Reseach article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Optimization Of Operational Cost of Machines In Three Stage Flow Shop Scheduling Using Branch and Bound Technique With Job Block

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ABSTRACT

In this paper, a study is made on scheduling of flow shop problem under uncertain environment and an efficient branch and bound optimization technique for three machines is proposed to find near-optimal schedule in a disturbance scenario. The processing time of all the jobs on machines are uncertain and are presented by triangular membership function. Further a job block for the three machines is introduced due to its applicability in the real world. The main aim of this paper is to attain an optimal schedule, which minimizes the total operational cost of the machines. Finally, computational results for this problem are provided to evaluate the performance of the proposed algorithm.

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INTRODUCTION

Scheduling is a decision-making procedure that concerns the allotment of constrained assets to an arrangement of assignments with the perspective of optimizing one or more objectives. In this day and age of worldwide rivalry, effectual scheduling has turned out to be essential keeping in mind the end goal to meet customer requirements as quickly as could be expected under the circumstances while maximizing the profits. Scheduling in manufacturing systems is classically associated with scheduling a set of jobs on a set of machines in order to maximize the profit. Manufacturing system is mainly categorized as flow shop, job shop, and open shop. A typical flow shop problem contains n jobs namely J_1, J_2, J_3,…, J_n to be prepared through m machines M_1, M_2, M_3, …M_m (Say). Technological constraints demand that each job should be processed through the machines in a precise order and gives a critical extraordinary case termed as flow shop. However in reality, due to the uncertainty of information as well as the variation of management scenario, it is often hard to get the correct job processing time of all jobs. Thus, the ordinary methodologies, both deterministic and random process, have a tendency to be less effective in conveying the imprecision or ambiguity nature of the linguistic values. One alternative option to deal with this type of problem associated with the processing time is to use fuzzy sets. In literature, several techniques were proposed for managing uncertainty but to solve vague situations in real problems, the first systematic approach related to fuzzy sets theory was recommended by Zadeh[1] in 1965.Fuzzy set theory because of its simplicity and similarity to human reasoning has numerous applications in various fields such as engineering, pharmaceutical, manufacturing and others. Fuzzy numbers are applied to represent the imprecise numerical measurements of different alternatives. McCahon and Lee[2] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee[3] addressed the formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Sanuja and Xueyan[4] optimized the makespan in a two-machine flow shop problem in the presence of uncertainty and proved that their approach of using different fuzzy sets determined by α-cut of processing times is better than McCahon and Lee[2].Hong and Chuang[5] developed a new triangular Johnson algorithm.Ignall and Scharge[6] introduced branch and bound technique in flow shop scheduling problems. Some of the noteworthy approaches are due toLomnicki[7], Bagga[8], Gupta JND[9], Yager[10], Marin and Roberto[11], Yao and Lin[12], Singh and Gupta[13], Gupta, Aggarwal and Sharma[14,15]etc.

In this paper we have introduced a new job block parameter for three machines to solve fuzzy scheduling problem under B & B approach. The idea of job block equivalent to a single job was first of all originated by Maggu & Das[16]in order to create a balance between the cost of providing priority in Service to the customer and the cost of giving service if no priority is considered i.e. how much is to be charged extra from the priority customer(s) as compared to non-priority customer(s). If J_1 and J_2 are two jobs, then ordered pair (J_1, J_2) is called a job block and designated by a single job. Further it is assumed that no more jobs can be processed in between J_1 and J_2 on all the machines and job J_2 can be processed before J_1 due to technological constraint.

There are many practical situations in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machines. In this regard, the decision maker has totake machines on rent in order to complete the assignments. For instance, care giving techniques often require hi-tech, expensive medical equipment which are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them.

The rest of this paper is organized as follows:

In section 2, we recall the basic concepts of fuzzy numbers and their arithmetic operations. In section 3, we have designed a mathematical model with the essential notations and assumptions used throughout the paper. Section 4 introduces the operational policy. In section 5 we have proposed a branch and bound algorithm with job block in order to minimize the operational cost of all the hired machines. In section 6, a numerical example is provided to illustrate the algorithm developed in this paper.Finally, we have given a conclusion at the end of this paper.

PRELIMINARIES

The aim of this section is to present some definitions based on fuzzy set and the various arithmetic operations used on it which are useful in further considerations.

Definition 2.1.A fuzzy set $\tilde{A}$defined on the set of real numbers R is said to be a fuzzy number if its membership function$\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics:

(i) $\tilde{A}$is convex, that is $\tilde{A}(\alpha x_1 + (1 - \alpha)x_2) \geq \min\{\tilde{A}(x_1),\tilde{A}(x_2)\}$, for all$x_1$, $x_2 \in$R and $\alpha \in [0,1]$.

(ii)$\tilde{A}$is normal i.e. there exists an $x \in$R such that $\tilde{A}$ (x) = 1.

(iii)$\tilde{A}$is piecewise continuous.

***Definition*2.2**.A fuzzy number$\tilde{A} = (a, b, c)$ on set of real numbersis said to be a triangular fuzzy number if its membership function$\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to closed interval [0,1].
2. $\mu_{\tilde{A}}(x) = 0$ for every x ε (-∞,a]
3. $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous on [a,b]

4. $\mu_{\tilde{A}}(x) = 1$ for every $x = b$
5. $\mu_{\tilde{A}}(x)$ is strictly decreasing and continuous on [b,c]
6. $\mu_{\tilde{A}}(x) = 0$ for every $x\ \varepsilon\ (c, \infty]$

2.3. *Fuzzy arithmetic operations*

Let $T_{F1} = (\alpha_1,\beta_1,\gamma_1)$ and $T_{F2} = (\alpha_2,\beta_2,\gamma_2)$ be two triangular fuzzy numbers. Then the arithmetic operations on these fuzzy numbers can be defined as follows:

1. Addition: $T_{F1}+T_{F2} = (\alpha_1+\alpha_2, \beta_1+\beta_2, \gamma_1+\gamma_2)$
2. Subtraction: $T_{F1}-T_{F2} = (\alpha_1-\alpha_2, \beta_1-\beta_2, \gamma_1-\gamma_2)$ if the following condition is satisfied

   $DP(T_{F1}) \geq DP(T_{F2})$, where $DP(T_{F1}) = (\gamma_1-\alpha_1)/2$ and $DP(T_{F2}) = (\gamma_2-\alpha_2)/2$. Here DP denotes difference point of a TFN.

   Otherwise, $T_{F1}-T_{F2} = (\alpha_1-\gamma_2, \beta_1-\beta_2, \gamma_1-\alpha_2)$
3. Equality : $T_{F1}=T_{F2}$ if $\alpha_1=\alpha_2, \beta_1=\beta_2, \gamma_1=\gamma_2$

   Multiplication: Suppose $A = (a_1, b_1, c_1)$ be any triangular fuzzy number and $B = (a_2, b_2, c_2)$ be non-negative triangular fuzzy number, then we define:

$$A \times B = \begin{cases} (a_1a_2, b_1b_2, c_1c_2), & a_1 > 0; \\ (a_1c_2, b_1b_2, c_1c_2), & a_1 < 0, c_1 \geq 0; \\ (a_1c_2, b_1b_2, c_1a_2), & c_1 < 0 \end{cases}$$

4. $\max[(T_{F1}), (T_{F2})] = T_{F1}$ if $\alpha_1>\alpha_2$; $\beta_1>\beta_2$; $\gamma_1>\gamma_2$

MODEL DESCRIPTION

This section provides some notations, assumptions and mathematical formulation of the proposed model.

3.1. *Model notations*

Before presenting the model, it is necessary to introduce the various notations used in our model. All are defined below:

Notations	**Description**
S_σ	Sequence obtained by applying branch and bound technique.

X	Machine index, $X = (A, B, C)$.
f_i^X	Fuzzy Processing time of i^{th} job on machine X.
h_i^X	AHR of fuzzy Processing time of i^{th} job on machine X.
J_r	Any partial schedule of $'r'$ jobs.
J'_r	Any job from the set of jobs other than J_r.
$L_b[J_{r,c}]$	Lower bound on rental cost for schedule J_r
$R_C[X]$	Rental Cost on machine X.

3.2. *Assumptions*

1. There are no unplanned windows of non-availability for e.g. breakdowns, unplanned maintenance, raw material shortage etc.
2. Splitting of job or job cancellation is not allowed.
3. Pre-emption is not allowed i.e. once a job is loaded on a machine, it cannot be removed until it is complete.

3.3. *Mathematical model formulation*

Consider a three-stage flow shop scheduling problem with n jobs. Let f_i^X be the processing time of i^{th} job (i=1,2,3….n) on machine X (X = A,B,C) described by triangular fuzzy number. The mathematical model of the problem in matrix form can be stated as:

Table 1: Jobs with uncertain processing time

Jobs I	Machine A	Machine B	Machine C
	f_i^A	f_i^B	f_i^C
1	$(\alpha_{11}, \beta_{11}, \gamma_{11})$	$(\alpha_{12}, \beta_{12}, \gamma_{12})$	$(\alpha_{13}, \beta_{13}, \gamma_{13})$
2	$(\alpha_{21}, \beta_{21}, \gamma_{21})$	$(\alpha_{22}, \beta_{22}, \gamma_{22})$	$(\alpha_{23}, \beta_{23}, \gamma_{23})$
3	$(\alpha_{31}, \beta_{31}, \gamma_{31})$	$(\alpha_{32}, \beta_{32}, \gamma_{32})$	$(\alpha_{33}, \beta_{32}, \gamma_{33})$
.	.	.	.
N	$(\alpha_{n1}, \beta_{n1}, \gamma_{n1})$	$(\alpha_{n2}, \beta_{n2}, \gamma_{n2})$	$(\alpha_{n3}, \beta_{n3}, \gamma_{n3})$

Our objective is to acquire an optimal/near optimal schedule of jobs which minimize the total operational cost of the machines using branch and bound technique.

OPERATIONAL POLICY

The operational policy is adopted when the machines are taken on rent or hired and the cost is associated with the operational time of the machines. According to this policy, the machines will be start its operation as and when they will be required and are stopped as and when they are no longer required .i.e. the first machine will start operating in the starting of the job processing, second machine will start its operation at time when first job is be completed on first machine and is in ready mode for processing on second machine.

BRANCH AND BOUND ALGORITHM

Step1: Evaluate <AHR> of the given fuzzy processing time for all the jobs using Yager's formula.

Step2: Calculate the processing time for the equivalent job δ on the given three machines for a job block (ω, τ) in which ω is given priority over job τ is given as follows:

	Machine A	Machine B	Machine C
	A_δ	B_δ	C_δ
Job δ	$A_\omega + A_\tau - \min(A_\tau, B_\omega)$	$Avg.(B_{\delta'}, B_{\delta''})$ where $B_{\delta'} = B_\omega + B_\tau - \min(A_\tau, B_\omega)$ $B_{\delta''} = B_\omega + B_\tau - \min(B_\tau, C_\omega)$	$C_\omega + C_\tau - \min(B_\tau, C_\omega)$

Step3: Reduce the given problem into new problem by replacing 'ω, τ' jobs by job block δ with processing times A_δ, B_δ & C_δ on machines A,B & C respectively as defined in step 2.

Step4: Evaluate the values l and L for all the Schedules that begin with schedule $J_r = J_1$ by the formula defined as below:

(a) Define and Calculate 'l' given by the following formula:

$$l = \max \left\{ \begin{array}{c} l_1 = t(J_r, 1) + \sum_{i \varepsilon J'_r} h_{i1} + \min_{i \varepsilon J'_r} h_{i2}, \\ l_2 = t(J_r, 2) + \sum_{i \varepsilon J'_r} h_{i2} \\ \end{array} \right\} - t_{i,1}$$

(b) Define and Calculate 'L' given by the following formula:

$$L = \max\begin{Bmatrix} L_1 = t(J_r,1) + \sum_{i\varepsilon J_r'} h_{i1} + \min_{i\varepsilon J_r'}(h_{i2} + h_{i3}), \\ L_2 = t(J_r,2) + \sum_{i\varepsilon J_r'} h_{i2} + \min_{i\varepsilon J_r'}(h_{i3}), \\ L_3 = t(J_r,3) + \sum_{i\varepsilon J_r'} (h_{i3}) \end{Bmatrix} - t_{i,2}$$

Where $t(J_r,1)$, $t(J_{r,}2)$, $t(J_r,3)$ be the times at which machine X complete processing on the last of the 'r' jobs in the sequence.

Step4: Obtain $L_b[J_{r,c}]$ = lower bound on rental cost for schedule J_r of $n\times 3$ flow shop problem which is defined as follows:

$$L_b[J_{r,c}] = l \times R_c[2] + L \times R_c[3]$$

Step 5: Choose minimum lower bound $L_b[J_{r,c}]$ among all the unbranched nodes. Now explore $L_b[J_{r,c}]$ for the partial schedule $J_r = J_2$ of two jobs for the $(n-1)$ subclasses starting with above searched node and again focus on the minimum $L_b[J_{r,c}]$ node. Continuing in this way, till the complete sequence is obtained and not reach at the end of the tree.

NUMERICAL ILLUSTRATION

To explain the working of the proposed algorithm, the following numerical illustration is carried out. Consider 4 jobs, 3 machine flow shop problem with processing time described by triangular fuzzy numbers as given in Table3. The operational costs per unit time for machines B and C are Rs4 and Rs6 respectively and jobs $(2,4)$ are to be processed as an equivalent group job. Our goal is to acquire an optimal/near optimal schedule, which minimizes the total operational cost of machines under the predetermined operational policy**.**

Table 3: Data set for fuzzy processing time

Jobs (i)	**Machine A**	**Machine B**	**Machine C**
	f_i^A	f_i^B	f_i^C
1	(4,5,7)	(6,9,18)	(5,7,12)
2	(20,25,26)	(19,23,28)	(22,24,25)
3	(8,10,17)	(10,12,16)	(11,14,17)
4	(9,14,18)	(12,15,18)	(10,20,22)

Solution:

The solution of the problem is formulated as follows:

The AHR of the given processing time on machines A, B and Cas per step 1 are shown in Table 4:

Table 4: AHR of fuzzy processing time

Jobs (i)	Machine A	Machine B	Machine C
	h_i^A	h_i^B	h_i^C
1	6	13	9
2	27	26	25
3	13	14	16
4	17	17	24

The processing time for the equivalent job δ for the job block (2,4) on the given machines are described as below:

Table 5: Reduced problem

Job (i)	Machine A	Machine B	Machine C
1	6	13	9
δ	27	26	32
3	9	14	16

As per step 4, the calculated values of lower bounds for $J_r = J_1$ using branch and bound technique are given in Table 6:

Table 6: Evaluation of rental cost for $J_r = J_1$

J_1	l_1	l_2	L_1	L_2	L_3	l	L	$L_b[j_{r},_c]$
1	56	59	72	75	76	53	57	554
δ	55	80	64	89	110	53	57	554
3	55	62	64	71	80	53	57	554

The corresponding scheduling tree for first level of nodes is shown as below:

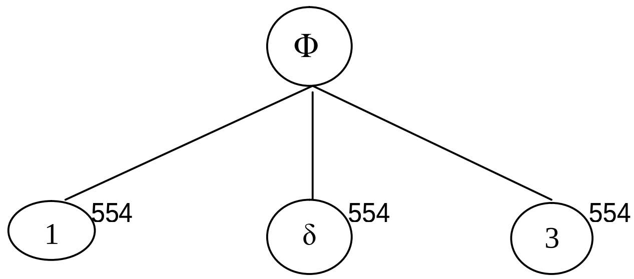

Fig.1: Scheduling tree for first level of nodes

Fig. 1 depict that all the lower bounds for first level of nodes produces the same operational cost i.e.$L_b[j_{r,c} = 1] = L_b[j_{r,c} = \delta] = L_b[j_{r,c} = 3] = 554$. Hence there is no node having the least lower bound on operational cost. That is the reason we have considered all the nodes of first level as the branching node in the scheduling tree for further proceeding.

Now take the partial schedule $j_r = j_2$ consisting of two jobs starting from the branching node.

The relevant operational costs estimation are given as in Table 7and graphically shown in Fig no.2

Table 7: Evaluations of operational cost for $J_r = J_2$

J_1	l_1	l_2	L_1	L_2	L_3	l	L	$L_b[J_{r,c}]$
1δ	56	73	72	89	107	40	48	448
13	68	59	100	91	51	53	67	614
δ1	56	80	72	96	110	47	44	452
δ3	55	80	64	89	110	44	43	434
31	68	62	100	94	106	53	70	632
3δ	55	75	64	84	103	39	41	402

The corresponding scheduling tree is depicted as below:

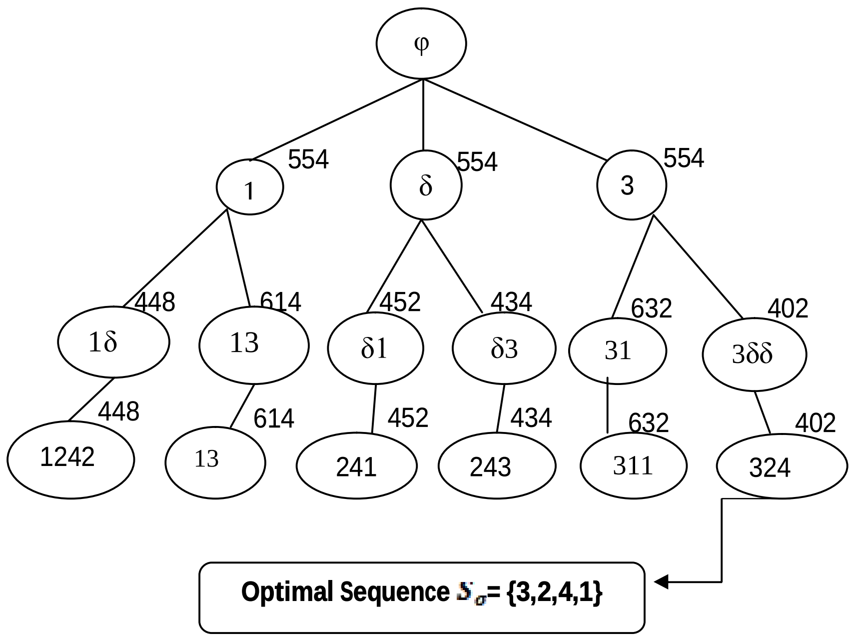

Fig.2: Scheduling Tree

Here, $\min\left[L_b\left(J_{r,c}\right) - (1\delta),(13),(\delta 1),(\delta 3),(31),(3\delta)\right] - 402$ which is correspond to $J_r = (3\delta)$. So the $J_r = (3\delta)$ is the next branching node but we have considered 'δ' as a equivalent job for a job block $(2,4)$. So by replacing the value of 'δ' by $(2,4)$ we obtained $J_r = (3\delta) = (324)$. Hence by the technique of branch and bound we have scheduled the three nodes in a scheduling sequence among the four nodes. In this way we acquired the complete sequence by placing the remaining node '1'in the last position that is (3241) with minimum operational Cost value402 Rs.

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