon_2 = \tau_0 \omega^{*(m-1)}, a_0 = 1 - \alpha^* T_0$.

Equation (7) can be written as

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} - \Omega^2 e, \tag{9}$$

We now define Laplace-Fourier double integral transform of a function $f(x, t)$ by

$$\bar{f}(x, s) = \int_0^\infty f(x, t) e^{-st} dt, \tag{10}$$

Applying Laplace transform defined by equation (10) to both sides of equations (6), (7) and (9) under homogeneous initial conditions, one obtains

$$\bar{\sigma} = a_0(\bar{e} - \bar{\theta}) - \frac{p}{s}, \tag{11}$$

$$D^2\bar{\theta} = g_1(s\bar{\theta} + \varepsilon_1 s\ \bar{e}), \tag{12}$$

$$D^2\bar{\sigma} = (s^2 - \Omega^2)\bar{e}, \tag{13}$$

Eliminating $\bar{e}$ from (16), we arrive at the following system of differential equations

$$\begin{aligned} D^2\bar{\theta} &= L_1\bar{\theta} + L_2\bar{\sigma} + L_3, \\ D^2\bar{\sigma} &= M_1\bar{\theta} + M_2\bar{\sigma} + M_3, \end{aligned} \tag{14}$$

where $D = \dfrac{d}{dx}$,

$$L_1 = g_1 s(1+\varepsilon_1),\ L_2 = \frac{g_1\varepsilon_1 s}{a_0},\ L_3 = \frac{g_1\varepsilon_1 p}{a_0},\ M_1 = s^2 - \Omega^2,\ M_2 = \frac{M_1}{a_0},\ M_3 = \frac{M_2 p}{s}.$$

STATE SPACE FORMULATION

Choosing temperature of heat conduction $\bar{\theta}$ and stress component $\bar{\sigma}$ in *x*-direction as state variables, set of equations (14) can be written in the matrix form as

$$D^2\bar{V}(x, s) = A(s)\bar{V}(x, s) + B(x, s), \tag{15}$$

where $\bar{V}(x, s) = \begin{bmatrix} \bar{\theta}(x, s) \\ \bar{\sigma}(x, s) \end{bmatrix}$, $A(s) = \begin{bmatrix} L_1 & L_2 \\ M_1 & M_2 \end{bmatrix}$ and $B(x, s) = \begin{bmatrix} L_3 \\ M_3 \end{bmatrix}$.

The formal solution of equation (15) can be expressed as

$$\bar{V}(x, s) = \exp\left[-\sqrt{A(s)}\ x\right]\left[\bar{V}(0, s) + A^{-1}(s)B(0, s)\right] - A^{-1}(s)B(x, s), \tag{16}$$

where

$\bar{V}(0, s) = \begin{bmatrix} \bar{\theta}(0, s) \\ \bar{\sigma}(0, s) \end{bmatrix} = \begin{bmatrix} \bar{\theta}_0 \\ \bar{\sigma}_0 \end{bmatrix}$ and *I* is a unit matrix of order 2.

With the view of getting the bounded solution for large *x*, we have retained the part of exponential that has a negative power.

We shall use well-known Cayley-Hamilton theorem to find the form of the matrix $\exp\left[-\sqrt{A(s)}\ x\right]$. The characteristic equation of the matrix $A(s)$ can be written as

$k^2-(L_1+M_2)k+(L_1M_2-M_1L_2)=0$.

The roots of the equation, namely, k_1 and k_2, satisfy the following relations

$k_1+k_2=L_1+M_2\ ,k_1k_2=L_1M_2-M_1L_2$.

The Taylor series expansion of the matrix exponential has the form

$$\exp\left[-\sqrt{A(s)}\ x\right]=\sum_{n=0}^{\infty}\frac{\left[-\sqrt{A(s)}\ x\right]^n}{n!}. \tag{17}$$

Using the well known Cayley-Hamilton theorem, we can express A^2 and higher orders of the matrix A in terms of I and A, where I is the unit matrix of second order.

Therefore, the infinite series in Eq. (17) can be reduced to the form $\exp\left[-\sqrt{A(s)}\ x\right]=\alpha_0 I+\alpha_1 A(s)$.

(18)

where α_0 and α_1 are the parameters depending on x and s.

As a consequence of Cayley-Hamilton theorem, the characteristic roots k_1 and k_2 of the matrix must satisfy equations

$$\exp\left[-\sqrt{k_1}\ x\right]=\alpha_0+\alpha_1 k_1, \tag{19}$$

$$\exp\left[-\sqrt{k_2}\ x\right]=\alpha_0+\alpha_1 k_2. \tag{20}$$

Solution of this system of linear equations yields

$$\alpha_0=\frac{k_1e^{-\sqrt{k_2}\,x}-k_2e^{-\sqrt{k_1}\,x}}{k_1-k_2}, \tag{21}$$

$$\alpha_1=\frac{e^{-\sqrt{k_1}\,x}-e^{-\sqrt{k_2}\,x}}{k_1-k_2}. \tag{22}$$

Hence, using (21) and (22) along with I and $A(s)$ into Eq. (18), we obtain $\exp\left[-\sqrt{A(s)}\ x\right]=L_{ij}(x,s)$

$(i,j=1,2)$ (23)

where

$$L_{11}=\frac{e^{-\sqrt{k_2}\,x}(k_1-L_1)-e^{-\sqrt{k_1}\,x}(k_2-L_1)}{k_1-k_2},\qquad L_{12}=\frac{L_2(e^{-\sqrt{k_1}x}-e^{-\sqrt{k_2}x})}{k_1-k_2},$$

$$L_{21}=\frac{M_1(e^{-\sqrt{k_1}x}-e^{-\sqrt{k_2}x})}{k_1-k_2}, \qquad L_{22}=\frac{e^{-\sqrt{k_1}x}(k_2-M_2)-e^{-\sqrt{k_2}x}(k_1-M_2)}{k_2-k_1}.$$

Now, the solution in Eq. (34) can be written in the form

$$\bar{V}(x,\, s)=L_{ij}\left[\bar{V}(0,\, s)+A^{-1}(s)B(0,\, s)\right]-A^{-1}(s)B(x,\, s). \tag{24}$$

Inserting the required values in Eq. (24), we obtain the following expressions for temperature and stress as

$$\bar{\theta}=\frac{e^{-\sqrt{k_2}\,x}((k_1-L_1)(\bar{\theta}_0+\gamma_1)-L_2(\bar{\sigma}_0+\gamma_2))-e^{-\sqrt{k_1}\,x}((k_2-L_1)(\bar{\theta}_0+\gamma_1)-L_2(\bar{\sigma}_0+\gamma_2))}{k_1-k_2}-\gamma_1, \tag{25}$$

$$\bar{\sigma}=\frac{e^{-\sqrt{k_2}\,x}(-M_1(\bar{\theta}_0+\gamma_1)+(k_1-M_2)(\bar{\sigma}_0+\gamma_2))+e^{-\sqrt{k_1}\,x}(M_1(\bar{\theta}_0+\gamma_1)-(k_2-M_2)(\bar{\sigma}_0+\gamma_2))}{k_1-k_2}-\gamma_2, \tag{26}$$

where $\gamma_1=\dfrac{T_2}{T_1}$, $\gamma_2=\dfrac{T_3}{T_1}$,

and $T_1=L_1M_2-M_1L_2$, $T_2=M_2L_3-L_2M_3$, $T_3=-M_1L_3+L_1M_3$.

BOUNDARY CONDITIONS

When the plane boundary is free from stress and is subjected to a thermal load, the boundary conditions on the surface x= 0 are

$$\sigma_{zz}+p=0, \theta=P(t), \tag{27}$$

where $P(t)$ is a given function of t.

Employing non-dimensional variables and Laplace transform on Eqs. (27), alongwith $P'=\dfrac{P}{T_0}$, we get

$$\bar{\sigma}(0,s)=\bar{\sigma}_0=-\frac{p}{s},$$

$$\bar{\theta}(0,s)=\bar{\theta}_0=\frac{P}{s}. \tag{28}$$

Using these values of $\bar{\sigma}_0$ and $\bar{\theta}_0$, the final expressions for temperature and stress and in the Laplace transform domain are obtained as

$$\bar{\theta}=\frac{e^{-\sqrt{k_2}x}\left((k_1-L_1)\left(P/s+\gamma_1\right)-L_2\left(-p/s+\gamma_2\right)\right)-e^{-\sqrt{k_1}x}\left((k_2-L_1)\left(P/s+\gamma_1\right)-L_2\left(-p/s+\gamma_2\right)\right)}{k_1-k_2}-\gamma_1, \tag{29}$$

$$\bar{\sigma}=\frac{e^{-\sqrt{k_2}x}\left(-M_1\left(P/s+\gamma_1\right)+(k_1-M_2)\left(-p/s+\gamma_2\right)\right)+e^{-\sqrt{k_1}x}\left(M_1\left(P/s+\gamma_1\right)-(k_2-M_2)\left(-p/s+\gamma_2\right)\right)}{k_1-k_2}-\gamma_2, \tag{30}$$

From equation (13), expression of displacement in the transformed domain is

$$\bar{u}=-\frac{1}{(s^2-\Omega^2)(k_1-k_2)}\left(\sqrt{k_2}\left(-M_1\left(P/s+\gamma_1\right)+(k_1-M_2)\left(-p/s+\gamma_2\right)\right)e^{-\sqrt{k_2}x}+\sqrt{k_1}\left(M_1\left(P/s+\gamma_1\right)-(k_2-M_2)\left(-p/s+\gamma_2\right)\right)e^{-\sqrt{k_1}x}\right), \tag{31}$$

LIMITING CASES

Following cases can be recognized from the preceding section as

$\Omega=0$ Fractional order theory of generalized thermo elasticity with initial stress and temperature dependent properties;

$p=0$ Fractional order theory of generalized thermo elasticity with rotation and temperature dependent properties;

$\alpha^*=0$ Fractional order theory of generalized thermo elasticity with rotation and initial stres;

$m=1$ Generalized theory of thermo elasticity with rotation, initial stress and temperature dependent properties.

INVERSION OF THE TRANSFORMS

It is difficult to find the analytical inverse Laplace transform of the complicated solutions for the displacement, temperature and stress in the Laplace transform domain. So we have to resort to numerical computations. In order to obtain the solution of the problem in the physical domain, we invert the Laplace transformation in equations (29)-(31) by adopting the methodology of Honig and Hirdes[15].

NUMERICAL RESULTS AND DISCUSSION

In this section, we perform some numerical calculations in order to illustrate the analytical results. The material chosen for this purpose is copper whose characteristics are as follows

$E = 10.8\times10^{10}\, kg\, m^{-1} s^{-2},\ \ \upsilon = 0.35,\ \rho = 8954\, kg\, m^{-3}, c_E = 383.1\, J\, kg^{-1} K^{-1},$
$T_0 = 293\, K, K = 386\, W\, m^{-1} K^{-1}, \alpha_t = 1.78\times10^{-5}\, K^{-1}, \tau_0 = 0.01 s,$
$t = 0.1 s, \alpha^* = 0.0005, \eta = 2.5, \Omega = 2, P = 1, p = 1, m = 0.5.$

Considering the above physical data, we compute the numerical values of displacement, temperature and stress component for a wide range of distance x. The results are displayed through various figures. From clarity point of view, we have divided the graphical representations into two categories. Category I (Figures 1-3) explores the influence of rotation and temperature dependent properties on field variables. By taking $\Omega = 0$, rotation effect is neglected and by taking $\alpha^* = 0$ temperature dependent properties are neglected. Pattern of different generalized thermo elastic fields in the presence and absence of initial stress and fractional order parameter is observed in Category II (Figures 4-6).

Category I

Figure 1 exhibits the effect of rotation and temperature dependent properties on displacement field. The profile of displacement field is very much similar in all the three cases attaining maximum value at x=0 and ultimately tending to zero. The numerical values of displacement field is less in the main case in comparison to that of other two cases. Thus rotation as well as temperature dependent properties has decreasing effect on displacement distribution.

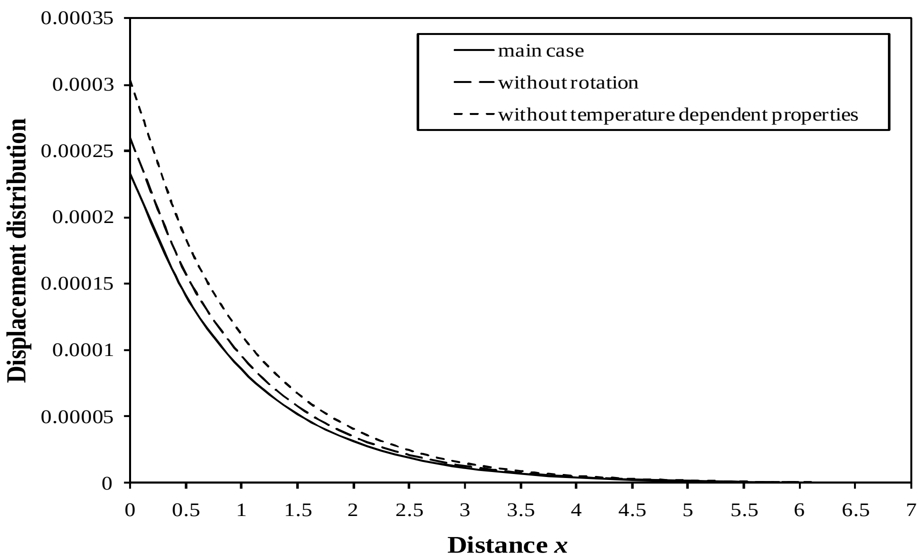

Fig. 1: Effect of rotation and temperature dependent properties on displacement distribution

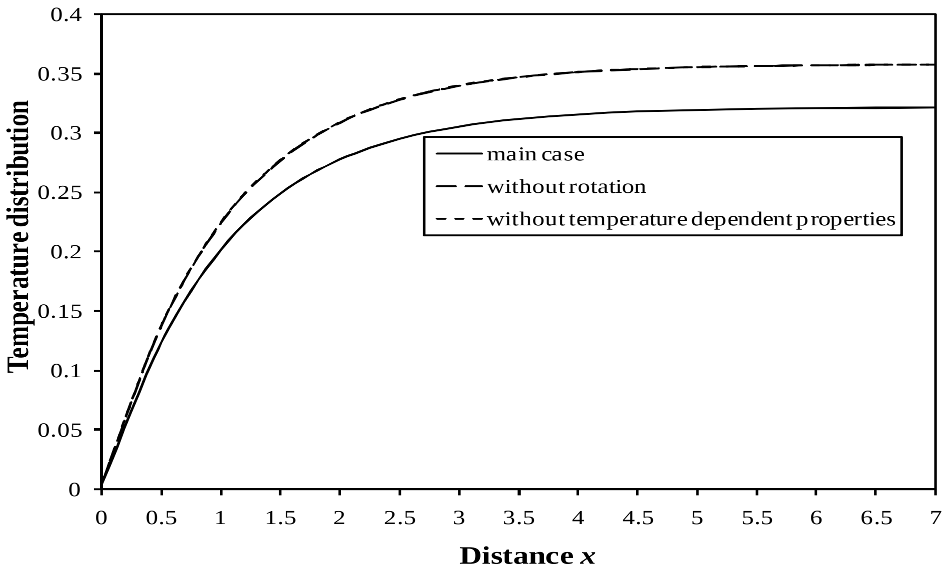

Fig. 2: Effect of rotation and temperature dependent properties on temperature distribution

Figure 2 displays the behavior of temperature distribution in the direction of wave propagation. It begins with non zero values on the boundary abiding by an increasing fashion of variations for all the considered cases. At x=0, non zero value of temperature is in accordance with boundary condition. Presence of rotation and temperature dependent properties affect the temperature field by decreasing its values.

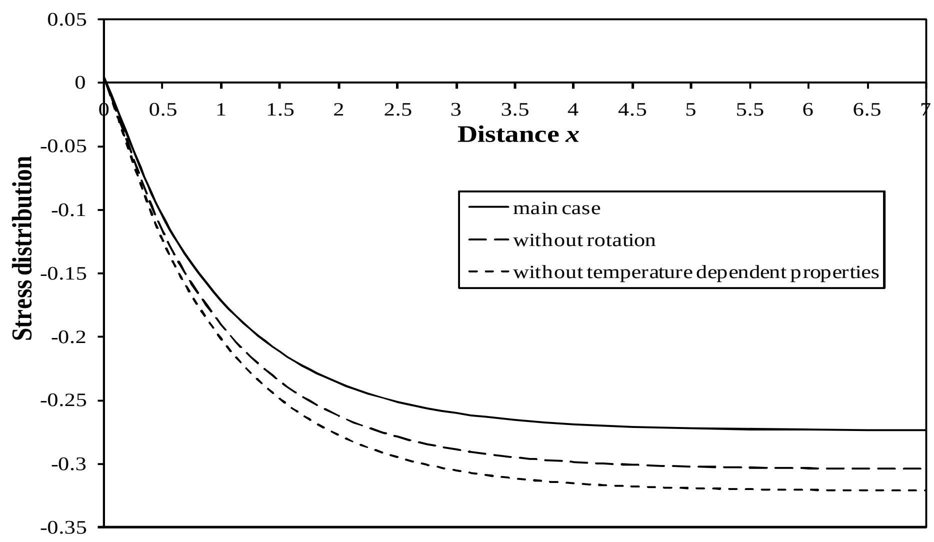

Fig. 3: Effect of rotation and temperature dependent properties on stress distribution

Figure 3 exhibits the variations of stress in the presence and absence of rotation and temperature dependent properties. We found that the behavior of stress component in all the three cases is alike. Due to the presence of initial stress, the curves begins with non zero positive value ultimately tending to non zero constant value. Presence of rotation and temperature dependent properties has decreasing effect on numerical values of stress component.

Category II

Figure 4 shows the effect of initial stress and fractional parameter on displacement distribution. Presence of initial stress increases the values of displacement distribution. Thus initial stress exhibits increasing effect on displacement. Presence of fractional parameter minifies the numerical values of displacement distribution before x=0.7 while magnifies after x=0.7. Thus fractional parameter has mix kind of effect on displacement distribution.

Figure 5 exhibits temperature distribution in the presence and absence of initial stress and fractional parameter. We can see that initial stress and fractional parameter have a noticeable effect on the temperature. Numerical values of temperature field are larger in magnitude in generalized thermo elasticity theory as compared to fractional order generalized thermo elasticity theory, which indicates that fractional parameter has a decreasing effect. On the other hand, initial stress has increasing effect on temperature. It is noticed that in the absence of initial stress temperature field ultimately tends to zero while in the other two cases, temperature is non zero in the whole range of x.

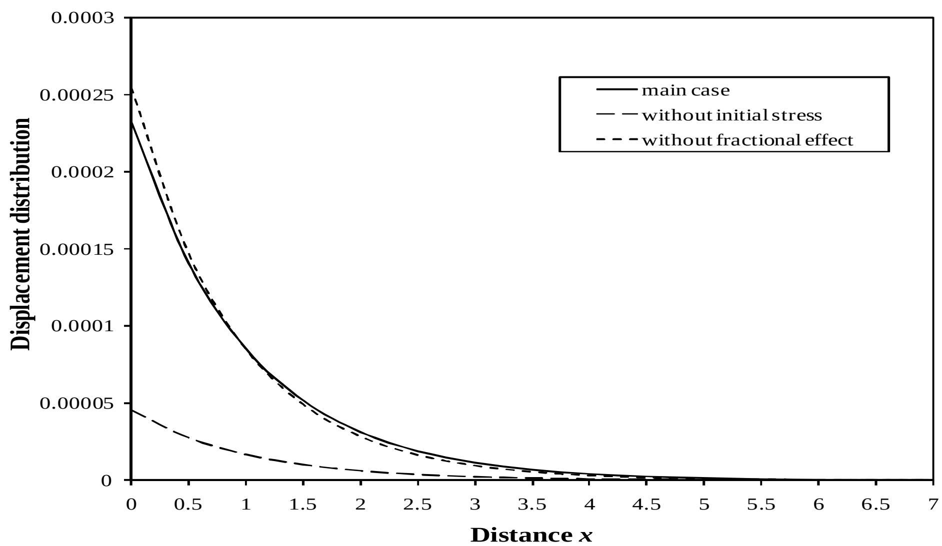

Fig. 4: Effect of initial stress and fractional parameter on displacement distribution

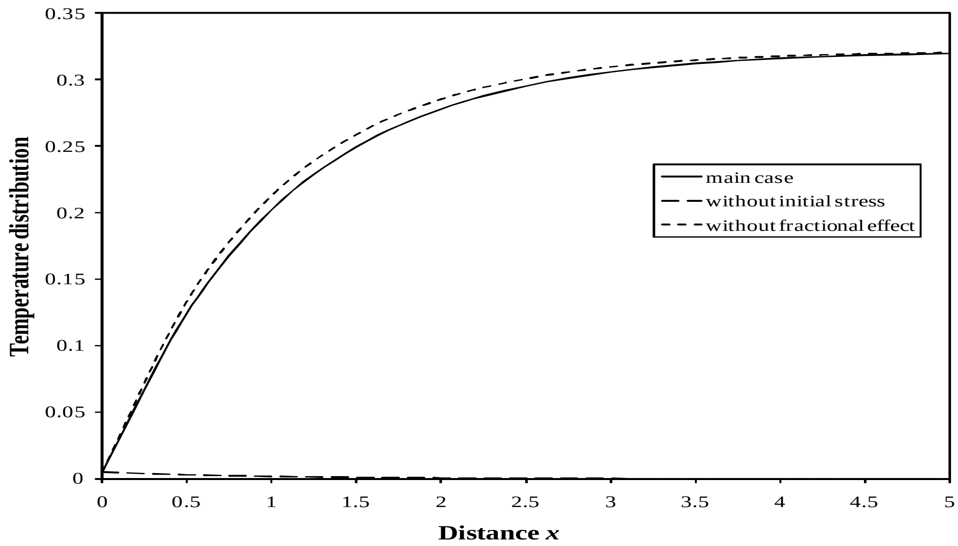

Fig. 5: Effect of initial stress and fractional parameter on temperature distribution

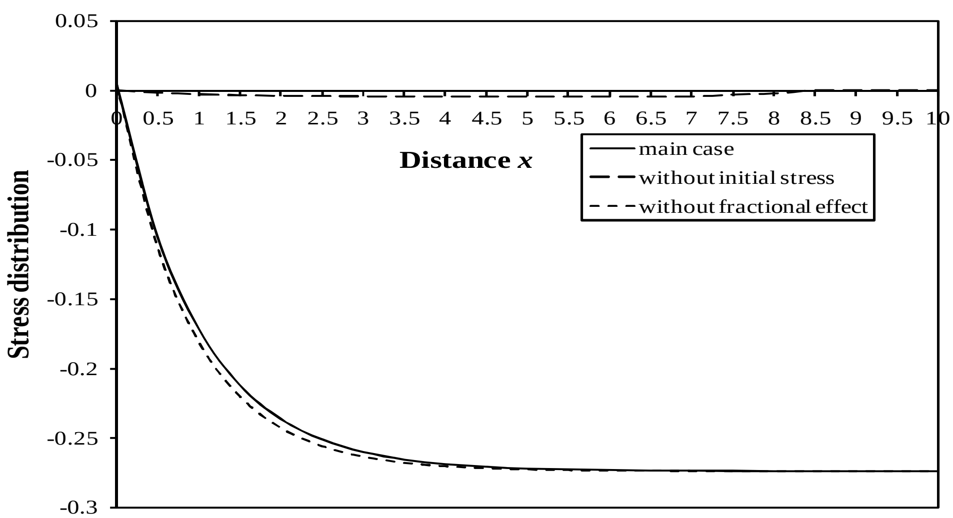

Fig. 6: Effect of initial stress and fractional parameter on stress distribution

Figure 6 is plotted to evince the effects of initial stress and fractional parameter on stress distribution. It is clear from the figure that in the absence of initial stress, stress field begins with zero value and ultimately tends to zero which is physically admissible. Initial stress enhances the numerical values of stress field while fractional parameter decreases the values of stress field numerically. Thus initial stress and fractional parameter have increasing and decreasing respectively on stress distribution.

CONCLUSIONS

Behavior of displacement, temperature and stress in a rotating initially stressed thermo elastic medium with temperature dependent properties due to thermal load has been examined within the framework of generalized thermo elasticity theory of fractional order heat conduction by using state space approach. Following observations emerge from the present investigation:

- Rotation and temperature dependent properties act as a decreasing agent for all the physical fields.
- Initial stress is also found to be the prominent factor affecting the variations of all the generalized fields pertaining to our considered model.
- It is interesting to notice from Figure 6 that in the absence of initial stress, stress component begins with zero value and temperature and stress distributions tend to zero.
- The effect of fractional parameter on all the studied fields is very much significant.

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