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Difference Cordial Labeling in context of Vertex Switching and Ringsum of a Graphs

Parsania Nidhi M.[1*], Rokad A. H.[2]

[1]Department of Applied Science & Humanities,VVP Engineering College, Rajkot, Gujarat, INDIA
[2]Department of Mathematics, School of Engineering, R. K. University, Rajkot - 360005, Gujarat, INDIA *Email: nidhi781992@gmail.com

ABSTRACT:

Suppose G be a (m, n) graph. Suppose f be a map from h(G) to {1,2,...,m}. For each edge xy assign, the label |h(x) – h(y)|. h is difference cordial if f is 1-1 and $|e_h(0) - e_h(1)| \leq 1$, where $e_h(1)$ and $e_h(0)$ denote the number of edges with labeled 1 except labeled with 1 respectively. A graph which admit difference cordial labeling is called a difference cordial graph.

In this paper we prove the following results.

1. Vertex switching of cycle is difference cordial.
2. Vertex switching of cycle with one chord is difference cordial.
3. Vertex switching of cycle with twin chords is difference cordial.
4. Vertex switching of path graph is difference cordial.
5. Ring sum of star graph and cycle graph is difference cordial.
6. Ring sum of star graph and cycle with one chord is difference cordial.
7. Ring sum of star graph and gear graph is difference cordial.
8. Ring sum of star graph and path graph is difference cordial.

KEYWORD: Difference cordial labelling , Vertex switching and Ring sum.

MSC AMS Classification:05C78

***Corresponding author**

Nidhi M. Parsania

Department of Applied Science & Humanities,

VVP Engineering College, Rajkot, Gujarat, INDIA

nidhi781992@gmail.com

INTRODUCTION:

The graph consider in this paper are finite, undirected and simple graphs only. The percept of Difference cordial labeling was brought out by R. Kala, S. SathishNarayanan andR. Ponraj[4].Notation and definitions not described here are used in the sense of Gross and Yellen[3].Gallian[2] published and updateda dynamic survey of graph labeling every year.Rokad and Ghodasara [5] proved that vertex switching of petersen graph, switching of wheel graph, switching of flower graph, switching of gear graph and switching of shell graph are cordial. Rokad and Ghodasara [6] proved that Ring sum of star graph and gear graph, Ring sum of star graph and cycle graph,Ring sum of star graph and cycle with one chord are 3-equitable graphs.

Definition 1 The vertex switching Gv of a graph G is get by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which is not adjacent to v in G.

Definition 2 Ring sum $G_1 \oplus G_2$ of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \oplus G_2 = (V_1 \cup V_2, (E_1 \cup E_2) - (E_1 \cap E_2))$.

Theorem 1: Switching of vertex of C_n is difference cordial.

Proof: Suppose G = C_n and $z_1, z_2,...,z_n$ be successive vertices of C_n. Suppose $(C_n)_{z1}$represent the switched vertex of C_n with respect to z_1 of C_n. Consider z_1 as the switched vertex and we start labeling pattern from z_1. Then $|E(G)| = 2n - 5$ and $|V(G)| = n$.

We define the labeling function $h:V((C_n)z_1) \rightarrow \{1,2,...,n\}$, as follows

$h(z_i) = i, i \in [1, n]$.

Since $e_h(1) = n - 2$ and $e_h(0)=n - 3$.

Hence, Switching of vertex of cycle $C_n$is difference cordial.

Example 1: Switching of vertex of cycle C_7 admitting difference cordial labeling is shown in Figure 1.

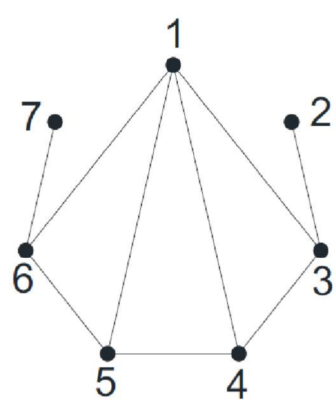


Figure 1

Theorem 2: Switching of vertex ofcycle C_n ($n \geq 4$, $n \in N$) having one chord admitsdivisor cordial labeling, where chord makea triangle with two edges of cycle C_n.

Proof: Suppose G be the cycle having one chord. Suppose$z_1, z_2,...,z_n$ be the successive vertices of cycle C_n and $e = z_nz_2$be the chord of cycle C_n. The edges $e = z_nz_2$, $e_1 = z_2z_1$, $e_2 = z_nz_1$ form a triangle.

Now the graph get by switching of vertices z_i and z_j of degree 2 areisomorphic to each other for all i and j. Similarly the graph get by switching of vertices z_i and z_j of degree 3 are isomorphic to each other for all i and j.Hence we need to talk about two cases: (i) switching of an arbitrary vertex ofG of degree 3, (ii) switching of an arbitrary vertex of G of degree 2. Without detriment of generality suppose the switched vertex be z_1 (of either degree 3 or degree 2) and suppose G_{z1} denote the switching of vertex of G with respect to vertex z_1.

To define labeling function h: V $(G_{z1}) \rightarrow \{1,2,...,n\}$ we consider the following cases.

Case I: Degree of z_1 is 2.

(Here the number of vertices is n and number of edges is $2n – 4$.)

$h(z_i) = i$, $i \in [1, n]$.

Since $e_h(1) = e_h(0) = n – 2$.

Case II: Degree of x_1 is3.

(Here the number of vertices is n and number of edges is $2n – 6$.)

$h(z_n) = n-1$,

$h(z_{n-1}) = n$

$h(z_i) = i$, $i \in [1, n-2]$.

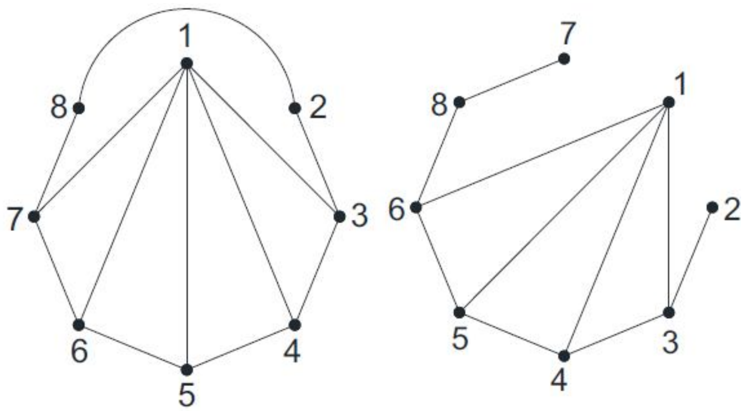
Since $e_h(1) = e_h(0) = n-3$.

Hence, Switching of vertex of cycle $C_n$having one chord admitsdivisor cordial labeling.

Example 2:

(a) Figure 2(a) shows switching of vertex of degree 2 of cycle C_8having one chord admitting difference cordial labeling.

(b) Figure 2(b) shows switching of vertex of degree 3 of cycle C_8having one chord admittingdifference cordial labeling.



(a) **(b)**

Theorem 3: Switching of vertex of cyclehaving twin chords $C_{n,3}$ admits difference cordial labeling.

Proof: Suppose G be the cycle having twin chords $C_{n,3}$. Supposex_1, x_2,...,x_n be the successive

vertices of G. Suppose $e_1 = x_n x_2$ and $e_2 = x_n x_3$ be the chords of cycle C_n which form two triangles and one cycle Cn-2.

Now the graph get by switching of vertices x_i and x_j of degree 2 areisomorphic to each other for all i and j. Similarly the graph get by switching of vertices x_i and x_j of degree 3 are isomorphic to each other and the graphget by switching of vertices x_i and x_j of degree 4 are isomorphic to eachother for all i and j. Hence we need to talk about three cases: (i) switching ofan arbitrary vertex of G of degree 2, (ii) switching of an arbitrary vertex of Gof degree 3, (iii) switching of an arbitrary vertex of G of degree 4. With out detriment of generality suppose the switched vertex be x_1 and suppose G_{x1} denote the switching of vertex of G with respect to vertex x_1.

To define labeling function h: $V(G_{x1})$ {1,2,...,n} we consider the following cases.

Case 1: Degree of x_1 is2

(Here the number of vertices is n and number of edges is 2n – 3.)

$h(x_i) = i$, $i \in [1, n]$.

Since $e_h(1) = n - 2$ and $e_h(0) = n - 1$.

Case 2: Degree of x_1 is 3

(Here the number of vertices is n and number of edges is 2n – 5.)

$h(x_n) = n-1$,

$h(x_{n-1}) = n$

$h(x_i) = i$, $i \in [1, n - 2]$.

Since $e_h(1) = n - 3$ and $e_h(0) = n - 2$.

Case 3: Degree of x_1 is 4

(Here the number of vertices is n and number of edges is 2n – 7.)

The labeling pattern is same as case - 2.

Since $e_h(1) = n – 3$ and $e_h(0) = n – 4$.

Thus, $|e_h (1) - e_h (0)| \leq 1$.

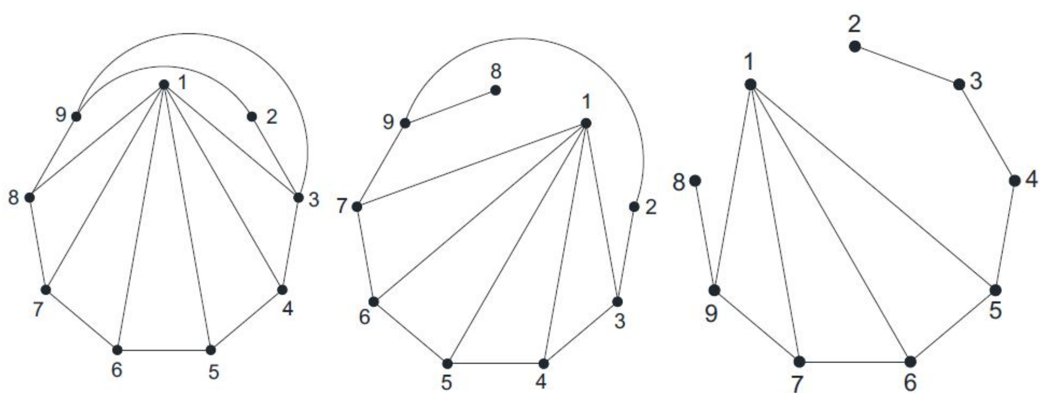
Hence Switching of vertex of cyclehaving twin chords $C_{n,3}$ admits difference cordial labeling.

Example3:

(a) Figure 3(a) shows switching of vertex of degree 2 of cycle C_9having twin chords admitting difference cordial labeling.

(b)Figure 3(b) shows switching of vertex of degree 3 of cycle C_9having twin chords admitting difference cordial labeling.

(c) Figure 3(c) shows switching of vertex of degree 4 of cycle C_9having twin chords admitting difference cordial labeling.



(a) **(b)** **(c)**

Theorem 4: Vertex switching of a pendant vertex of pathP_n is difference cordial labeling.

Proof: Suppose $z_1, z_2, ..., z_n$ be the vertices of path P_n. The graph G get by Switching of a pendant vertexx_1 in the path P_n. Then $|E(G)| = 2n – 4$& $|V(G)| = n$.

We define labeling function h: V(G) {1,2,...,n} as follows.

$h(z_i) = i$, $i\in[1, n]$.

Then we have $e_h(1) = n – 2$ and $e_h(0) = n – 2$.

Therefore $|e_h (1) - e_h (0)| \leq 1$.

Hence switching of a pendant vertex in path P_n isdifference cordial labeling.

Example 4: Switching of pendant vertex of pathP_7admitting difference cordial labeling is shown in Figure 4.

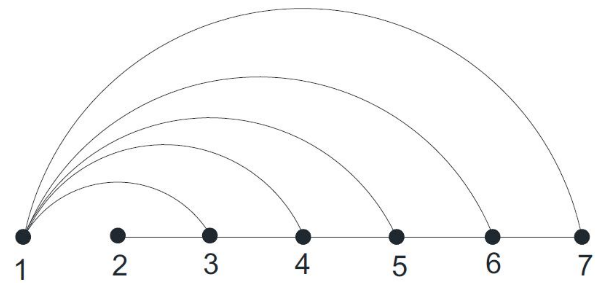


Figure 4

Theorem 5: Ring sum of star graph and cycle graph is difference cordial for all n.

Proof: Suppose V (G) = X ∪Y, where X = $\{z_1, z_2, . . . ,z_n\}$ be the vertex set of C_n and

Y = $\{y = z_1, y_1, y_2, . . . ,y_n\}$ be the vertex set of $K_{1,n}$. Here $y_1, y_2, . . .,y_n$ are pendent vertices.

Also|E (G)| = |V (G)| = 2n.

We define labeling function h: V(G) {1,2,...,2n} as follows.

$h(z_1) = 2$,

$h(z_2) = 4$,

$h(z_i) = i +2, i \in [3, n]$.

$h(y_1) = 1$,

$h(y_2) = 3$,

$h(y_i) = n+i, i \in [3, n]$.

Then in each case we have $e_h (1) = e_h (0) = n$.

Therefore $|e_h (1) - e_h (0)| \leq 1$.

Hence Ring sum of star graph and cycle graph is difference cordial.

Example 5: Ring sum of star graph$K_{1,7}$and cycleC_7admitting difference cordial labeling is shown in Figure 5.

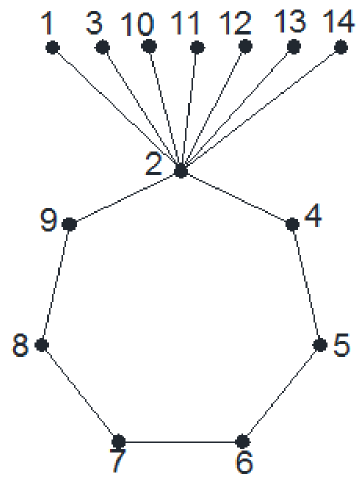


Figure 5

Theorem6. Ring sum of star graph and cycle with one chord is difference cordial, where chord forms a triangle with two edges of thecycle.

Proof. Suppose G be the cycle having one chord and suppose e $=z_2z_n$ be the chord in G.Suppose V = X∪Y, where X = $\{z_1, z_2, . . . ,z_n\}$ be the vertexset of G andY = $\{y = z_1, y_1, y_2, . . . ,y_n\}$ be the vertex set of star graph $K_{1,n}$. Here$y_1, y_2, . . . ,y_n$ are pendent vertices.Also $|E (G)| = 2n + 1$ &

$|V (G)| = 2n$.

We define labeling function h: V(G) {1,2,...,2n} as follows.

The labeling pattern is same as **Theorem-5.**

Then we have $e_h (1) = n$ and$e_h (0) = n + 1$.

Therefore $|e_h (1) - e_h (0)| \leq 1$.

Hence, Ring sum of star graph and cycle with one chord is difference cordial.

Example 6: Ring sum of $K_{1,8}$ and cycle C_8 with one chord having difference cordial labeling is shown in Figure 6.

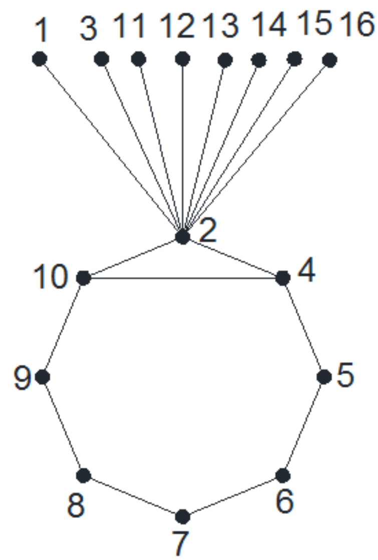


Figure 6

Theorem 7: Ring sum of star graph and gear graph is difference cordialfor all n.

Proof: SupposeV(G) = X∪Y, where X = {x_0, x_1, x_2, ..., x_{2n}}with apex x_0 and x_1, x_2,….., x_{2n} are other vertices of G_n,where deg(x_i) = 2 when i is even and deg(x_i) = 3 when i is odd andY = {x_1= y,y_1, y_2, ... , y_n} be the vertex set of $K_{1, n}$. Here y_1, y_2,... ,y_n are pendent vertices and y is the apex vertex of $K_{1,n}$. Also|E(G)| = 4n & |V (G)| = 3n + 1.

We define labeling function h: V(G) {1,2,..., 1 + 3n} as follows.

$h(u_0) = 2n + 1$,

$h(u_{2n}) = 1$,

$h(u_1) = 2$,

$h(u_i) = i+1, i \in [2, n]$.

$h(v_i) = 2n + i + 1, i \in [1, n]$.

Then we have $e_h(1) = e_h(0) = 2n$.

Therefore $|e_h(1) - e_h(0)| \leq 1$.

Hence,Ring sum of star graph and gear graph is difference cordial.

Example 7: Ring sum of $K_{1,7}$ and G_7 is difference cordial is shown in Figure 7.

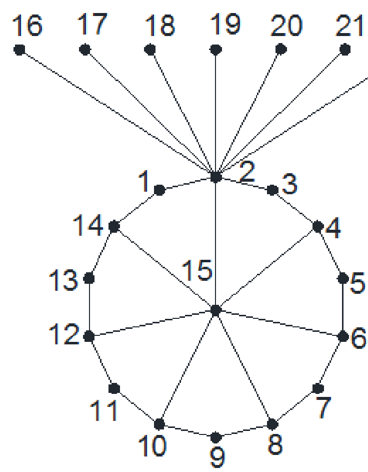


Figure 7

Theorem 8: Ring sum of star graph and path graph is difference cordial for all n.

Proof: Suppose $V(P_n \oplus K_{1,n}) = X \cup Y$, where $X = \{x_1, x_2, \ldots, x_n\}$ is the vertex set of P_n and $Y = \{y = x_1, y_1, y_2, \ldots, y_n\}$ is the vertex set of $K_{1,n}$. Here $y_1, y_2, \ldots, y_n$ are the pendant vertices and y is the apex vertex. Also $|E(P_n \oplus K_{1,n})| = 2n - 1$ and $|V(P_n \oplus K_{1,n})| = 2n$.

We define labeling function h: V(G) {1,2,...,2n} as follows.

$h(x_i) = i, i \in [1, n]$.

$h(y_i) = n + i, i \in [1, n]$.

Then we have $e_h(1) = n - 1$ and $e_h(0) = n$.

Therefore $|e_h(1) - e_h(0)| \leq 1$.

Hence, Ring sum of star graph and path graph is difference cordial.

Example 8: Ring sum of $K_{1,7}$ and P_7 is difference cordial is shown in Figure 8.

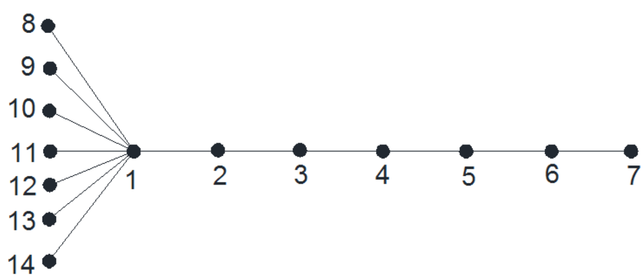


Figure 8

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