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Generalized Separation Axioms in Topology

Rashid Md. Mamun Ar*

Department of Mathematics, Santipur College, P.O. Santipur ,Dist.Nadia, West Bengal, India.741404
Email: mamunrashid1971@gmail.com, Mob. No. 09475258698

ABSTRACT

In this paper, we introduce and study a new class of separation axioms called generalized separation axioms using generalized open sets due to Levine. Introducing the definition of ξ- open sets in a topological space we define a new separation axioms. The connections between these separation axioms and other existing well-known related separation axioms are also investigated.

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***Corresponding author**

Md. Mamun Ar Rashid

Department of Mathematics, Santipur College, P.O. Santipur ,

Dist.Nadia, West Bengal, India.741404

Email: mamunrashid1971@gmail.com

Mob. No. 09475258698

INTRODUCTION

In 1970, Levine generalized the concept of closed sets to generalized closed sets. The complement of a open (resp. *g*-closed[1]) set is called a closed (resp. *g*-open [1]) set. Recently, there is a vast progression in the field of generalized closed sets. After then, there are many works on separation axioms has been done.. In this paper, we introduce the generalized forms of separation axioms using the concepts of generalized open sets called generalized - T_i (briefly denoted by g-T_i) spaces. Also, we define the concepts of ξ-open sets in topology to define the another class of separation axioms called ξ-separation axioms and also a few properties of these separation axioms are investigated.

PRELIMINARIES

Through out in this paper, we denote X *as a* topological space.

Definition 2.1: The closure (resp. preclosure) of a subset A of X is the intersection of all closed (resp. preclosed) sets that contains A and is denoted by clA (resp. $pclA$). The union of all open subsets of A is called the interior of A and is denoted by int A.

Definition 2.2: Generalized closure of a subset A of a space X is the intersection of all $g$closed sets containing A and is denoted by $gcl(A)$.

Definition 2.3:A point x of a space X is called a generalized limit point(g-limit point) of a subset A of X, if for each g-open set U containing x, $A \cap (U - \{x\}) \neq \Phi$ and the set of all g-
limit points of A, denoted by $gd(A)$, is called generalized derived set of A .

Definition 2.4: Let A be a subset of a space X then A is said to be a generalized closed (*i.e.* g-closed) set if $clA \subset U$ whenever $A \subset U$ and U is open set.

Definition 2.5: A space X is called a $T_{1/2}$ space if every g-closed set is closed.

Definition 2.6: A space X is called T_1 iff to each pair of distinct points x, y of X, there exists a pair of open sets, one containing x but not y, and the other containing y but not x.

Definition 2.7: A space X is called R_o iff for each open set G and $x \in G$ implies $gcl\{x\} \subset G$.

Definition 2.8: A space X is called R_1 space iff for x, $y \in X$ with $gcl\{x\} \neq gcl\{y\}$, there exist disjoint open sets U and V such that $gcl\{x\} \subset U$ and $gcl\{y\} \subset V$.

Definition 2.9: A space X is called T_2 space iff to each pair of distinct points x, y of X there exists a pair of disjoint open sets, one containing x and the other containing y.

SEMI-GENERALIZED SEPARATION AXIOMS

In this section, we define and study some new separation axioms using g-open sets .

Definition 3.1: A space X is called generalized $-T_0$ (briefly g-T_0) iff to each pair of distinct points x, y of X, there exists a g-open set containing one but not the other.

Clearly, every T_0 space is g-T_0 space since every open set is g-open set but converse is not true.

Theorem 3.1: If in any topological space X, generalized closures of distinct points are distinct, then X is g-T_0.

Proof: Let x, y X, $x \neq y$ imply $gcl\{x\} \neq gcl\{y\}$. Then there exists a point $z \in X$ such that z belongs one of two sets, say, $gcl\{y\}$ but not to $gcl\{x\}$. If we suppose that $z \in gcl\{x\}$, then $z \in gcl\{y\} \subset gcl\{x\}$, which is contradiction. So, $y \in X\text{-}gcl\{x\}$, where $X\text{-}gcl\{x\}$ is g-open set which does not contain x. This shows that X is g-T_0.

Theorem 3.2: In any topological space X, generalized closures (gclosures) of distinct points are distinct.

Proof is simple.

Definition 3.2: A space X is called generalized $-T_1$ (briefly g-T_1) iff to each pair of distinct points x, y of X, there exists a pair of g-open sets, one containing x but not y, and the other containing y but not x.

Clearly, every T_1 space is g-T_1 space since every open set is g-open set.

Definition 3.3: A subset A of a space X is called a generalized neighbourhood (i.e. *gnbd.*) of a point x of X if there exists g-open set U containing x such that $U \subset \Phi\, A$.

Definition 3.4: The union of all g-open sets which are contained in A is called the generalized interior of A and is denoted by gint A.

Since the union of g-open sets is g-open and hence g int A is g-open.

Lemma 3.1: A subset of a space X is g-open iff it is a g nbd. of each of its points.

Proof is omitted.

Definition 3.5: A point x of X is called a generalized interior point (i.e.g-interior point) if there is a g open set A such that x $\in$A.

Lemma 3.2: Let X be a space and $A \subset X$, $x \in X$. Then x is a g-interior point of A iff A is a *gnbd*. of x.

Theorem 3.3: For a topological space X, each of the following statement is equivalent:

(a) X is g-T_1

(b) Each one point set is g-closed set in X

Proof is simple.

Lemma 3.3: A point $x \in gcl(A)$ iff every g-open set containing x contains a point of A.

Theorem 3.4: If X is g-T_1 and $p \in gd(A)$ for some subset A of X, then every $gnbd$. of p contains infinitely many points of A.

Proof is simple.

Theorem 3.5: In a g-T_1 space X, $gd(A)$ is g-closed for any subset A of X.

Definition 3.6: A space X is called generalized – T_2 space (briefly written as g-T_2 space) iff to each pair of distinct points x, y of X there exists a pair of disjoint g-open sets, one containing x and the other containing y.

Clearly, every T_2 space is g-T_2 space since every open set is g-open set.

Definition 3.7: A space X is called generalized -R_o(i.e. written as g-R_o) iff for each g-open set G and $x \in G$ implies $gcl\{x\} \subset G$.

Clearly, every R_o space is g-R_o.

Definition 3.8: A space X is called generalized – R_1 (i.e. written as g-R_1) space iff for x, $y \in X$ with $gcl\{x\} \neq gcl\{y\}$, there exist disjoint g-open sets U and V such that $gcl\{x\} \subset U$ and $gcl\{y\} \subset V$.

Clearly, every R_1 space is g-R_1.

Theorem 3.4: The following are equivalent.

(a) X is g-T_2 space

(b) X is g-R_1 and g-T_1 space

(c) X is g-R_1 and g-T_o.

Proof is easy and hence omitted.

ξ-SEPARATION AXIOMS

In this section, we define and study some new separation axioms by defining ξ-open sets which are stronger than generalized separation axioms.

Definition 4.1: A subset A of X is called ξ-open set of X if $F \subset \text{int } A$ whenever F is g-closed and $F \subset A$.

Clearly, every open set is ξ-open and every ξ-open set is g-open set.

Definition 4.2: A subset A of a space X is called a ξ-neighbourhood of a point x of X if there exists ξ-open set U containing x such that $U \subset A$.

Definition 4.3: The union of all ξ□open sets which are contained in A is called the ξ-interior of A and is denoted by ξ-int$A$.

Since the union of ξ-open sets is ξ-open and hence ξ-int$A$ is ξ-open.

Lemma 4.1: A subset of a space X is ξ-open iff it is a ξ-nbd. of each of its points.

Proof is omitted.

Definition 4.4: A point x of X is called a ξ-interior point of $A \subset X$ if there is ξ-open set containing $x \in A$.

Lemma 4.2: Let X be a space and $A \subset X$, $x \in X$. Then x is a ξ - interior point of Aiff A is a ξ-*nbd*. of x.

Definition 4.5: The ξ-closure of a subset A of X is the intersection of all ξ-closed sets that contains A and is denoted by ξ-*clA*.

Definition 4.6: A point x of a space X is called a ξ--limit point of a subset A of X, if for each ξ--open set U containing x, $A \cap (U - \{x\}) \neq \Phi$

Definition 4.7: The set of all ξ--limit points of A, denoted by ξ--$d(A)$, is called ξ--derived set of A

Definition 4.8: A space X is called ξ- – T_0 iff to each pair of distinct points x, y of X, there exists a ξ--open set containing one but not the other.

Clearly, every T_0 space is ξ--T_0 space and every ξ--T_0 is g-T_0 since every open set is ξ--open and every ξ--open set is g-open set.

Theorem 4.1: If in any topological space X, ξ-closures of distinct points are distinct, then X is ξ-T_0:

Definition 4.9: A space X is called ξ--T_1 iff to each pair of distinct points x, y of X, there exists a pair of ξ--open sets, one containing x but not y, and the other containing y but not x.

Clearly, every T_1 space is ξ-T_1 space and ξ-T_1 space is g-T_1 space , since every open set is ξ--open and every ξ--open set is g-open set.

Theorem 4.2: For a topological space X, each of the following statement are equivalent:

(a) X is ξ-–T_1

(b) Each one point set is ξ--closed set in X

Proof is simple.

From the definition of ξ-limit point and ξ--$d(A)$, the following can be easily proved.

Lemma 4.2: A point $x \in$ξ-$cl(A)$ iff every ξ-open set containing x contains the point of A.

Theorem 4.3: If *X* is ξ-T_1 and $p \in$ξ-*d*(*A*) for some subset *A* of *X*, then every ξ-nbd. of *p* contains infinitely many points of *A*.

Theorem 4.4: In a ξ--T_1 space *X,* ξ--*d*(*A*) is ξ-closed for any subset *A* of *X*.

Definition 4.10: A space *X* is called ξ- T_2 space iff to each pair of distinct points *x*, *y* of *X* there exists a pair of disjoint ξ-open sets, one containing *x* and the other containing *y*.

Clearly, every T_2 space is ξ--T_2 and ξ-T_2 space is *g*-T_2 space since every open set is ξ--open and every ξ--open set is *g*-open set.

Definition 4.11: A space *X* is called ξ--*Ro* iff for each ξ--open set *G* and $x \in G$ implies ξ--*cl*{*x*} ⊂ *G*.

Clearly, every R_o space is ξ-*Ro* and every ξ-*Ro* space is *g*-R_o.

Definition 4.12: A space *X* is called ξ-R_1 space iff for *x*, $y \in X$ with ξ-*cl*{*x*} ≠ ξ-*cl*{*y*}, there exist disjoint ξ-open sets *U* and *V* such that ξ-*cl*{*x*} ⊂ *U* and ξ-*cl*{*y*}⊂V.

Clearly, every R_1 space is ξ-R_1 and every ξ-R_1 space is *g*-R_1.Clearly, every ξ-R_1 space is ξ-R_o space.

Theorem 4.4: The following are equivalent.

(*a*) *X* is ξ-T_2 space

(*b*) *X* is ξ-R_1 and ξ-T_1space

(*c*) *X* is ξ-R_1 and ξ- T_o.

Proof is easy and hence omitted.

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