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A Study of Fuzzy Topological Spaces

Kaur Shivdeep

Dept. of Mathematics, Mata Gujri College, Fatehgarh Sahib-*140406*, Punjab, India.

ABSTRACT

Dubiety and vagueness are the major problems in the field of Engineering and Technology.Solutions to such problems can be obtained using fuzzy topology. The researchers look into the fuzzy topological ideas using the adequate complement functions. The fuzzy topological concepts can be theorized using the other complement functions which are in the fuzzy literature.The purpose of this paper is to study the basic concepts of fuzzy topological spaces.

KEYWORDS: Fuzzy topological space (FTS), fuzzy continuous map, fuzzy semi-compact spaces, fuzzy semi-connectedness, fuzzy weakly-compact spaces.

*Corresponding Author

Shivdeep Kaur

Department of Mathematics

Mata Gujri College,

Fatehgarh Sahib-*140406*, Punjab, India.

Email: sadsrdmath@gamil.com, Mobile No-8437747771

1. INTRODUCTION

Topology has its roots in geometry and analysis. From a geometric point of view, topology was the study of properties preserved by a certain group of transformations, namely the homeomorphisms. Certain notions of topology are also abstractions of classical concepts in the study of real or complex functions. These concepts include open sets, continuity, connectedness, compactness, and metric spaces. They were a basic part of analysis before being generalized in topology[7].

2. FUZZY TOPOLOGICAL SPACES

A topological space is an ordered pair (X, τ), where X is a set and τ is a collection of subsets of X, satisfying the axioms:- (i)The empty set and X itself belongs to τ(ii) Any(finite or infinite) union of members of τ still belongs to τ(iii) The intersection of any finite number of members of τ still belongs to τ. A fuzzy topology on a set X is a collection δ of fuzzy sets in X such that: (i) $0,1 \in \delta$ (ii) $\mu, v \in \delta \Rightarrow \mu \wedge v \in \delta$ (iii) $\forall (\mu_i)_{i \in I} \in \delta \Rightarrow v_{i \in I}(\mu_i) \in \delta$; δ is called as fuzzy topology for X, and the pair (X, δ) is a fuzzy topological space, or FTS in short. Every member of δ is called a T-open fuzzy set. Fuzzy sets of the form $1 - \mu$, where μ is an open fuzzy set, are called closed fuzzy sets. Few examples of fuzzy topologies are :- Any topology on a set X (subsets are identified with their characteristic functions), The indiscrete fuzzy topology {0, 1} on a set X (= indiscrete topology on X),The discrete fuzzy topology on X containing all fuzzy sets in X ,The collection of all crisp fuzzy sets in X (= discrete topology on X),The collection of all constant fuzzy sets in X ,The intersections of any family of fuzzy topologies on a set X [12].

3. BASE AND SUBBASE FOR FTS

A base for a fuzzy topological space (X , τ) is a sub collection β of τ such that each member A of τ can be written as $V_{j \in A} A_j$, where each $A_j \in \beta$. A subbase for a fuzzy topological space (X , τ) is a sub collection S of τ such that the collection of infimum of finite subfamilies of S forms a base for (X , τ) . Let (X , τ) be an FTS. Suppose A is any subset of X. Then (A, τ_A) is called a fuzzy subspace of(X , τ) ,Where, $\tau_A = \{B_A : B \in \tau\}$, $B = \{(x, \mu_B(x)) : x \in X, B_A = \{(x, \mu_{B/A}(x)) : x \in A\}$.

4. FUZZY POINT

A fuzzy point L in X is a special fuzzy set with membership function defined by L(x) = { $\forall x = y$; 0 $\forall x \neq y$}; where 0 <$\mu \leq 1$.L is said to have support y, value μ and is denoted by P_y^{μ} or $P(y, \mu)$.Let A be a fuzzy set in X, then $P_y^{\alpha} \subset$ A $\Leftrightarrow \alpha \leq$ A(y).In particular $P_y^{\alpha} \subset P_z^{\beta} \Leftrightarrow y = z, \alpha \leq \beta$. A fuzzy point P_y^{α} is said to be in A, denoted by $P_y^{\alpha} \in$ A $\Leftrightarrow$nα ≤ A(y). The complement of the fuzzy point P_x^{α} is denoted either by $P_x^{\lambda-1}$ or by$(P_x^{\lambda})^c$.The fuzzy pointP_x^{λ} is said to be contained in a fuzzy set A, or to belong to A,denoted by $P_x^{\alpha} \in$ A if and only if λ < A(x). Every fuzzy set A can be expressed as the union of all the fuzzy points which belong to A. That is, if A(x) is not zero for x $\in X$, then A(x) = sup {λ: P_x^{λ}, 0 < λ ≤A(x)}.Two fuzzy sets A, B in X are said to be intersecting if and only if there exists a point x $\in X$ such that (A ∧ B)(x) ≠0. For such a case, we say that A and B intersect at x. Let A, B ∈ I X. Then A = B if and only if P ∈ A ⇔ P ∈ B for every fuzzy point P in X.

5. CLOSURE AND INTERIOR OF FUZZY SETS

The closure A' and the interior A^0 of a fuzzy set A of X are defined as

$A' = \inf\{K : A \leq K,\ K^c \in \tau\}$

$A^0 = \sup\{O : O \leq A,\ O \in \tau\}$

6. NEIGHBORHOOD

A fuzzy point P_x^{α} is said to be quasi-coincident with A, denoted by $P_x^{\alpha}\, qA$ if and only if $\lambda > A^c(x)$, or λ + A(x) > 1.

7. FUZZY CONTINUOUS MAP

Given fuzzy topological space and , a function is fuzzy a function f : X→Y is fuzzy continuous if the inverse image under f of any open fuzzy set in Y is an open fuzzy set in X ; that is if $f^{-1}(\nu) \in \tau$ whenever $\nu \in \gamma$.

8. GENERALIZED LOCALLY CLOSED SETS AND GLC-CONTINUOUS FUNCTION

Fuzzy G-Closed sets : $S \in (X, \tau)$ is Fuzzy G-closed, $\Leftrightarrow cl(S) \subset G, S \subset G, G$ is open in (X , τ). Fuzzy G-open Sets : $S \in$ (X , τ) is fuzzy G-open,$\Leftrightarrow$ $(X - S)$ is fuzzy g-closed. Fuzzy Locally Closed sets :∈ is fuzzy locally closed $\Leftrightarrow S = G \cap F$,Where, $G \in \tau$ and F is closed in (X, τ). Fuzzy G-Locally closed sets:∈ (X, τ) is fuzzy G-locally closed $\Leftrightarrow S = G \cap F$, Where, G is fuzzy g-open in (X, τ).Fuzzy Generalized Locally Closed Functions: Fuzzy GLC-irresolute: f: (X, τ) $\rightarrow (Y, \sigma) \Leftrightarrow f^{-1}(V) \in GLC$ (X, τ) $\forall V \in GLC$ (Y, σ). Fuzzy GLC-continuous: f: (X, τ) $\rightarrow (Y, \sigma) \Leftrightarrow f^{-1}(V) \in GLC$ (X, τ) $\forall V \in \sigma$ [1].

9. FUZZY SEMI-COMPACT SPACES

A FTS X is said to be a fuzzy semi-compact space if every fuzzy cover of X by fuzzy semi-open sets (such a cover will be called a fuzzy semi-open cover of X) has a finite sub- cover.A direct consequence of the above definition yields the following alternative formulation of a fuzzy semi-compact space.

10.FUZZY SEMI-COMPACT SETS

A fuzzy set A in a FTS X is said to be a: (i) fuzzy compact set, if every fuzzy open cover of A has a finite sub-cover for A .(ii) fuzzy nearly compact set, if every fuzzy regular open cover of A has a finite sub-cover forA .(iii)fuzzy s-closed set, if every fuzzy semi-open cover of A has a semi-proximate sub-cover forA .(iv)fuzzy almost compact set, if every fuzzy open cover of A has a finite proximate sub-cover for A. (v)fuzzyθ-rigid set, if for every fuzzy open cover U of A , there exists a finite subfamily U_0 of U such that $A \leq \text{int } cl(\cup U_0)$.(vi) fuzzy θ^* -rigid, if for every semi-open cover U of A , there exists a finite subfamily U_0 of U such that $A \leq scl(\cup\{sclU : U \in U_0\})$ [5].

11.SEMI*-CONNECTEDNESS IN FUZZY TOPOLOGICAL SPACES

Let A bea subset of a fuzzy topologicalspace X . The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by Cl *(A) . A subset B of a fuzzy topological space X is called g-closed, if $Cl(B) \subseteq U$whenever$B \subseteq U$and U is open in X .A subset A of a fuzzy topological space X is called semi*-open if $A \subseteq Cl^{*}(Int(A))$.A subset A of a fuzzy topological space X is called semi*-regular ifnit is both semi*-open and semi*closed. Let A be a subset of X . Then the semi*-closure of A is defined as the intersection of all semi*-closed sets containing A and is denoted by s * Cl(A) .A subset A of a fuzzy topological spaces X , the semi*-frontier of A is defined by $s * Fr(A) = s * Cl(A) \setminus s * Int(A)$.A function f : X→Y is said to be (i) semi*-continuous if $f^{-1}(V)$is semi*-open in X for every open set V in Y .(ii) semi*-irresolute if $f^{-1}(V)$is semi*-open in X for every semi*-open set V in Y. A fuzzy topological space X is said to be semi*-connected if X cannot be expressed as the union ofntwo disjoint non-empty semi*-open sets in X [9].

12.FUZZY WEAKLY-COMPACT SPACES

A fuzzy subset S is said to be fuzzy regular open (resp. fuzzy regular closed) ifnint(cl(S))= S (resp. cl(int(S))=S). A fuzzy open cover $\{V_\alpha : \alpha \in L\}$ of an FTS is said to be fuzzy regular if for each $\alpha \in L$ there exists a nonempty fuzzy regular closed set F_α in Xsuch that $F_\alpha \subset V\alpha$and $X = \cup\{\text{int}(F_\alpha) : \alpha \in L\}$. An FTS X is said to be fuzzy weakly-compact (resp. fuzzy alomost- compact) if every fuzzy regular (resp. fuzzy open) cover of X has a finite subfamily whose fuzzy closures cover X. It is clear that every fuzzy almost-compat space is fuzzy weakly-compact. A fuzzy subset S of the fts X is said to be fuzzy weakly-compact if S is fuzzy weakly- compact as a fuzzy subspace of X. A fuzzy subset S of an fts X is said to be fuzzy weakly-compact relative to X if for each cover $\{V_\alpha : \alpha \in L\}$of S by fuzzy open sets of X satisfying the condition:-For each $\alpha \in L$, there exists a nonempty fuzzy regular closed set F_α of X such that $F_\alpha \subset V_\alpha$ and $S \subset \cup\{\text{int}(F_\alpha) : \alpha \in L\}$, there exists a finite subset Lo *of* L such that $S \subset \{\text{cl}(V_\alpha) : \alpha \in Lo\}$. An FTS X is said to be fuzzy nearly compact if every regular fuzzy open cover of X has a finite fuzzy subcover [8].

13.CONCLUSION

This paper presented the study of fuzzy topological spaces. After acquiring the basic knowledge of fuzzy topology one can go for its detailed study i.e theorems and their proofs and finally concepts of fuzzy topological spaces can be used to solve real life engineering problems.

REFERENCES

1. Hamid Reza Moradi and Anahid Kamali, On fuzzy strongly g*_closed sets and g**_closed sets, International Journal of Advances in Applied Mathematics and Mechanics, 2015; 2: 13-17.
2. Pious Missier, S., and Robert, A., On Semi*-open Sets, International Journal of Scientific and Innovative Mathematical Research, Vol. 2, Issue. 2014; 5: 460-468.

3. Robert, A. and Pious Missier, S., On Functions Associated with Semi*-open Sets, International Journal of modern sciences and Engineering Technology, Issue. 2014; 1(2):39-46.
4. Robert, A. and Pious Missier, S., On Semi*-closed sets, Asian Journal of Engineering & Maths, 2012; 1(4).
5. R.P. Chakarborty, Anjana Bhattacharya and M.N Mukherjee, Semicompactness in Fuzzy topological spaces, Bull. Malays. Math. Sci. Soc, 2005; 28(2): 205-213.
6. S. Willard, General Topology, Dover Publications. New York, 2004.
7. John N. Mordeson and Premchand S.Nair ,Fuzzy Mathematics, An introduction for Engineers and scientists, Springer-Verlag Berlin Heidelberg,2001; 67-113.
8. E.E. Kerre and A.A. Nough, Operations on the class of Fuzzy sets on a Universe Endowed with a Fuzzy Topology, Journal of Mathematical Analysis and Applications, 1993; 180: 325-341.
9. U.V Fatteh and D.S. Bassan, Fuzzy Connectedness and its stronger forms, Journal of mathematical Analysis and applications, 1985; 111(2): 449-464.
10. M. Sarkar, On fuzzy topological spaces, Journal of Mathematical Analysis and Applications, 1981; 79(2): 384–394.
11. Levine,N., Generalized closed sets in topology , Rend. Circ. Mat. Palermo, 1970; 19(2): 89-96.
12. C. L. Chang, Fuzzy topological spaces, Journal of Mathematical Analysis and Applications, 1968; 24(1):182-190.
13. L.A. Zadeh, Fuzzy Sets, Information and Control, 1965; 8(3): 338-353.
14. Levine,N., Semi-open sets and semi-continuity in topological space , American Mathematical, 1963: 70(1):36-41.