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Total Dominator Chromatic Number of Grid Graphs

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ABSTRACT

Let G be a graph with minimum degree at least one. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number of G is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G. In this paper, we obtain total dominator chromatic number of grid graphs.

Mathematics Subject Classification : 05C15, 05C69

KEY WORDS : Total dominator chromatic number, ladder graph, grid graph.

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INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definiton of graph theory as found in [1]. Let G =(V, E) be a graph of order n with minimum degree atleast one . The open neighborhood N(v) of a vertex v∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v]=N (v)∪ {v}. The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H , to be the graph with vertex set V(G)×V(H) and edges between two vertices

(u_1 ,v_1) and (u_2,v_2) iff either $u_1=u_2$ and $v_1v_2 \in E(H)$ or $u_1u_2 \in E(G)$ and $v_1=v_2$. A grid graphs can be defined as $P_m \times P_n$ where m, n ≥2. A ladder graph can be defined as $P_2 \times P_n$ where n≥2 and is denoted by L_n. A subset S of V is called a total dominating set if every vertex in V is adjacent to some vertex in S. The total dominating set is minimal total dominating set if no proper subset of S is a total dominating set of G. The total domination number γ_t is the minimum cardinality taken over all minimal total dominating set of G. A γ_t-set is any minimal total dominating set with cardinality γ_t.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G). A total dominator coloring (td- coloring) of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by χ_{td} (G) and is defined by the minimum number of colors needed in a total dominator coloring of G .This concept was introduced by A.Vijiyalekshmi in[2]. This notion is also referred as a smarandachely k-dominator coloring of G (k≥1) and was introduced by A.Vijiyalekshmi in[4]. For an integer k≥1, a smarandachely k-dominator coloring of G is a proper coloring of G such that every vertex in G properly dominates a k color class. The smallest number of colors for which there exist a smarandachely k-dominator coloring of G is called the smarandachely k-dominator chromatic number of G, and is denoted by $\chi^s_{td}(G)$.

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C^1 be a minimal td-coloring of G. We say that a color class $c_i \in C^1$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes.

The total dominator chromatic number of paths, cycles and ladder graphs were found in [3].

We have the following observations from [3].

Theorem A [3] Let G be p_n or C_n. Then

$$\chi_{td}(p_n) = \chi_{td}(C_n) = \begin{cases} 2\left\lfloor \frac{n}{4} \right\rfloor + 2 & if\ n \equiv 0 (mod\ 4) \\ 2\left\lfloor \frac{n}{4} \right\rfloor + 3 & if\ n \equiv 1 (mod\ 4) \\ 2\left\lfloor \frac{n+2}{4} \right\rfloor + 2 & otherwise \end{cases}$$

Theorem B [3] For every n ≥ 2, the total dominator chromatic number of a ladder graph L_n is

$$\chi_{td}(L_n) = \begin{cases} 2\left\lfloor \frac{p}{6} \right\rfloor + 2 & if\ p \equiv 0 (mod 6) \\ \begin{cases} 2\left\lfloor \frac{p-2}{6} \right\rfloor + 4 \\ 2\left\lfloor \frac{p-4}{6} \right\rfloor + 4 \end{cases} & otherwise \end{cases}$$

In this paper, we obtain the least value for total dominator chromatic number for grid graphs.

Main Results

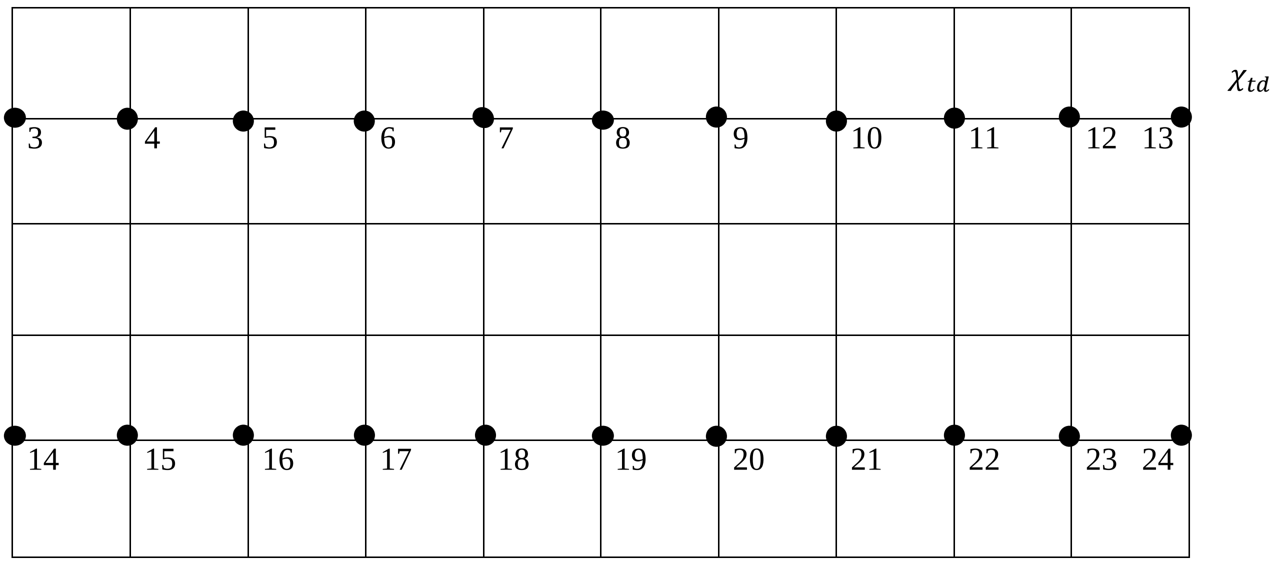
Notations: We denote $G_{m,n} = P_m \times P_n$ and let V($G_{m,n}$)= $\{u_{ij} / 1 \leq i \leq m\ and\ 1 \leq j \leq n\}$.

Theorem 1 $\chi_{td}(G_{m,n}) = \frac{mn}{3} + 2$ if either m or n≡0 (mod3).

Proof: Let m=3p, p∈Z^+.Proof is by using induction on p.For 1≤ i≤ p, let Di,n =$\{u_{(i+1,j)} / {}^{1 \leq j \leq n}_{1 \leq i \leq k+1}\}$ be a γ_t-set of $G_{3,n}$. We assign n distinct colors say 3,4,5 ,……,(n+2) to all vertices of $D_{i,n}$. Also we assign two repeated colors say 1, 2 to the vertices u_{ij} and $u_{kl} \in V(G_{3,n}) - D_{i,n}$ such that $|i-k| + |j-l| = 1$. So $\chi_{td}(G_{3,n})$ = n+2 = $\frac{mn}{3} + 2$. By induction hyphothesis, we assume that the theorem is true for p=k and so $\chi_{td}(G_{3k,n})$=kn+2= $\frac{mn}{3} + 2$. For p=k+1, first for td-coloring of $G_{3k,n}$,we need kn+2 colours, by induction hyphothesis. Since in a td-coloring of $G_{3(k+1),n}$,we can already used repeated colors 1 and 2 in the vertices $V(G_{3k,n}) - D_{i,n}$ followed by $G_{3(k+1),n}$ as earlier and we assign n(k+1) different colors to the vertices of $D_{i,n}$ for 1≤ $i \leq k+1$. So $\chi_{td}(G_{3(k+1),n})$ = n(k+1)+2 = $\frac{mn}{3} + 2$.

Illustration:

Consider $\mathbf{G_{6,11}}$



$(G_{6,11})=24$ Fig.1

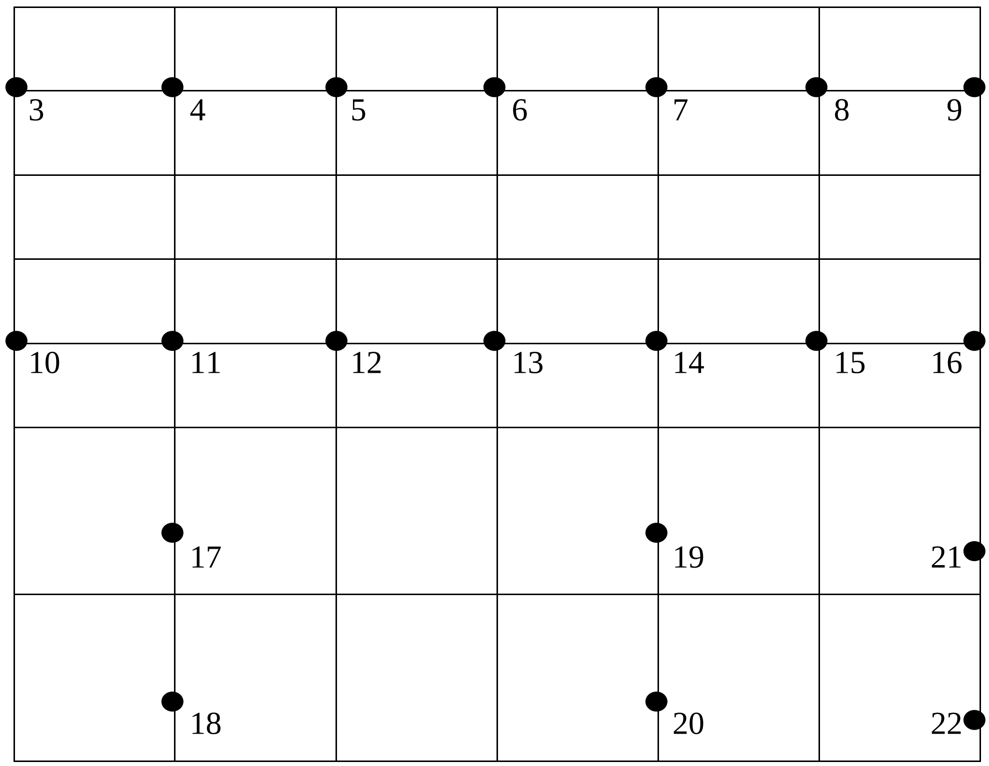
Theorem 2 $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m-2,n}) + \chi_{td}(L_n) - 2$ if $m \equiv 2(mod\ 3)\ and\ n \equiv 1,2(mod 3)$.

Proof: We have $G_{m,n}$ is obtained by $G_{m-2,n}$ followed by $G_{2,n}$ .Since in a td-coloring of $G_{m,n}$, we cannot use the non-repeated colors of vertices in $G_{m-2,n}$, for the $G_{2,n}$ and we can use the same repeated colors of vertices in the graphs $G_{m-2,n}$ and $G_{2,n}$ .Since $m-2 \equiv 0(mod\ 3)$ and

$\chi_{td}(G_{m-2,n}) = \frac{(m-2)n}{3} + 2$. Thus $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m-2,n}) + \chi_{td}(L_n) - 2$.□

Illustration:

Consider $G_{8,7}$



$\chi_{td}(G_{8,7})=22$ Fig.2

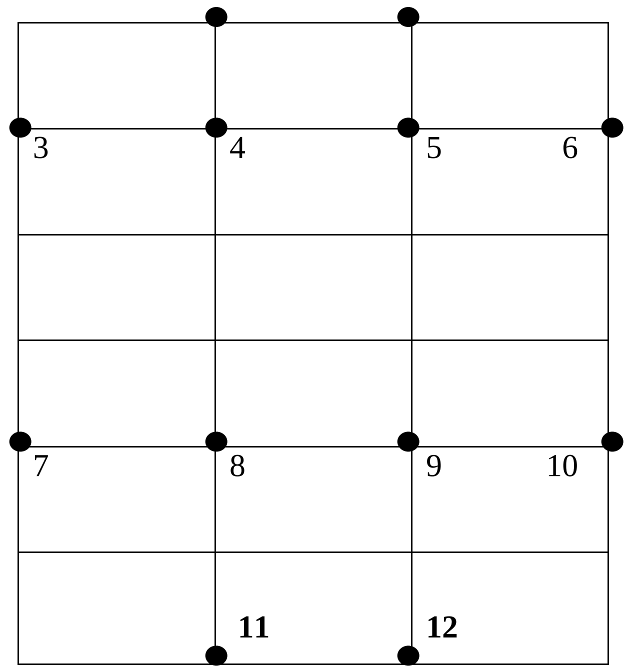
Theorem 3 For m,n$\equiv 1(mod\ 3)$,

$$\chi_{td}(G_{m,n})=\begin{cases}\chi_{td}(G_{m,n-1})+\chi_{td}(P_m)-2 & if\ m\le n\\ \chi_{td}(G_{m-1,n})+\chi_{td}(P_n)-2 & if\ m\ge n\end{cases}$$

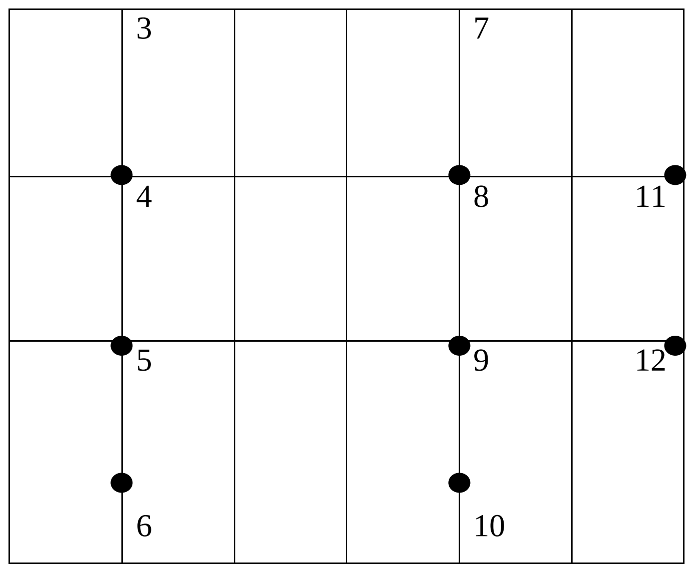
Proof: Let m, n $\equiv 1(mod\ 3)$, so (m-1) ,(n-1) $\equiv 0(mod\ 3)$. Let $D_{m,n-1}$ be the γ_t- set of $G_{m,n-1}$ and $|D_{m,n-1}| = \frac{m(n-1)}{3}$ .Suppose that $|V(G_{m,n})\cap D_{m,n-1}| = \frac{m(n-1)}{3}$ holds for $\frac{m(n-1)}{3} -$ layer P_{n-1}. We now assign $\frac{m(n-1)}{3}$ distinct colors to the vertices of $D_{m,n-1}$ and two repeated colors say 1 and 2 to the remaining vertices such that adjacent vertices receive different colors. Since the graph $G_{m,n}$ is $G_{m,n-1}$ followed by P_m ,$\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-1})+\chi_{td}(P_m)$ .Also the already used repeated colors are used in the coloring of P_m. So $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-1}) + \chi_{td}(P_m)$ -2. Proof is similar for the case $m\ge n$. □

Illustration:

Consider $G_{4,7}$ and $G_{7,4}$



χ_{td} ($G_{4,7}$) =12 Fig.3



χ_{td} ($G_{7,4}$)=12 Fig.4

Theorem 4

χ_{td} ($G_{m,n}$)= χ_{td}($G_{m,n-2}$)+ χ_{td} (L_m) -2 if $m \equiv 1(mod\ 3)\ and\ \ n \equiv 2(mod 3)$.

Proof: Since $n-2 \equiv 0(mod\ 3)$, χ_{td} ($G_{m,n-2}$)= $\frac{m(n-2)}{3} + 2$. $G_{m,n}$ is got from $G_{m,n-2}$ followed by L_m . From theorem 2, χ_{td} ($G_{m,n}$)= χ_{td} ($G_{m,n-2}$)+ χ_{td} (L_m) -2. □

Illustration:

Consider $G_{7,8}$

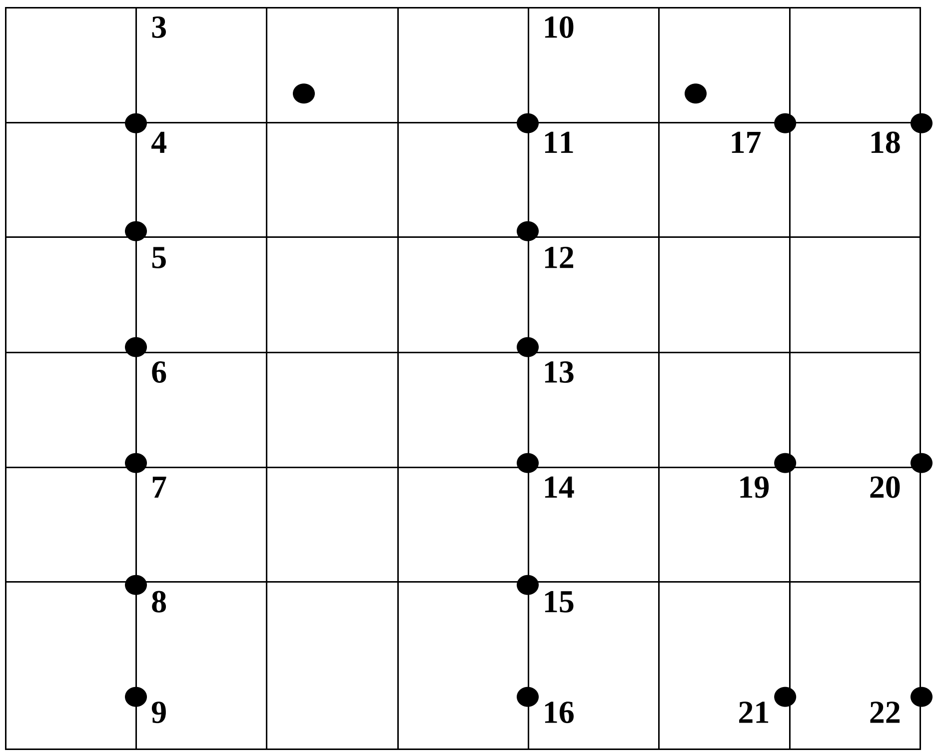


Fig .5

χ_{td} ($G_{7,8}$)=22

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