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Odd Prime Labeling of Various Snake Graphs

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ABSTRACT

For a graph G (V, E), a function f is called an odd prime labeling, given that f is a bijection from V to $\{1, 3, 5, \ldots, 2|V|-1\}$, satisfying $gcd\ (f(u), f(v)) = 1$, for each $uv \in E$. A graph admitting this labeling is called an odd prime graph. Various snake graphs like n-polygonal snake, double n-polygonal snake, alternate n-polygonal snake, double alternate triangular snake, irregular triangular snake, irregular quadrilateral snake are proved to be odd prime graphs.

KEYWORDS: Odd prime graph, n-polygonal snake, double n-polygonal snake, alternate n-polygonal snake, double alternate triangular snake, irregular triangular, irregular quadrilateral snake

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INTRODUCTION

Finite, simple, undirected, non-trivial and connected graphs have been considered in this paper. V or $V(G)$ and E or $E(G)$ of a graph $G(V, E)$ are the vertex and edge set respectively, where $|V|$ and $|E|$ are the number of elements in V and E respectively. Gross and Yellen[1] is referred for graph theoretical notations and terminologies. Burton[2]is referred for number theoretical notations. Assignment of integers to vertices and/or edges of a graph subject to certain conditions is known as graph labeling.[3] Gallian[3] is referred for the latest survey of graph labeling. Different labeling has been proved on various snake graphs including prime labeling. Odd prime labeling is a variation of prime labeling. This paper attempts to prove various snake graphs to have odd prime labeling. We start with few notations and definitions required in the paper.

Notation: 1 For each natural number n, $[n]$ and O_n respectively are the set of first n natural numbers and the set of first n odd natural numbers. i.e $[n]=\{1, 2, 3, \ldots, n\}$ and $O_n=\{1, 3, 5, \ldots, 2n-1\}$.[4, 5]

Definition: 2 Let P_k $(k \geq 2)$ be a path with consecutive vertices $v_1, v_2, \ldots, v_k$. An ***n*-polygonal snake** $(n \geq 3)$ is obtained from the path P_k whose vertex set $V = V(P_k) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$ and the edge set is $E = E(P_k) \cup \{v_i u_{i,1}, u_{i,j-1} u_{i,j}, u_{i,n-2} v_{i+1} \mid i \in [k-1], j \in [n-2]-\{1\}\}$. We denote it as $S_k(C_n)$. i.e. a snake of path P_k where each edge of P_k is replaced by a cycle C_n.[6]

Definition: 3 An **alternate triangular snake** is obtained from the path P_n by replacing each alternate edge of the path by a triangle C_3. It is denoted as $A(T_n)$.[7]

Similar to alternate triangular snake, we define the following:

Definition: 4 An **alternate *n*-polygonal snake** $(n \geq 3)$ is obtained from the path P_k $(k \geq 2)$ by replacing each alternate edge of the path by C_n. We denote it as $AS_k(C_n)$.

Definition: 5 A **double triangular snake** consists of two triangular snakes that have a common path P_n. It is denoted by $D(T_n)$.[8]

Similar to double triangular snake, we define the following:

Definition: 6 A **double *n*-polygonal snake** $(n \geq 3)$ consists of two n-polygonal snakes that have a common path P_k $(k \geq 2)$. We denote it as $DS_k(C_n)$.

Definition: 7 A **double alternate triangular snake** consists of two similar alternate triangular snakes that have a common path P_n. It is denoted as $DA(T_n)$.[8]

Definition: 8 Let P_n $(n \geq 3)$ be a path with consecutive vertices $v_1, v_2, \ldots, v_n$. An **irregular triangular snake** $(n \geq 3)$ is obtained from the path P_n whose vertex set $V = V(P_n) \cup \{u_i \mid i \in [n-2]\}$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i v_{i+2} \mid i \in [n-2]\}$. It is denoted as IT_n.[9]

Definition: 9 Let P_n $(n \geq 3)$ be a path with consecutive vertices $v_1, v_2, \ldots, v_n$. An **irregular quadrilateral snake** $(n \geq 3)$ is obtained from the path P_n whose vertex set $V = V(P_n) \cup \{u_i, w_i \mid i \in [n-2]\}$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i w_i, w_i v_{i+2} \mid i \in [n-2]\}$. It is denoted as IQ_n.[9]

Prime labeling was originated by Entringer and first introduced by Tout et al[10] in their paper. This labeling is defined as follows:

Definition: 10 For a graph $G(V,E)$ with n vertices, a bijection $f : V \to [n]$ is called **prime labeling** if $\gcd(f(u), f(v)) = 1$, for each $uv \in E$. The graph admitting prime labeling is called a **prime graph**.[10]

Carlson[11] proved that generalized books are prime graphs. Seoud and Youssef[12] proved that C_m-snakes are prime graphs. Ganesan et al[13] proved that cycle-cactus $C_k^{(n)}$ and triangular book $B_{3,n}$ are prime graphs. Vaidya and Prajapati[14] proved that tadpole, also called kite or dragon is a prime graph.

A variation of prime labeling, called odd prime labeling was introduced by Prajapati and Shah.[5]

Definition: 11 For a graph $G(V,E)$ with n vertices, a bijective function $f : V \to O_n$ is called **odd prime labeling** if for each $uv \in E$, $\gcd(f(u), f(v)) = 1$. The graph admitting this labeling is called an **odd prime graph**.[5]

Prajapati and Shah[5] proved that graphs like path, ladder graph, complete graph K_n iff $n \leq 4$, complete bipartite graphs under certain conditions, wheel graph, helm graph, fan graph, friendship graph, Petersen graph $P(n,2)$ and many more are odd prime graphs. Graphs obtained by duplicating each vertex by an edge and each edge by a vertex in a path, star graph, cycle and wheel graph are all odd prime graphs is also proved by them.[15]

MAIN RESULTS

Theorem: 1 $S_k(C_n)$, $n \geq 3,\ k \geq 2$ is an odd prime graph.

Proof: Consider an *n*-polygonal snake $S_k(C_n)$ on a path with *k* consecutive vertices $v_1, v_2, \ldots, v_k$. Let the vertex set of $S_k(C_n)$ be $V = V(P_k) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence, $|V| = nk-(n+k)+2$ and its edge set is $E = E(P_k) \cup \{v_i u_{i,1}, u_{i,j-1}u_{i,j}, u_{i,n-2}v_{i+1} \mid i \in [k-1], j \in [n-2]-\{1\}\}$. Let $f : V \to O_{|V|}$ be defined as

$$f(x) = \begin{cases} (2n-2)i-2n+3, & \text{if } x = v_i, i \in [k]; \\ (2n-2)i-2n+2j+3, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$$

For each $e \in E$, if

1. $e = v_i v_{i+1}$, $i \in [k-1]$, $\gcd(f(v_i), f(v_{i+1})) = \gcd((2n-2)i-2n+3, (2n-2)i+1) = 1$;
2. $e = v_i u_{i,1}$, $i \in [k-1]$, $\gcd(f(v_i), f(u_{i,1})) = \gcd((2n-2)i-2n+3, (2n-2)i-2n+5) = 1$;
3. $e = u_{i,n-2}v_{i+1}$, $i \in [k-1]$, $\gcd(f(u_{i,n-2}), f(v_{i+1})) = \gcd((2n-2)i-1, (2n-2)i+1) = 1$;
4. $e = u_{i,j-1}u_{i,j}$, $i \in [k-1]$, $j \in [n-2]-\{1\}$, $\gcd(f(u_{i,j-1}), f(u_{i,j}))$

$$= \gcd((2n-2)i-2n+2j+1, (2n-2)i-2n+2j+3) = 1.$$

This shows that *f* admits odd prime labeling on $S_k(C_n)$ and hence it is an odd prime graph.

Theorem: 2 $DS_k(C_n)$, $n \geq 3,\ k \geq 2$ is an odd prime graph.

Proof: Consider a double *n*-polygonal snake on path of *k* consecutive vertices $v_1, v_2, \ldots, v_k$. Let the vertex set of $DS_k(C_n)$ be $V = V(P_k) \cup \{u_{i,j}, w_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence, $|V| = 2nk-2n-3k+4$ and its edge set is $E = E(P_k) \cup \{v_i u_{i,1}, u_{i,j-1}u_{i,j}, u_{i,n-2}v_{i+1}, v_i w_{i,1}, w_{i,j-1}w_{i,j}, w_{i,n-2}v_{i+1} \mid i \in [k-1], j \in [n-2]-\{1\}\}$.

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} (4n-6)i-4n+7, & \text{if } x = v_i, i \in [k]; \\ (4n-6)i-4(n-j)+5, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]; \\ (4n-6)i-4(n-j)+7, & \text{if } x = w_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

For each $e \in E$, if

1. $e = v_i v_{i+1}$, $i \in [k-1]$, $\gcd(f(v_i), f(v_{i+1})) = \gcd((4n-6)i - 4n + 7, (4n-6)i + 1) = 1$;

2. $e = v_i u_{i,1}$, $i \in [k-1]$, $\gcd(f(v_i), f(u_{i,1})) = \gcd((4n-6)i - 4n + 7, (4n-6)i - 4n + 9) = 1$;

3. $e = u_{i,n-2} v_{i+1}$, $i \in [k-1]$, $\gcd(f(u_{i,n-2}), f(v_{i+1})) = \gcd((4n-6)i - 3, (4n-6)i + 1) = 1$;

4. $e = u_{i,j-1} u_{i,j}$, $i \in [k-1]$, $j \in [n-2] - \{1\}$, $\gcd(f(u_{i,j-1}), f(u_{i,j}))$

   $$= \gcd((4n-6)i - 4(n-j) + 1, (4n-6)i - 4(n-j) + 5) = 1;$$

5. $e = v_i w_{i,1}$, $i \in [k-1]$, $\gcd(f(v_i), f(w_{i,1})) = \gcd((4n-6)i - 4n + 7, (4n-6)i - 4n + 11) = 1$;

6. $e = w_{i,n-2} v_{i+1}$, $i \in [k-1]$, $\gcd(f(w_{i,n-2}), f(v_{i+1})) = \gcd((4n-6)i - 1, (4n-6)i + 1) = 1$;

7. $e = w_{i,j-1} w_{i,j}$, $i \in [k-1]$, $j \in [n-2] - \{1\}$, $\gcd(f(w_{i,j-1}), f(w_{i,j}))$

   $$= \gcd((4n-6)i - 4(n-j) + 3, (4n-6)i - 4(n-j) + 7) = 1.$$

This shows that *f* admits odd prime labeling on $DS_k(C_n)$ and hence it is an odd prime graph.

Theorem: 3 $AS_m(C_n)$, $m, n \geq 3$ is an odd prime graph.

Proof: Consider an alternate *n*-polygonal snake $AS_m(C_n)$ on a path with *m* consecutive vertices $v_1, v_2, \ldots, v_m$.

1. Let *m* be odd. In this case, $AS_m(C_n)$ is obtained from a path of $m = 2k-1, (k \geq 2)$ consecutive vertices $v_1, v_2, \ldots, v_{2k-1}$. Consider the vertex set $V = V(P_{2k-1}) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$. Hence $|V| = n(k-1)+1$ and the edge set $E = E(P_{2k-1}) \cup \{v_{2i-1}u_{i,1}, u_{i,n-2}v_{2i}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2] - \{1\}\}$.

   Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2ni-1, & \text{if } x = v_{2i}, i \in [k-1]; \\ 2n(i-1)+2j-1, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

2. Let *m* be even. In this case, $AS_m(C_n)$ is obtained from a path of $m = 2k, (k \geq 2)$ consecutive vertices $v_1, v_2, \ldots, v_{2k}$. Here two non-isomorphic graphs are obtained:

   (a) When the polygon starts with the first edge, the vertex set $V = V(P_{2k}) \cup \{u_{i,j} \mid i \in [k], j \in [n-2]\}$ with $|V| = nk$ and

the edge set $E = E(P_{2k}) \cup \{v_{2i-1}u_{i,1}, u_{i,n-2}v_{2i}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2]-\{1\}\}$

Let $f: V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2ni-1, & \text{if } x = v_{2i}, i \in [k]; \\ 2n(i-1)+2j-1, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

(b) When the polygon starts with the second edge,

the vertex set $V = V(P_{2k}) \cup \{u_{i,j} \mid i \in [k-1], j \in [n-2]\}$ with $|V| = n(k-1)+2$ and

the edge set $E = E(P_{2k}) \cup \{v_{2i}u_{i,1}, u_{i,n-2}v_{2i+1}, u_{i,j-1}u_{i,j} \mid i \in [k-1], j \in [n-2]-\{1\}\}$

Let $f: V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 2n(i-1)+1, & \text{if } x = v_{2i-1}, i \in [k]; \\ 2n(i-1)+3, & \text{if } x = v_{2i}, i \in [k-1]; \\ 2n(i-1)+2j+3, & \text{if } x = u_{i,j}, i \in [k-1], j \in [n-2]. \end{cases}$

In each of the cases, it is easy to check that if $n-1=2^t$, then the functions defined in the respective cases will admit odd prime labeling on $AS_m(C_n)$.

If $n-1 \neq 2^t$, assume that *d* is an odd divisor of *n* – 1.

For each $v_l \in V(P_{2k-1})$, whenever $f(v_l) = qd$ for some $q \in \mathbb{N}$, $\gcd(f(v_l), f(v_{l+1})) \neq 1$. In this case, either $\gcd(f(v_{l-1}), f(v_{l+1})) = 1$ or $\gcd\left(f\left(u_{\lceil \frac{l}{2} \rceil, 1}\right), f(v_{l+1})\right) = 1$. Interchange $f(v_l)$ by $f(v_{l-1})$ or $f\left(u_{\lceil \frac{l}{2} \rceil, 1}\right)$ accordingly and the function thus obtained will admit odd prime labeling on $AS_m(C_n)$ and it is an odd prime graph.

Theorem: 4 $DA(T_n)$, $n \geq 3$ is an odd prime graph.

Proof: Consider a double alternate triangular snake graph $DA(T_n)$ on a path with *m* consecutive vertices $v_1, v_2, \ldots, v_m$.

1. Let n be odd. In this case, $DA(T_n)$ is obtained from the path of $n=2k-1,(k\geq 2)$ consecutive vertices $v_1,v_2,\ldots,v_{2k-1}$. Consider the vertex set $V=V(P_{2k-1})\cup\{u_i,w_i \mid i\in[k-1]\}$. Hence $|V|=4k-1$ and the edge set $E=E(P_{2k-1})\cup\{v_{2i}u_i,u_iv_{2i+1},v_{2i}w_i,w_iv_{2i+1} \mid i\in[k-1]\}$.

Let $f:V\to O_{|V|}$ be defined as
$$f(x)=\begin{cases} 24i-23, & \text{if } x=v_{6i-5}, i\in\left[\left\lfloor \frac{k+2}{3}\right\rfloor\right];\\ 24i-19, & \text{if } x=v_{6i-4}, i\in\left[\left\lfloor \frac{2k+3}{6}\right\rfloor\right];\\ 24i-17, & \text{if } x=v_{6i-3}, i\in\left[\left\lfloor \frac{k+1}{3}\right\rfloor\right];\\ 24i-13, & \text{if } x=v_{6i-2}, i\in\left[\left\lfloor \frac{2k+1}{6}\right\rfloor\right];\\ 24i-7, & \text{if } x=v_{6i-1}, i\in\left[\left\lfloor \frac{k}{3}\right\rfloor\right];\\ 24i-5, & \text{if } x=v_{6i}, i\in\left[\left\lfloor \frac{2k-1}{6}\right\rfloor\right];\\ f(v_{2i+1})-4, & \text{if } x=u_i, i\in[k-1];\\ f(v_{2i})+4, & \text{if } x=w_i, i\in[k-1]. \end{cases}$$

2. Let n be even. In this case, $DA(T_n)$ is obtained from a path of $n=2k,(k\geq 2)$ consecutive vertices $v_1,v_2,\ldots,v_{2k}$. Here two non-isomorphic graphs are obtained:

   (a) When the triangle starts from the first edge, the vertex set $V=V(P_{2k})\cup\{u_i,w_i \mid i\in[k]\}$ with $|V|=4k$ and the edge set $E=E(P_{2k})\cup\{v_{2i-1}u_i,u_iv_{2i},v_{2i-1}w_i,w_iv_{2i} \mid i\in[k]\}$

   Let $f:V\to O_{|V|}$ be defined as
$$f(x)=\begin{cases} 16i-13, & \text{if } x=v_{4i-3}, i\in\left[\left\lfloor \frac{2k+3}{4}\right\rfloor\right];\\ 16i-11, & \text{if } x=v_{4i-2}, i\in\left[\left\lfloor \frac{k+1}{2}\right\rfloor\right];\\ 16i-3, & \text{if } x=v_{4i-1}, i\in\left[\left\lfloor \frac{2k+1}{4}\right\rfloor\right];\\ 16i-5, & \text{if } x=v_{4i}, i\in\left[\left\lfloor \frac{k}{2}\right\rfloor\right];\\ 8i-7, & \text{if } x=u_i, i\in[k];\\ 8i-1, & \text{if } x=w_i, i\in[k]. \end{cases}$$

   (b) When the triangle starts from the second edge, the vertex set $V=V(P_{2k})\cup\{u_i,w_i \mid i\in[k-1]\}$ with $|V|=4k-2$ and the edge set $E=E(P_{2k})\cup\{v_{2i}u_i,u_iv_{2i+1},v_{2i}w_i,w_iv_{2i+1} \mid i\in[k-1]\}$.

Let $f: V \to O_{|V|}$ be defined as $$f(x)=\begin{cases} 24i-23, & \text{if } x=v_{6i-5}, i\in\left[\left\lceil\frac{k+2}{3}\right\rceil\right]; \\ 24i-19, & \text{if } x=v_{6i-4}, i\in\left[\left\lfloor\frac{2k+3}{6}\right\rfloor\right]; \\ 24i-17, & \text{if } x=v_{6i-3}, i\in\left[\left\lfloor\frac{k+1}{3}\right\rfloor\right]; \\ 24i-13, & \text{if } x=v_{6i-2}, i\in\left[\left\lfloor\frac{2k+1}{6}\right\rfloor\right]; \\ 24i-7, & \text{if } x=v_{6i-1}, i\in\left[\left\lfloor\frac{k}{3}\right\rfloor\right]; \\ 24i-5, & \text{if } x=v_{6i}, i\in\left[\left\lfloor\frac{2k-1}{6}\right\rfloor\right]; \\ 8k-5, & \text{if } x=v_{2k}; \\ f(v_{2i+1})-4, & \text{if } x=u_i, i\in[k-1]; \\ f(v_{2i})+4, & \text{if } x=w_i, i\in[k-1]. \end{cases}$$

In each of the cases, it is easy to check that the function defined in the respective cases admit odd prime labeling and so $DA(T_n)$ is an odd prime graph.

Theorem: 5 IT_n, $n \geq 3$ is an odd prime graph.

Proof: Consider an irregular triangular snake graph IT_n on a path with *n* consecutive vertices $v_1, v_2, \ldots, v_n$. Let the vertex set of IT_n be $V = V(P_n) \cup \{u_i \mid i \in [n-2]\}$. Hence $|V| = 2n-2$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i v_{i+2} \mid i \in [n-2]\}$.

Let $f: V \to O_{|V|}$ be defined as $$f(x)=\begin{cases} 12i-11, & \text{if } x=v_{3i-2}, i\in\left[\left\lceil\frac{n+1}{3}\right\rceil\right]; \\ 12i-7, & \text{if } x=v_{3i-1}, i\in\left[\left\lfloor\frac{n}{3}\right\rfloor\right]; \\ 12i-5, & \text{if } x=v_{3i}, i\in\left[\left\lfloor\frac{n-1}{3}\right\rfloor\right]; \\ 4n-3, & \text{if } x=v_n; \\ 12i-9, & \text{if } x=u_{3i-2}, i\in\left[\left\lceil\frac{n}{3}\right\rceil\right]; \\ 12i-3, & \text{if } x=u_{3i-1}, i\in\left[\left\lfloor\frac{n-1}{3}\right\rfloor\right]; \\ 12i-1, & \text{if } x=u_{3i}, i\in\left[\left\lfloor\frac{n-2}{3}\right\rfloor\right]. \end{cases}$$

It is easy to check that the function defined here admits odd prime labeling and hence IT_n is an odd prime graph.

Theorem: 6 IQ_n, $n \geq 3$ is an odd prime graph.

Proof: Consider an irregular quadrilateral snake graph IQ_n on a path with *n* consecutive vertices $v_1, v_2, \ldots, v_n$. Let the vertex set of IQ_n be $V = V(P_n) \cup \{u_i, w_i \mid i \in [n-2]\}$. Hence $|V| = 3n-4$ and the edge set is $E = E(P_n) \cup \{v_i u_i, u_i w_i, w_i v_{i+2} \mid i \in [n-2]\}$.

Let $f : V \to O_{|V|}$ be defined as $f(x) = \begin{cases} 6i-5, & \text{if } x = v_i, i \in [n-1]; \\ 6n-9, & \text{if } x = v_n; \\ 6i-3, & \text{if } x = u_i, i \in [n-2]; \\ 6i-1, & \text{if } x = w_i, i \in [n-2]. \end{cases}$

It is easy to check that the function defined here admits odd prime labeling and hence IQ_n is an odd prime graph.

CONCLUSION

Various snake graphs like *n*-polygonal snake, double *n*-polygonal snake, alternate *n*-polygonal snake, double alternate triangular snake, irregular triangular snake and irregular quadrilateral snake have been shown to be odd prime graph in this paper.

OPEN PROBLEMS

Investigating double alternate *n*-polygonal snake and irregular *n*-polygonal snakes for odd prime labeling can be considered as open problems for research work in future.

REFERENCES

3. Gross JL, Yellen J., Zhang P. (edited), "*A handbook of graph theory*", 2nd edition, CRC Press Taylor & Francis Group 2014; 1-1633.
4. Burton DM, "*Elementary number theory*", 6th edition, 2007; 1-451.
5. Gallian JA, "*A dynamic survey of graph labeling*", The Electronic Journal of Combinatorics, 2016 # DS6: 1 - 502.
6. Pikhurko O., "*Trees are almost prime*", Discrete Math. 2007; 307: 1455-1462.
7. Prajapati UM, Shah KP, "*On odd prime labeling*", IJRAR 2018; 5(4): 284-294. Availabe from URL: http://doi.one/10.1729/Journal.18822

8. Sekar C., "*Studies in graph theory*", Ph.D. Thesis, Madurai Kamaraj University, 2002.

9. Ponraj R., Narayanan SS, "*Difference cordiality of some graphs obtained from double alternate snake graphs*", Global Journal of Mathematical Sciences: Theory and Practical 2013; 5: 167-175.

10. Ponraj R., Narayanan SS, Kala R., "*Difference cordial labeling of graphs obtained from double snakes*", International Journal of Mathematics Research 2013; 5: 317-322.

11. Ponraj R., Narayanan SS, "*Difference cordiality of some snake graphs*", J. Appl. Math. and Informatics 2014; 32: 377-387.

12. Tout A., Dabboucy AN, Howalla K., "*Prime labeling of graphs*", Nat. Acad. Sci. Letters 1982; 11: 365-368.

13. Carlson K, "*Generalized books and C_m-snakes are prime graphs*", Ars. Combin. 2006; 80: 215-221.

14. Seoud MA, Youssef MZ, "*On prime labelings of graphs*", Congr. Numer. 1999; 141: 203-215.

15. Ganesan V., Mahalakshmi S., Sathya MP, "*Prime labeling of triangular book and cycle-cactus graphs*", International Journal of Scientific Research and Modern Education. 2017; 2(2): 16-20.

16. Vaidya SK, Prajapati UM, "*Some results on prime and k-prime labeling*", J. Math. Research 2011; 3(1): 66-75. URL: http://www.ccsenet.org/journal/index.php/jmr/

17. Prajapati UM, Shah KP, "*Odd prime labeling of graphs with duplication of graph elements*", JASC 2018; 5(12): 1281-1288. DOI:16.10089.JASC.2018.V5I12.453459.1500267, Available from URL: http://j-asc.com/gallery/183-december-1553.pdf