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Phase Vacation Queuing model with Server Breakdown

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ABSTRACT

This article manages a Queuing framework during which one server offers support to all the showing up clients. Clients show up in batch follows a Poisson conveyance though the administration follows a general dispersion. Administration interference happens aimlessly. It gets into a fix procedure right away. In the wake of completing of the repair system, administration proceeds. Clients because of eagerness, they leave the framework during the hour of repair process. After fruition of the administration, if there aren't any clients inside the line, server will take an obligatory vacation of two stages. In stage I vacation stage, fundamental upkeep work for the server will be managed. In stage II, significant upkeep work will be directed. During these stage vacations, clients may join the line. To stay away from the clog inside the framework, an idea of limited suitability is taken among the appearance of customers during the hour of stage get-aways. Confined suitability might be played dependent on two distinct reasons in every one of the stage excursion. This lining issue is examined through a birth passing procedure of Queuing hypothesis and it's explained by one among the lining issue strategy known to be beneficial variable method. The model is all around clarified very well by methods for reasonable application. The model is all around bolstered by techniques for numerical portrayal and graphical strategy.

KEYWORDS: Phases of Vacation, Reneging, Restricted admissibility, Supplementary variable technique

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1. INTRODUCTION

Choudhury A and Medhi P[1] made a study on Balking and reneging in multi ever markovian queuing systems. Madan, K. C. and Ebrahim Malalla [2] investigated the work on a single server bulk input queue with Random failures. Maragathasundari S[3] examined a Queuing System of General Service Distribution with an Establishment Time and Second Discretionary Administration Maragathasundari S and Dhanalakshmi[4] studied the Mobile adhoc networks problem through aqueueing approach. A study on the performance measures of the non-Markovian model of optional types of service with extended vacation is well determined by Maragathasundari S and Manikandan P[5]. Montazer-Haghighi et.al[6] investigated a multiserver Markovian queuing system with balking and reneging. Radha S and Maragathasundari S.[7] investigated a mathematical modeling in non markovian queue. Santhanamahalingam and Maragathasundari S [8] analyzed the F-Dematel method to evaluate criteria for affecting productivity in HP valve production industries. Balking and reneging of batches in vod applications was well investigated by Vanalakshmi R et. al[9]. S. Vignesh and S. Maragathasundari[10] made a analysis on a non markovian single server batch arrival queuing system of compulsory three stages of services with fourth optional stage service.

2. DIAGRAMMATIC VERSION OF THE MODEL

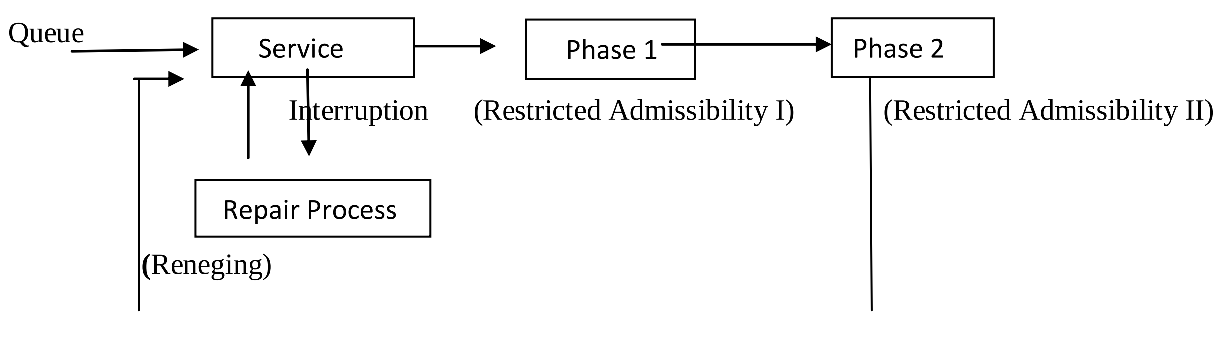

Figure 1: Queuing model defined in the system

3. MATHEMATICAL ASSUMPTION OF THE MODEL

Clients arrival follows a Poisson distribution with arrival rate $\lambda_c > 0$. Service time, vacation time and repair process follows a general distribution.

Let $U_1^*(x)$ and $u_1^*(x)$ be the distribution function and the density function of phase service of stage 1.

Let $\gamma_1(x)$ be the conditional probability of a completion of service and it is given by,

$\gamma_1(x) = \frac{u_1^*(x)}{1-U_1^*(x)}$, $u_1^*(x) = \gamma_1(x)e^{-\int_0^x \gamma_1(t)dt}$

Similarly, the process is repeated for Phase 1, Phase 2, vacations and Repair process. Hence we have the following:

In case of Phase 1, we have, $\gamma_{S_1}(x) = \frac{u_2^*(x)}{1-U_2^*(x)}$ and $u_2^*(x) = \gamma_{S_1}(x)e^{-\int_0^x \gamma_{S_1}(t)dt}$

For the Phase 2, we have, $\gamma_{S_2}(x) = \frac{u_3^*(x)}{1-U_3^*(x)}$ and $u_3^*(x) = \gamma_{S_2}(x)e^{-\int_0^x \gamma_{S_2}(t)dt}$

For the Repair process, $\gamma_l(x) = \frac{u_4^*(x)}{1-U_4^*(x)} u_4^*(x) = \gamma_l(x)e^{-\int_0^x \gamma_l(t)dt}$

4. GOVERNING EQUATIONS OF THE MODEL

The equations governing the system are as follows:

In case of service, we have

$$\frac{\partial}{\partial x} R_n(x) + (\lambda_c + \gamma_1(x) + \theta)R_n(x) = \lambda_c \sum_{p=1}^{n} e_p R_{n-p}(x) \quad (1)$$

$$\frac{\partial}{\partial x} R_0(x) + (\lambda_c + \gamma_1(x) + \theta)R_0(x) = 0 \quad (2)$$

For vacation phases, we have the following:

$$\frac{\partial}{\partial x} S_n^{(1)}(x) + \left(\lambda_c + \gamma_{S_1}(x)\right) S_n^{(1)}(x) = \lambda_c(1-d_1)S_n^{(1)}(x) + \lambda_c d_1 \sum_{p=1}^{n} e_p S_{n-p}^{(1)}(x) \quad (3)$$

$$\frac{\partial}{\partial x} S_0^{(1)}(x) + \left(\lambda_c + \gamma_{S_1}(x)\right) S_0^{(1)}(x) = \lambda_c(1-d_1)S_0^{(1)}(x) \quad (4)$$

$$\frac{\partial}{\partial x} S_n^{(2)}(x) + \left(\lambda_c + \gamma_{S_2}(x)\right) S_n^{(2)}(x) = \lambda_c(1-d_2)S_n^{(2)}(x) + \lambda_c d_2 \sum_{p=1}^{n} e_p S_{n-p}^{(2)}(x) \quad (5)$$

$$\frac{\partial}{\partial x} S_0^{(2)}(x) + \left(\lambda_c + \gamma_{S_2}(x)\right) S_0^{(2)}(x) = 0 \quad (6)$$

In case of repair, we have

$$\frac{\partial}{\partial x} L_n(x) + (\lambda_n + \gamma_l(x) + \xi)L_n(x) = \lambda_n \sum_{p=1}^{n} e_p L_{n-p}(x) + \xi L_{n+1}(x) \quad (7)$$

$$\frac{\partial}{\partial x} L_0(x) + (\lambda_n + \gamma_l(x) + \xi)L_0(x) = \xi L_1(x) \quad (8)$$

Considering the idle time of the server, we consider

$$\lambda_c M = \int_0^\infty L_0(x)\gamma_l(x)dx + \int_0^\infty S_0^{(2)}(x)\gamma_{S_2}(x)dx \quad (9)$$

5. INITIAL AND BOUNDARY CONDITIONS

$$R_n(0) = \int_0^\infty L_{n+1}(x)\gamma_l(x) + \int_0^\infty S_{n+1}^{(2)}(x)\gamma_{S_2}(x)dx + \lambda_{c_{n+1}} M \quad (10)$$

$$S_n^{(1)}(0) = \int_0^\infty R_n(x)\gamma_1(x)dx \quad (11)$$

$$S_n^{(2)}(0) = \int_0^\infty S_n^{(1)}(x)\gamma_2(x)dx \quad (12)$$

$$L_n(x) = \theta \int_0^\infty R_{n-1}(x)dx = \alpha R_{n-1} \quad (13)$$

6. DISTRIBUTION OF THE QUEUE LENGTH AT ANY POINT OF TIME

We apply the concept of supplementary variable technique for (1) to (13). As a result we get the probability queue size of the system defined.

$$\frac{\partial}{\partial x}R(x,z) + (\lambda_c - \lambda_c E(z) + \gamma_1(x) + \theta)R(x,z) = 0 \quad (14)$$

$$\frac{\partial}{\partial x}S^{(1)}(x,z) + \left(\lambda_c d_1 - \lambda_c d_1 E(z) + \gamma_{S_1}(x)\right)S^{(1)}(x,z) = 0 \quad (15)$$

$$\frac{\partial}{\partial x}S^{(2)}(x,z) + \left(\lambda_c d_2 - \lambda_c d_2 E(z) + \gamma_{S_2}(x)\right)S^{(2)}(x,z) = 0 \quad (16)$$

$$\frac{\partial}{\partial x}L(x,z) + \left(\lambda_c - \lambda_c E(z) + \gamma_1(x) + \xi - \frac{\xi}{z} + \gamma_l(x)\right)L(x,z) = 0 \quad (17)$$

$$zR(0,z) = \int_0^\infty R(x,z)\gamma_1(x)dx + \int_0^\infty S^{(2)}(x,z)\gamma_{S_2}(x)dx + \lambda_c M(E(z) - 1) \quad (18)$$

$$S^{(1)}(0,z) = \int_0^\infty R(x,z)\gamma_1(x)dx \quad (19)$$

$$S^{(2)}(0,z) = \int_0^\infty S^{(1)}(x,z)\gamma_{S_1}(x)dx \quad (20)$$

$$K(0,z) = \theta_z R(z) \quad (21)$$

Integrating equation (14) from 0 to x yields

$$R(x,z) = R(0,z)e^{-(\lambda_c - \lambda_c E(z) + \theta)x - \int_0^x \gamma_1(t)dt} \quad (22)$$

Integrating equation (22) by parts with respect to x yields,

$$R(z) = R(0,z)\left[\frac{1-U_1^*(a)}{a}\right], \ a = \lambda_c - \lambda_c E(z) + \theta \tag{23}$$

$U_1^*(a) = \int_0^\infty e^{-(\lambda_c - \lambda_c E(z)+\theta)x} dU_1(x)$is the Laplace Stieltjes transform of the service time $U_1(x)$.

Multiplying equation (22) by $\gamma_1(x)$ on both sides, and integrating, we get

$$\int_0^\infty R(x,z)\gamma_1(x)dx = R(0,z)U_1^*(a) \tag{24}$$

Applying the same procedure for equation (15) to (18), we get

$$S^{(1)}(z) = S^{(1)}(0,z)\left[\frac{1-U_2^*(b)}{b}\right], \ b = \lambda_c d_1 - \lambda_c d_1 E(z) \tag{25}$$

$$\int_0^\infty S^{(1)}(x,z)\gamma_{S_1}(x)dx = S^{(1)}(0,z)U_2^*(b) = R(0,z)U_1^*(a)U_2^*(b) \tag{26}$$

$$S^{(2)}(z) = S^{(2)}(0,z)\left[\frac{1-U_3^*(f)}{f}\right], \ f = \lambda_c d_2 - \lambda_c d_2 E(z) \tag{27}$$

$$\int_0^\infty S^{(2)}(x,z)\gamma_{S_2}(x)dx = R(0,z)U_1^*(a)U_2^*(b)U_3^*(f) \tag{28}$$

$$L(z) = \theta_z R(0,z)\left[\frac{1-U_1^*(a)}{a}\right]\left[\frac{1-U_4^*(g)}{g}\right], \ g = \lambda_c - \lambda_c E(z) + \xi - \frac{\xi}{z} \tag{29}$$

$$\int_0^\infty K(x,z)\gamma_l(x)dx = \theta_z R(0,z)\left[\frac{1-U_1^*(a)}{a}\right]U_4^*(g) \tag{30}$$

Now from equation (18) , we have

$$R(0,z) = \frac{\lambda_c M(E(z)-1)}{z-U_1^*(a)[1+U_2^*(b)U_3^*(f)]} \tag{31}$$

Substituting R(0,z) in equations (23), (25), (27), and (29), we get

$$R(z) = \frac{\lambda_c M(E(z)-1)}{z-U_1^*(a)[1+U_2^*(b)U_3^*(f)]}\left[\frac{1-U_1^*(a)}{a}\right] \tag{32}$$

$$S^{(1)}(z) = \frac{\lambda_c M(E(z)-1)}{z-U_1^*(a)[1+U_2^*(b)U_3^*(f)]}U_1^*(a)\left[\frac{1-U_2^*(b)}{b}\right] \tag{33}$$

$$S^{(2)}(z) = \frac{\lambda_c M(E(z)-1)}{z-U_1^*(a)[1+U_2^*(b)U_3^*(f)]}U_1^*(a)U_2^*(b)\left[\frac{1-U_3^*(f)}{f}\right] \tag{34}$$

$$L(z) = \theta_z \frac{\lambda_c M(E(z)-1)}{z-U_1^*(a)[1+U_2^*(b)U_3^*(f)]}\left[\frac{1-U_1^*(a)}{a}\right]\left[\frac{1-U_4^*(g)}{g}\right] \tag{35}$$

7. PROBABILITY GENERATING FUNCTION OF THE QUEUE SIZE

Let $K_q(z)$ be the probability generating function of the queue size.

$$K_q(z) = R(z) + S^{(1)}(z) + S^{(2)}(z) + L(z)$$

$$K_q(z) = \frac{\lambda_c M(E(z)-1)\left\{\begin{matrix}\left[\frac{1-U_1^*(a)}{a}\right] + U_1^*(a)\left[\frac{1-U_2^*(b)}{b}\right] + U_1^*(a)U_2^*(b) \\ \left[\frac{1-U_3^*(f)}{f}\right] + \theta_z\left[\frac{1-U_1^*(a)}{a}\right]\left[\frac{1-U_4^*(g)}{g}\right]\end{matrix}\right\}}{z - U_1^*(a)[1 + U_2^*(b)U_3^*(f)]}$$

(36)

8. IDLE TIME AND UTILIZATION FACTOR

Using the Normalization Condition, $K_q(1) + M = 1$ (37)

We have $K_q(z)$ at z=1 becomes a indeterminate form $\frac{0}{0}$.

Hence we apply L' Hopital's rule to get

$$\lim_{z\to 1} K_q(z) = \frac{N'(1)}{D'(1)}$$

From this, the idle time S can be formed

$$S = \frac{D'(1)}{N'(1)+D'(1)} \quad (38)$$

Also, the utilization factor $\rho = 1 - M$ is determined.

9. RECITAL MEASURES OF THE QUEUING SYSTEM

To find the steady state average queue length, L_q, we adopt the following method

$$L_q = \frac{d}{dz} K_q(z) \text{at} z = 1$$

This attains indeterminate form $\frac{0}{0}$. Consider (36) as $K_q(z) = \frac{N(z)}{D(z)}$

$N(z)$ and $D(z)$ are the numerator and denominator of the R.H.S. of (36)

Apply L'Hopital's rule twice on (36) we obtain

$$L_q = \lim_{z\to 1}\frac{D'(z)N''(z)-D''(z)N'(z)}{2(D'(z))^2} = \frac{D'(1)N''(1)-D''(1)N'(1)}{2(D'(1))^2} \quad (39)$$

Where

$$D(z) = z - U_1^*(a)[1 + U_2^*(b)U_3^*(f)]$$

$$N(z) = \lambda_c M(E(z) - 1)\left\{\left[\frac{1-U_1^*(a)}{a}\right] + U_1^*(a)\left[\frac{1-U_2^*(b)}{b}\right] + U_1^*(a)U_2^*(b)\left[\frac{1-U_3^*(f)}{f}\right] + \theta_z\left[\frac{1-U_1^*(a)}{a}\right]\left[\frac{1-U_4^*(g)}{g}\right]\right\}$$

$$D'(1) = 1 - \left\{-2\lambda_c U_1^{*'}(\theta) + \lambda_c E(U_2)d_1 + \lambda_c d_2 E(U_3)\right\} \quad (40)$$

$$D''(1) = -\lambda_c^2\{2U_1^{*''}(\theta) + 2(-U_1^{*'}(\theta))[d_1E(U_2) + d_2E(U_3)] + d_1E(U_2^2) + (d_1 + d_2)E(U_3)E(U_2) + d_2E(U_3^2)\}$$
(41)

$$N'(1) = \lambda_c\left[\frac{1-U_1^*(\theta)}{\theta}\right] \quad (42)$$

$$N''(1) = \lambda_c M\left[\frac{1-U_1^*(\theta)}{\theta}\right] + \lambda_c M\left\{-U_1^{*'}(\theta) + E(U_2) + E(U_3)\right\} \quad (43)$$

Substituting equations 40,41,42 and 43 in equation 39, we get Lq in closed form.

Other performance measures of the queuing system can be found using little's law.

10. ARITHMETICAL RATIONALIZATION OF THE MODEL

Consider the following:

$$U_1^*(\theta) = \frac{\gamma_1}{\gamma_1 + \theta}, U_1'(\theta) = \frac{\gamma_1}{(\gamma_1 + \theta)^2}, U_1''(\theta) = \frac{\gamma_1}{(\gamma_1 + \theta)^3}$$

$$E(U_2) = \frac{1}{\gamma_{s_1}}, E(U_3) = \frac{1}{\gamma_{s_2}}, E(U_4) = \frac{1}{\gamma_l}$$

$$E(U_2{}^2) = \frac{2}{(\gamma_{s_1})^2}, E(U_3{}^2) = \frac{2}{(\gamma_{s_2})^2}, E(U_4{}^2) = \frac{2}{(\gamma_l)^2}$$

$\gamma_{s_1} = 3.6, \gamma_{s_2} = 4, \gamma_l = 4.5$, $\gamma_1 = 3.4$,$\lambda_c = 3, d_2 = 0.6$

Table 1: $d_1 = 0.2, 0.4, 0.6, 0.8, 1$

d_1	M	ρ	L_q	W_q	W	L
0.2	0.6609	0.3391	0.8839	0.2946	0.4077	1.223
0.4	0.6225	0.3775	1.1518	0.3839	0.5098	1.5293
0.6	0.5743	0.4257	1.5969	0.5323	0.6742	2.0226
0.8	0.5119	0.4881	2.4359	0.8119	0.9747	2.924
1	0.4283	0.5717	4.3759	1.4586	1.6492	4.9476

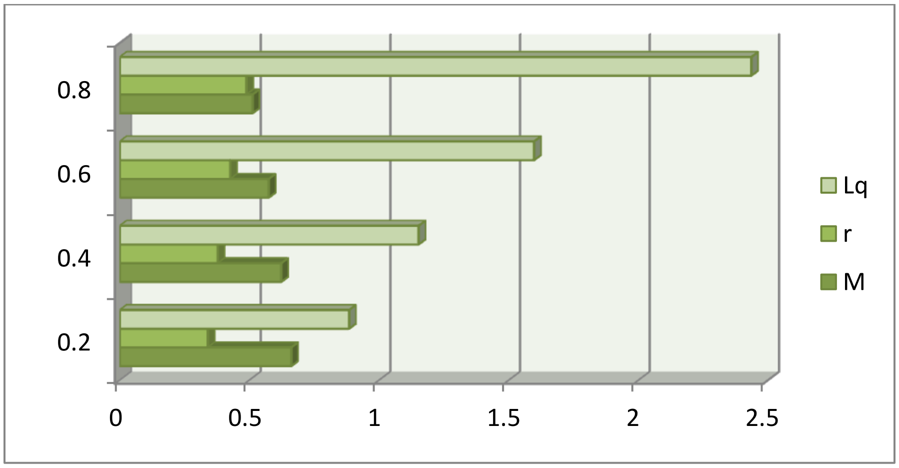

Figure 2: Graphical study on the effect of confined suitability over the queuing system

Table 2 :$\theta = 2, 2.5, 3, 3.5, 4$

θ	M	ρ	L_q	W_q	W	L
2	0.6609	0.3391	0.8839	0.2946	0.4077	1.223
2.5	0.6559	0.3441	1.1419	0.3806	0.4953	1.486
3	0.6528	0.3472	1.2986	0.4329	0.5486	1.6458
3.5	0.6512	0.3488	1.4421	0.4807	0.5969	1.7909
4	0.6509	0.3491	1.5725	0.5242	0.6405	1.9216

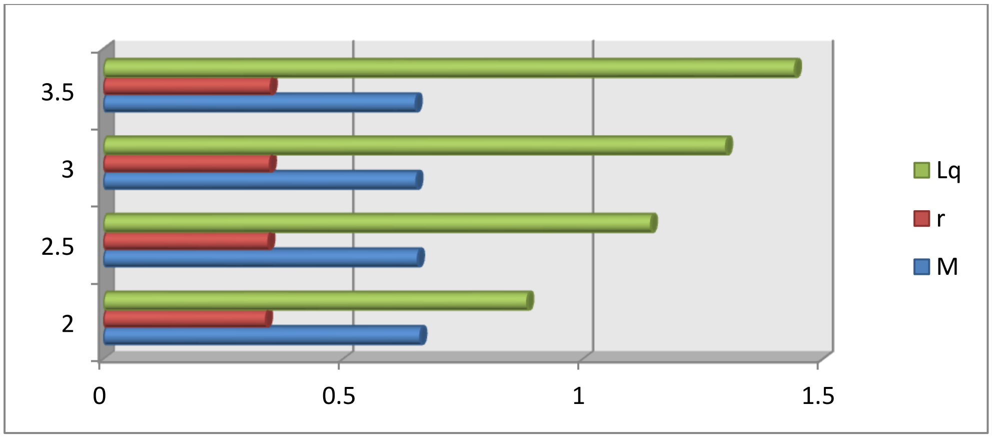

Figure 3: Graphical study on the effect of service interruption over the queuing system

11.ALGEBRAIC INVESTIGATION DESCRIPTION

From the above table 1,it is obvious that,as the probability of confined suitability increments during the hour of stage I get-away, it prompts a lessening out of gear time and expands the use factor.In addition,average number of clients holding up time in the line just as in the framework increases.Also holding up time of the clients likewise increases.Table II shows the way that as the probability of administration interference builds, there is an expansion in all the execution proportions of the framework. Additionally inert time diminishes and henceforth the use factor increments.

12.CONCLUSION

In the above article, we considered a lining arrangement of two stage get-aways, administration interference and fix process. Restricted admissibility happens during phase vacation and reneging happens during fix process. Numerical and graphical image progress the model to a great extend. As a future work, shying away, phases of administration, shut down planning, defer stage and so forth can be thought of.

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