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EOQ Inventory Model for Time Varying Deteriorating Products with Quadratic time varying demand, Weibull Deterioration and Salvage Value

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ABSTRACT

In the present paper, an inventory model for deteriorating products is developed in which the demand rate is time varying and follows the quadratic pattern and deterioration having a two-parameter Weibull distribution deterioration rate. The salvage value is incorporated in the model and shortages are not allowed. The mathematical formulation of the model has been done to obtain the optimal solution of the problem. The result is demonstrated with the help of mathematical example and the sensitivity study is carried out with respect to the change in the major parameters.

KEYWORDS: Inventory, Deteriorating items,Quadratic demand,Weibull deterioration, Salvage value.

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INTRODUCTION

Controlling and maintaining inventory of deteriorating items has become a major challenge for decision makers in today's world as the deteriorating items are subject to an endless loss in their quantity or utility throughout their life time as a result of decay, damage, spoilage and many other different reasons. The literature on constantly decaying items has been reviewed by Raafat[1]. For the most part, when the things are kept in stock, they don't start disintegrating as soon as they are received, rather deterioration begins after some time. The two-parameter Weibull distribution deterioration may be applied for items that have a specific life time before they start to deteriorate. Therefore, the two-parameter Weibull distribution deterioration is reasonable for items with any underlying value of the rate of deterioration and for items, which start deteriorating only after a certain time frame. The study on Weibull deterioration with constant demand was begun by Covert and Philip[2]. Baten and Kamil[3]have continued with the analysis of inventory-producing systems with Weibull distributed deterioration. Researchers like Ghosh and Chaudhuri[4], Kalamet al.[5],Venkateswarlu and Mohan[6], Babu Krishna raj and Ramasamy[7] have discussed the Weibull deterioration in their article. Palani and Maragatham[8] have built a stock model for deteriorating items using the preservation technology by which the retailer can diminish the deterioration rate which may lead to decrease in the financial misfortunes, enhancement of client benefit and increase in business intensity.

Earlier, in the traditional EOQ model, the demand rate of an item was consistent. Many models on inventory were developed in the inventory literature considering the demand rate as constant. But in genuine circumstances demand rate of any product is dynamic in nature depending on the show of stock level, selling price and time. Silver and Meal[9]were the earliest researchers who modified the EOQ model with demand varying with time. Goyal and Gunasekaran[10] have studied the impact of different marketing policies such as the cost per unit item and the advertisement recurrence on the demand of a perishable item. Lately, few models have been developed with an exponential demand rate that varies with time. Demands for spare parts of new cars, advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete cars, computers etc. decrease very rapidly with time. Some researchers have suggested that this kind of rapid change in demand can be represented by an exponential function of time. Shukla et al.[11] have analyzed the exponential demand rate in their inventory model for deteriorating items permitting deficiencies in stock which are partially backlogged. Devyani and Gothi[12] have built up an EPQ model with exponentially varying demand and two-parameter Weibull disintegration rate.Chaitanya kumar and Deepa[13]also studied the exponential demand with two parameter Weibull deterioration and linear holding cost. The rapid change due to the exponential demand was not appropriate in the original market. Wu[14]has considered the ramp type demand rate along with Weibull deterioration and partial backlogging in his model. Bhunia and Maiti[15] have applied the linear pattern in demand with shortages, in the model for deteriorating items which are stocked in two-warehouses.Bose et.al.[16] ,Teng[17] and Mishra et. al.[18], havealso has studied the demand pattern having the linear trend. The merchandise that undergoes seasonal variation like clothes worn in summer and winter, crackers used during a few Hindu festivals, etc. can be well addressed by the quadratic demand .Quadratic demand is suitable for the seasonal fashion goods, cosmetics and high-tech products whose decay rate begins after some time when the items are stocked.Kumar et al.[19] have considered a stock model of two-stockroom with quadratically increasing demand and deterioration, fluctuating with time. Smrutirekhaet.al.[20]have analyzed the inventory model for decaying items with the demand rate being a piecewise quadratic function under constant deterioration rate. Bhandari and Sharma[21] have considered a solitary period inventory issue with quadratic demand distribution affected by the market policies. Babu Krishna raj and Nagarani[22] have contemplated an inventory model for two parameter Weibull distribution deterioration with quadratic demand rate, in which deficiencies are allowed and are partly backlogged. Sarkar et al.[23] have dealt with an ideal inventory replenishment strategy for a deteriorating item considering that the quadratic demand with partial backlogging varies with time.

Sarkar[24] has dealt with the model in which the retailers are permitted an exchange credit offer by the suppliers to purchase more items with various discount rates on the purchasing costs. During the period of credit, the retailers can procure more by selling their products. The interest on purchasing cost is charged for the deferral of payments by the retailers. This can increase the net income from the merchandise. The vast majority of the articles addressed, tended to expect that the deterioration of a unit is a total misfortune to the inventory framework and that these deteriorated units have no sale value. In order to sell the deteriorated stock, the cost of the stock is reduced to motivate the clients to buy the deteriorated items. The reduced cost is called the salvage value. Researchers like Jaggi and Aggarwal[25], Mishra and Shah[26] andAnnadurai[27] have considered the salvage value in their article. Venkateswarlu and Mohan[28] have considered the salvage value in their model along with time variant deterioration and cost dependent quadratic demand. Pradhan and Tripathy[29] have presented the model in which the demand rises in the beginning and becomes steady after some time called ramp type demand with salvage value. This salvage value is required for the calculation of optimal total cost. So in the present article we have inserted the salvage value in the model along with quadratic demand and Weibull deterioration without permitting shortages.

II ASSUMPTIONS

The following assumptions are used for the development of the model:

- A single item is considered over the prescribed period of time.
- Replenishment rate is infinite and lead time is negligible.
- The demand $D(t)$ at time t follows quadratic pattern and is assumed to be $D(t) = a + bt + ct^2;\ a \geq 0, b \neq 0, c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases, c is the rate which the change in the demand rate increases
- No repair or replenishment of the deteriorated items takes place during a given cycle.
- Deterioration rate $\theta(t) = \alpha\beta t^{\beta-1}$ follows a two parameter Weibull distribution, where $0 \leq \alpha < 1$ is the scale parameter, $\beta \geq 1$ is the shape parameter.
- The salvage value $\gamma C, 0 \leq \gamma < 1$ is associated with deteriorated units during a cycle time.
- Once a unit of the product is produced, it is available to meet the demand.
- Shortages are not permitted.

III NOTATIONS

- $I(t)$:The inventory level at time t.
- A : The fixed ordering cost per order during the cycle period.
- Q : The order quantity for each ordering cycle.
- C : The cost per unit.
- $C * i$: The holding cost per unit.
- γ : Salvage value
- T : Duration of a cycle.
- α : Scale parameter.
- β :Shape parameter
- TC : Total cost per unit time.

IV MATHEMATICAL FORMULATION

In this model quadratic demand is considered with Weibull distributed deterioration, depletion of inventory occurs due to deterioration and demand in each cycle. The instantaneous states of $I(t)$during the interval $[0,T]$ is given by the governing differential equation.

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}I(t)=-(a+bt+ct^2),\ 0\le t\le T. \tag{1}$$

Solving equation (1) with boundary condition $I(t)=0$, we get

$$I(t)e^{\int \alpha\beta t^{\beta-1}}=\int -(a+bt+ct^2)e^{\int \alpha\beta t^{\beta-1}}\,dt$$

$$I(t)\ e^{\alpha t^\beta}=-\left[\int (a+bt+ct^2)\left(1+\frac{\alpha t^\beta}{1!}\right)dt\quad\right]$$

$$=-\int\left[a+bt+ct^2+a\alpha t^\beta+b\alpha t^{\beta+1}+c\alpha t^{\beta+2}\right]dt$$

$$=-\left[at+\frac{bt^2}{2}+\frac{ct^3}{3}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}+\frac{c\alpha t^{\beta+3}}{\beta+3}\right]$$

$$=-\left[\left(aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{a\alpha T^{\beta+1}}{\beta+1}+\frac{b\alpha T^{\beta+2}}{\beta+2}+\frac{c\alpha T^{\beta+3}}{\beta+3}\right)\right.$$
$$\left.\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}+\frac{c\alpha t^{\beta+3}}{\beta+3}\right)\right]$$

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$$=\left[\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}+\frac{c\alpha t^{\beta+3}}{\beta+3}\right)\right.$$
$$\left.-\left(aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{a\alpha T^{\beta+1}}{\beta+1}+\frac{b\alpha T^{\beta+2}}{\beta+2}+\frac{c\alpha T^{\beta+3}}{\beta+3}\right)\right]$$

$$=\left[a\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}\right)+b\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}\right)+c\left(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3}\right)\right]$$
$$-\left[a\left(T+\frac{\alpha T^{\beta+1}}{\beta+1}\right)+b\left(\frac{T^2}{2}+\frac{\alpha T^{\beta+2}}{\beta+2}\right)+c\left(\frac{T^3}{3}+\frac{\alpha T^{\beta+3}}{\beta+3}\right)\right]$$

$$I(t)=\left\{\left[a\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}\right)+b\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}\right)+c\left(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3}\right)\right]\right.$$
$$\left.-\left[a\left(T+\frac{\alpha T^{\beta+1}}{\beta+1}\right)+b\left(\frac{T^2}{2}+\frac{\alpha T^{\beta+2}}{\beta+2}\right)+c\left(\frac{T^3}{3}+\frac{\alpha T^{\beta+3}}{\beta+3}\right)\right]\right\}\left(1-\frac{\alpha t^\beta}{1!}\right)$$

$$= \left\{ \left[a\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}\right) + c\left(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3}\right) \right] - \left[a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) + c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) \right] \right\} - \left\{ \left[a\left(\alpha t^{\beta+1} + \frac{\alpha^2 t^{\beta+2}}{\beta+1}\right) + b\left(\frac{\alpha t^{\beta+2}}{2} + \frac{\alpha^2 t^{\beta+3}}{\beta+2}\right) + c\left(\frac{\alpha t^{\beta+3}}{3} + \frac{\alpha^2 t^{\beta+4}}{\beta+3}\right) \right] - \left[a\left(\alpha t^{\beta} T + \frac{\alpha^2 t^{\beta} T^{\beta+1}}{\beta+1}\right) + b\left(\frac{\alpha t^{\beta} T^2}{2} + \frac{\alpha^2 t^{\beta} T^{\beta+2}}{\beta+2}\right) + c\left(\frac{\alpha t^{\beta} T^3}{3} + \frac{\alpha^2 t^{\beta} T^{\beta+3}}{\beta+3}\right) \right] \right\}$$

$$= \left[a\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}\right) + c\left(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3}\right) - a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) - c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) - a\left(\alpha t^{\beta+1}\right) - b\left(\frac{\alpha t^{\beta+2}}{2}\right) - c\left(\frac{\alpha t^{\beta+3}}{3}\right) + a\left(\alpha t^{\beta} T\right) + b\left(\frac{\alpha t^{\beta} T^2}{2}\right) + c\left(\frac{\alpha t^{\beta} T^3}{3}\right) \right]$$

(by neglecting the higher powers of α as $0 < \alpha < 1$).

$$I(t) = \left[a\left(t + \frac{\alpha t^{\beta+1}}{\beta+1} - \alpha t^{\beta+1}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{\alpha t^{\beta+2}}{2}\right) + c\left(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3} - \frac{\alpha t^{\beta+3}}{3}\right) - a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1} - \alpha t^{\beta} T\right) - b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2} - \frac{\alpha t^{\beta} T^2}{2}\right) - c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3} - \frac{\alpha t^{\beta} T^3}{3}\right) \right]$$

$$I(t) = \left[a\left(t - \frac{\beta \alpha t^{\beta+1}}{\beta+1}\right) + b\left(\frac{t^2}{2} - \frac{\beta \alpha t^{\beta+2}}{2(\beta+2)}\right) + c\left(\frac{t^3}{3} + \frac{\beta \alpha t^{\beta+3}}{3(\beta+3)}\right) - a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) - c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) + a\alpha t^{\beta} T + \frac{b\alpha t^{\beta} T^2}{2} + \frac{c\alpha t^{\beta} T^3}{3} \right] \quad (2)$$

The optimum order quantity is given by putting $I(0) = Q$ in equation (2).Therefore

$$I(0) = Q = -\left[a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) + c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) \right] \quad (3)$$

The total cost per unit time consists of the following costs:

(i) The number of deteriorated units (NDU) during one cycle time is given by

$$NDU = Q - \int_0^T D(t)\,dt$$

$$= -\left[a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) + c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) \right] - \left[aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right]$$

$$NDU = -\left[2aT + bT^2 + \frac{2cT^3}{3} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3}\right] \quad (4)$$

(ii) Carrying cost or holding cost per cycle $= C * i \int_0^T I(t)\, dt$

$$= \frac{C*i}{T}\int_0^T \left[a\left(t - \frac{\beta\alpha t^{\beta+1}}{\beta+1}\right) + b\left(\frac{t^2}{2} - \frac{\beta\alpha t^{\beta+2}}{2(\beta+2)}\right) + c\left(\frac{t^3}{3} - \frac{\beta\alpha t^{\beta+3}}{3(\beta+3)}\right) - a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) - c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right) + a\alpha t^{\beta}T + \frac{b\alpha t^{\beta}T^2}{2} + \frac{c\alpha t^{\beta}T^3}{3}\right] dt$$

$$= \frac{C*i}{T}\left[a\left(\frac{T^2}{2} - \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(\frac{T^3}{6} - \frac{\beta\alpha T^{\beta+3}}{2(\beta+2)(\beta+3)}\right) + c\left(\frac{T^4}{12} - \frac{\beta\alpha T^{\beta+4}}{3(\beta+3)(\beta+4)}\right) - a\left(T^2 + \frac{\alpha T^{\beta+2}}{\beta+1}\right) - b\left(\frac{T^3}{2} + \frac{\alpha T^{\beta+3}}{\beta+2}\right) - c\left(\frac{T^4}{3} + \frac{\alpha T^{\beta+4}}{\beta+3}\right) + \frac{a\alpha T^{\beta+2}}{\beta+1} + \frac{b\alpha T^{\beta+3}}{2(\beta+1)} + \frac{c\alpha T^{\beta+4}}{3(\beta+1)}\right]$$

$$= \frac{C*i}{T}\left[a\left(-\frac{T^2}{2} - \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(-\frac{T^3}{3} - \frac{\beta\alpha T^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\beta\alpha T^{\beta+3}}{2(\beta+1)(\beta+2)}\right) + c\left(-\frac{T^4}{4} - \frac{\beta\alpha T^{\beta+4}}{3(\beta+3)(\beta+4)} - \frac{2\beta\alpha T^{\beta+4}}{3(\beta+1)(\beta+3)}\right)\right]$$

$$= \frac{C*i}{T}\left[a\left(-\frac{T^2}{2} - \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(-\frac{T^3}{3} + \frac{\beta\alpha T^{\beta+3}}{(\beta+1)(\beta+2)^2(\beta+3)}\right) + c\left(-\frac{T^4}{4} + \frac{(\beta^2+7\beta)\alpha T^{\beta+4}}{3(\beta+1)(\beta+3)^2(\beta+4)}\right)\right] \quad (5)$$

(iii) Cost due to deterioration $= \frac{C}{T}\left[Q - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right]$

$$= \frac{C}{T}\left[-\left[a\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right) + c\left(\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right)\right] - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right]$$

$$= -\frac{C}{T}\left[2aT + bT^2 + \frac{2cT^3}{3} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3}\right] \quad (6)$$

(iv) Ordering cost per cycle $= \frac{A}{T}$ (7)

(v) Salvage value per cycle $= \frac{\gamma C}{T}\left[Q - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right]$

$$= -\frac{\gamma C}{T}\left[2aT + bT^2 + \frac{2cT^3}{3} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3}\right] \quad (8)$$

Total cost (TC) = Ordering cost + Carrying cost + Cost due to deterioration – Salvage value

$$= \frac{A}{T} + C * i \int_0^T I(t)\, dt + \frac{C}{T}\left[Q - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right] - \frac{\gamma C}{T}\left[Q - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right]$$

$$= \frac{A}{T} + C * i \int_0^T I(t)\, dt + \frac{C}{T}(1-\gamma)\left[Q - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)\right]$$

$$= \frac{A}{T} + \frac{C*i}{T}\left[a\left(-\frac{T^2}{2} - \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(-\frac{T^3}{3} + \frac{\beta\alpha T^{\beta+3}}{(\beta+1)(\beta+2)^2(\beta+3)}\right) + c\left(-\frac{T^4}{4} + \frac{(\beta^2+7\beta)\alpha T^{\beta+4}}{3(\beta+1)(\beta+3)^2(\beta+4)}\right)\right] - \frac{C}{T}(1-\gamma)\left[2aT + bT^2 + \frac{2cT^3}{3} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3}\right] \quad (9)$$

The necessary condition for total cost to be minimum is $\frac{\partial(TC)}{\partial T} = 0$, i.e.,

$$\frac{\partial(TC)}{\partial T} = -\frac{A}{T^2} - \frac{C*i}{T^2}\left[a\left(-\frac{T^2}{2} - \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(-\frac{T^3}{3} + \frac{\beta\alpha T^{\beta+3}}{(\beta+1)(\beta+2)^2(\beta+3)}\right) + c\left(-\frac{T^4}{4} + \frac{(\beta^2+7\beta)\alpha T^{\beta+4}}{3(\beta+1)(\beta+3)^2(\beta+4)}\right)\right] + \frac{C*i}{T}\left[a\left(-T - \frac{\beta\alpha T^{\beta+1}}{(\beta+1)}\right) + b\left(-T^2 + \frac{\beta\alpha T^{\beta+2}}{(\beta+1)(\beta+2)^2}\right) + c\left(-T^3 + \frac{(\beta^2+7\beta)\alpha T^{\beta+3}}{3(\beta+1)(\beta+3)^2}\right)\right] + \frac{C}{T^2}(1-\gamma)\left[2aT + bT^2 + \frac{2cT^3}{3} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3}\right] - \frac{C}{T}(1-\gamma)\left[2a + 2bT + 2cT^2 + a\alpha T^{\beta} + b\alpha T^{\beta+1} + c\alpha T^{\beta+2}\right] = 0 \quad (10)$$

Solving the above equation we get the minimum total cost, providedthat the sufficient condition $\frac{\partial^2(TC)}{\partial T^2} > 0$ is satisfied.

V NUMERICAL EXAMPLE

Let $A = 150$, $a = 250$, $b = 20$, $c = 3$, $C = 3$, $i = 0.2$, $\alpha = 0.1$, $\beta = 0.01$, $\gamma = 0.1$, in appropriate units. Solving the equations we obtain the optimum values of T and TC as $T = 13.0326$ and $TC = 481.1604$.