

International Journal of Scientific Research and Reviews

Deformation Behaviour of Graded Material Plate Under Thermo mechanical Load For Varying Aspect Ratio

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ABSTRACT

Gradually varying the volume fraction of the constituents rather than abruptly changing them over an interface can resolve the problems of crack and stress concentration in composites. The gradual variation results in functionally graded materials (FGM). The gradation in properties of the material reduces thermal stresses, residual stresses and stress concentration factors. A lot of work have been carried out for the stress and deformation analyses of FGM plate when it is subjected to mechanical and thermo-mechanical load conditions. The FGM are very much useful in applications where high load as well as high variation in temperature occur. Hence it is vital to study the behavior of FGM under such circumstances. The studies considered in this paper are concerned with deformation and stress problems of FGM plates accounting for various effects, such as geometric and physical non-linearity and transverse shear deformability subjected to thermo-mechanical loads. The results are presented in terms of non dimensional characteristics such as non dimensional deflection, non dimensional tensile stress and non dimensional shear stress.

KEYWORDS:FGM, Thermo mechanical, Deflection, Stress.

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1. INTRODUCTION

In a composite material two dissimilar materials are bonded together, there is a very high chance that problems such as cracks will occur at some extreme loading conditions, be it static, dynamic, or thermal loads. Cracks are likely to initiate at interfaces and grow into the weaker material section. Another problem is the presence of residual stresses due to the difference in coefficients of thermal expansion of two dissimilar materials. These problems can be resolved by gradually varying the volume fraction of the constituents rather than abruptly changing them over an interface. The gradual variation results in a very efficient material tailored to suit our needs. Such materials are known as functionally graded materials (FGM). The gradation in properties of the material reduces thermal stresses, residual stresses and stress concentration factors. A lot of work has been carried out for the stress and deformation analyses of FGM plate when it is subjected to mechanical and thermo-mechanical load conditions. Suresh and Mortensen¹ investigated the thermo-mechanical deformation of FGM plate and identified the geometry and loading conditions which promote instabilities, shape changes and bifurcation. Qian and Batra² analyzed transient thermo-mechanical deformations of an FGM thick plate loaded by either a thermal load or a combined thermal and mechanical load on the top surface. Liviu and Daniel³ investigated the 2-D steady state heat conduction problems for both isotropic and anisotropic, single and composite, non-linear functionally graded materials (FGMs) with the continuity conditions for the temperature and heat flux at the interfaces. Hui and Qing⁴ developed a mesh-less algorithm for analyzing two-dimensional (2D) thermo-mechanical problems in FGMs. It was concluded that the appropriate graded parameter can lead to low stress concentration and little change in the distribution of stress fields. Ashraf and Daoud⁵ investigated that thermal deflection analysis for FGM plates under uniform, linear and non-linear thermal loading through the thickness and found that the critical buckling temperature difference is proportional to the plate aspect ratio. Mostapha et. al.⁶ presented the buckling analysis of FGM plate under thermal loading and derived closed form solutions for the critical buckling temperatures of plates. Srinivas G et.al.⁷ reported the variation in stresses and deflection under thermo-mechanical loading. Bhandari and Purohit⁸ studied FGM plate when subjected to various types of mechanical loadings e.g. point load, uniform distributed loading. Dai et. al.⁹ and Alshorbagy et. al.¹⁰ developed the equations based on the combination of the first order plate theory and the Von Karman strains. The Von Karman plate theory accounts for moderately large deflections and small strains. It was assumed that the transverse strain components are negligible as compared to the other strain components. Qian and Batra² studied Aluminum- Silicon Carbide FGM, Reddy¹¹ and Bhandari and Purohit¹² studied Aluminum- Zirconia FGM, Ashraf and Daoud⁵ used Aluminum-Alumina as FGM. Sharma¹³ revealed that material gradation affects the stability and failure behavior of FGM

plate considerably and it is concluded that FGM plate with elastic material properties exhibits stable equilibrium path. Xiaohui¹⁴ analyzed the thermoelastic behaviors of functionally graded material plates with various configurations. Senthil and Batra¹⁵, Mostapha et. al.¹⁶ adopted the uncoupled quasi-static thermo-elasticity theory. They derived equilibrium and stability equations of a rectangular plate made of functionally graded material (FGM) under thermal loads based on the higher order shear deformation plate theory.

The studies considered in this paper are concerned with deformation and stress problems of FGM plates accounting for various effects, such as geometric and physical non-linearity and transverse shear deformability subjected to thermo-mechanical loads. The results are determined numerically using finite element method and presented in terms of non dimensional characteristics such as non dimensional deflection, non dimensional tensile stress and non dimensional shear stress.

1. FINITE ELEMENT MODELING TECHNIQUE

The material properties of the FGM vary throughout the thickness as per Power law (P-FGM) and Sigmoid law (S-FGM). The numerical model is developed by breaking up the FGM plate into sufficient no. of "layers" in order to grasp the change in properties. Material properties are calculated using the various volume fraction distribution laws. To ascertain the accuracy and proficiency of the present finite element formulation, two examples have been analysed for thermo-mechanical deformations of the FGM plates Bhandari et.al.¹⁶. It is concluded that the present finite element formulation gives satisfactory accuracy level.

2. THERMAL AND THERMOMECHANICAL ANALYSIS

The thermo-mechanical analysis is conducted for FGM made of combination of metal and ceramic. The metal and ceramic chosen are Aluminum and Zirconia respectively. The Young's modulus for Aluminium is 70 GPa and that for Zirconia is 151 GPa. The coefficient of thermal expansion for Aluminium is $23 \times 10^{-6} / ^\circ\text{C}$ and that for Zirconia is $10 \times 10^{-6} / ^\circ\text{C}$. The Poisson's ratio for both the materials was chosen to be 0.3. The effect of Poisson's ratio on the deformation is much less as compared to that of Young's modulus [Reddy²¹]. The FGM plate is simply supported at all of its edges (SSSS). The thickness of the plate (h) is taken 0.02m. The ratio of the plate side lengths is termed as aspect ratio (a/b). Thermal analysis was performed by applying thermal load on the FGM plate. The ceramic top surface is exposed to a temperature of 100 °C. The lower metallic surface and all the edges are kept at a temperature of 0 °C. The thermomechanical analysis has been performed by applying uniformly distributed load (udl) along with thermal load with varying aspect ratio (a/b). The value of udl (p_0) chosen was equal to $1 \times 10^6 \text{ N/m}^2$. The analysis is performed for various values of the volume fraction exponent (n) in P-FGM and S-FGM. The results are presented in terms of

non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}=u_z/h$), non-dimensional tensile stress ($\overline{\sigma_x}=\sigma_x/p_o$) and non-dimensional shear stress ($\overline{\sigma_{xy}}=\sigma_{xy}/p_o$) where 'u_z' is deflection, 'σ' is stress, 'a' and 'b' are side lengths of plate, and p_o is applied load (N/m²).

3. NUMERICAL RESULTS

The results have been presented for variation in aspect ratio in constant thermal environment and in constant thermal environment under mechanical load:

3.1 Effect of aspect ratio (a/b) in constant thermal environment

In this section, the results of the analysis performed on FGM plate with varying aspect ratio subject to constant thermal environment are discussed. The FGM plate is considered to be simply supported. The effect of volume fractions laws i.e. P-FGM and S-FGM are presented. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}$), non-dimensional tensile stress ($\overline{\sigma_x}$) and non-dimensional shear stress ($\overline{\sigma_{xy}}$).

a. Non-Dimensional Deflection ($\overline{u_z}$)

Fig.1 and Fig. 2 show the effect of variation of aspect ratio (a/b) on non-dimensional deflection (u_z) for simply supported plate under constant thermal environment for P-FGM and S-FGM respectively. The comparison of results reveals the following informations:

- i. In both P-FGM and S-FGM initially upto aspect ratio equals to 2, the deflection increases rapidly and thereafter its value is almost constant.
- ii. In case of pure metallic plate (i.e. $n=\infty$) the deflection is maximum ($\overline{u_z} = 0.87$) while in case of pure ceramic plate, its value is about 0.83 which is incidentally near to the pure metal plate and it is much higher than that of FGM of different configuration ($0 < n < \infty$).
- iii. The non-dimensional deflection in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2, in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional deflection is approximately 0.15 whereas in metal rich region (P-FGM- $n=100$), it is 0.55.
- iv. Comparison of P-FGM and S-FGM for various value of volume fraction exponent 'n' shows that order of deflection in S-FGM is found to lower than P-FGM for a constant value of aspect ratio.

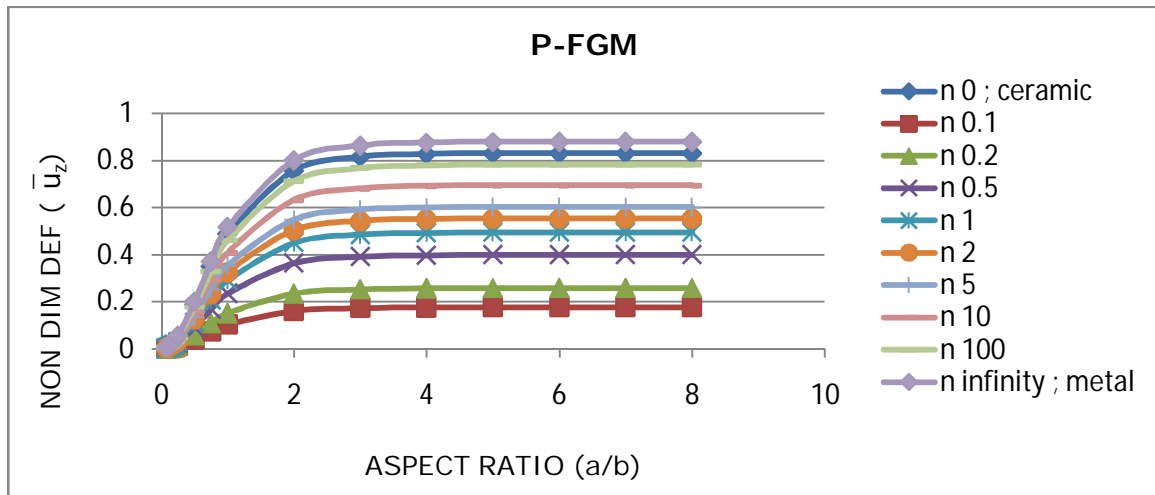


Fig.1:Effect of aspect ratio (a/b) on non-dim.def. (\bar{u}_z) under constant thermal environment(P-FGM)

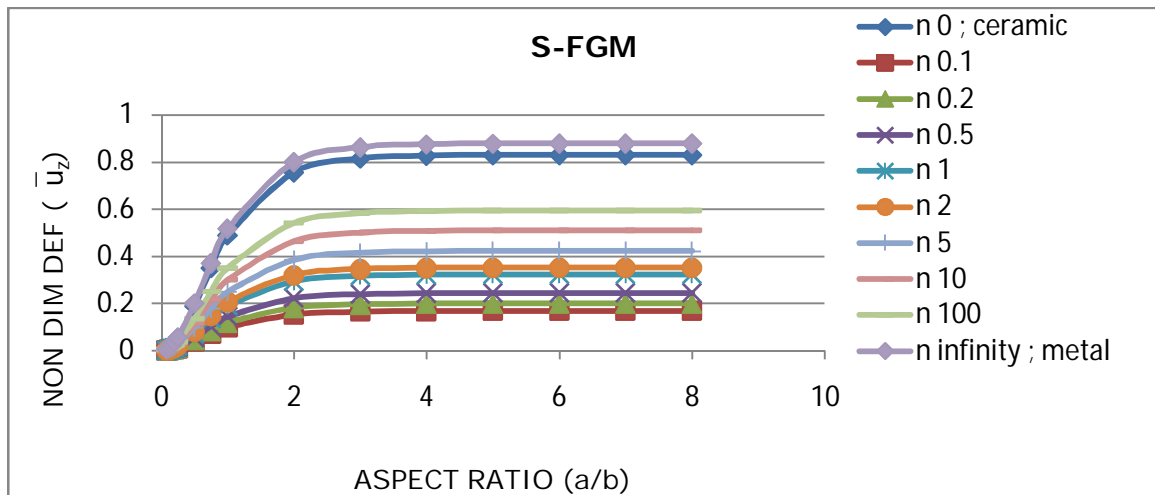


Fig.2:Effect of aspect ratio (a/b) on non-dim. def. (\bar{u}_z) under constant thermal environment(S-FGM)

b. Non-Dimensional Tensile Stress ($\bar{\sigma}_x$)

The numerical results for variation of non-dimensional tensile stress ($\bar{\sigma}_x$) with aspect ratio (a/b) for simply supported plate under constant thermal environment for P-FGM and S-FGM are shown in Fig. 3 and Fig. 4 respectively.

The following observations are made while studying the effect of aspect ratio on tensile stress:

- i. Tensile stress has reducing trend for aspect ratio between 0.25 and 0.75 but for aspect ratio 1 i.e. square plate, the tensile stress increases to the maximum value.
- ii. For aspect ratio more than 1, the tensile stress reduces. Firstly upto aspect ratio 2, decrease is sharp. Beyond this decrease in stress is moderate.
- iii. The non-dimensional tensile stress in metal and ceramic plate is less in value as compared to the FGMs. For example at aspect ratio 1, in ceramic plate (n=0) the non-dimensional tensile stress is

approximately 8 whereas in metal plate ($n=\infty$), it is 12.3. Further the non-dimensional tensile stress in P-FGM-n0.1 is approximately 19 whereas in P-FGM-n100, it is 106.

iv. The non-dimensional tensile stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2, in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional tensile stress is approximately 9 whereas in metal rich region (P-FGM- $n=100$), it is 52.

v. No significant change in non-dimensional tensile stress has been noticed for the aspect ratio up to 0.25.

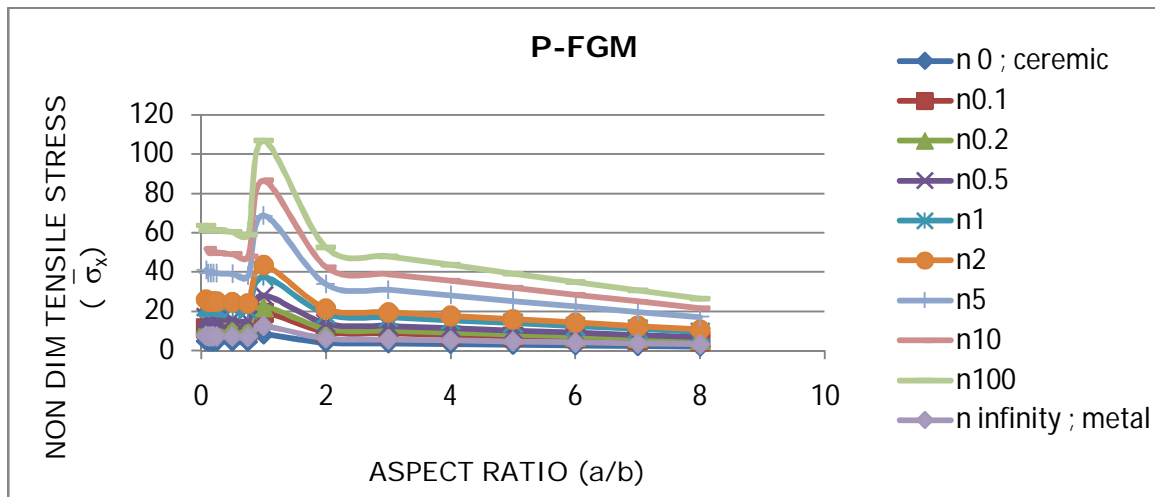


Fig.3: Effect of aspect ratio (a/b) on non-dim. tensile stress $(\bar{\sigma}_x)$ under constant thermal environment(P-FGM)

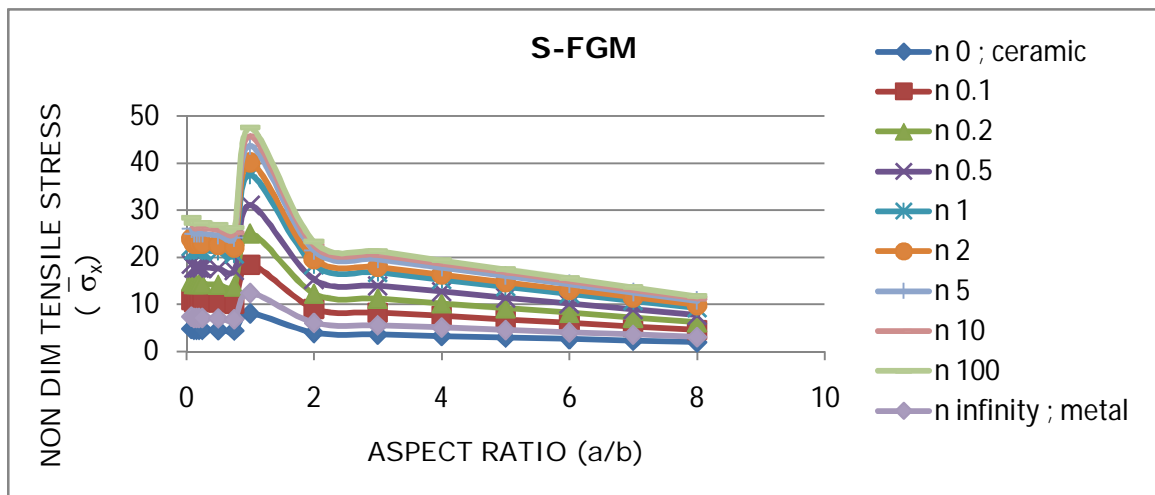


Fig.4: Effect of aspect ratio (a/b) on non-dim. tensile stress $(\bar{\sigma}_x)$ under constant thermal environment(S-FGM)

c. Non-dimensional Shear Stress $(\bar{\sigma}_{xy})$

Fig.5 and Fig. 6 show the effect of variation of aspect ratio (a/b) on non-dimensional shear stress $(\bar{\sigma}_{xy})$ for simply supported plate under constant thermal environment for P-FGM and S-FGM respectively.

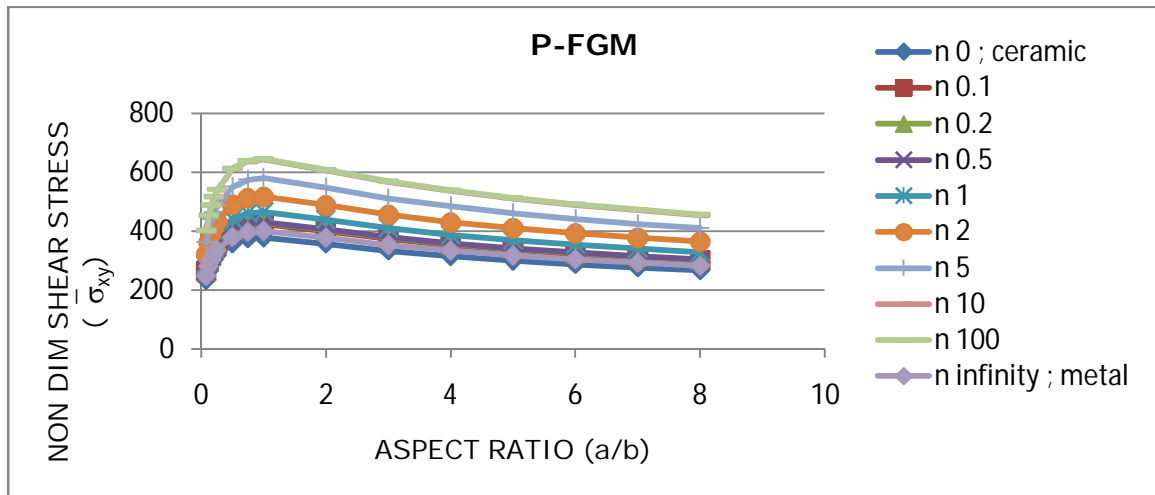


Fig.5: Effect of aspect ratio (a/b) on non-dim. shear stress ($\overline{\sigma_{xy}}$) under constant thermal environment (P-FGM)

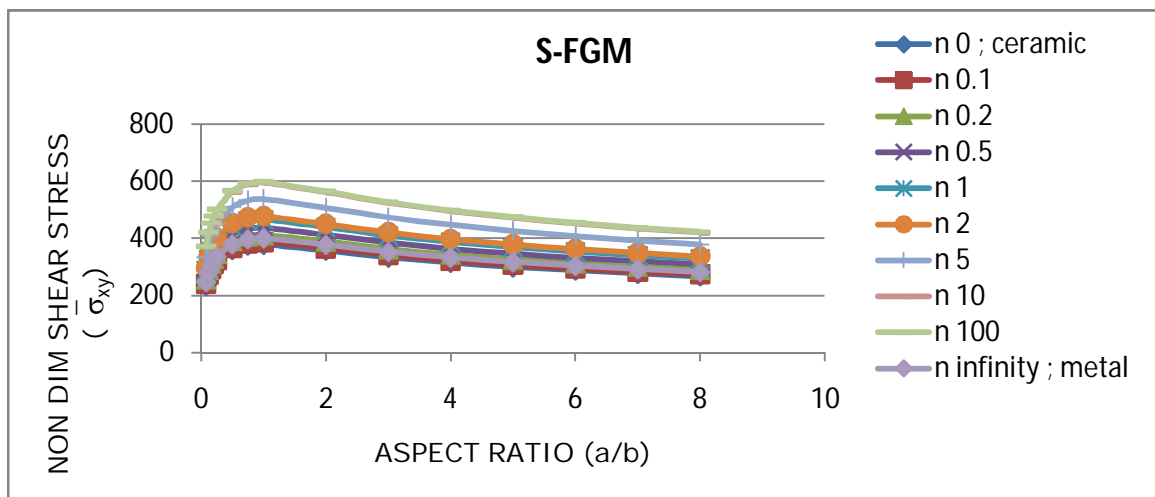


Fig.6: Effect of aspect ratio (a/b) on non-dim. shear stress ($\overline{\sigma_{xy}}$) under constant thermal environment (S-FGM).

The following observations are made by comparing the non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various values of aspect ratio and types of FGM:

- i. There is a steep rise of non-dimensional shear stress upto aspect ratio 1, beyond that further increase in aspect ratio have diminishing effect on non-dimensional shear stress. Hence in the case of square plate the non-dimensional shear stress is maximum.
- ii. The non-dimensional shear stress is found to be minimum in metal and ceramic plate for all values of aspect ratio.
- iii. The non-dimensional shear stress in metal and ceramic plate is less in value as compared to the FGMs. For example at aspect ratio 1, in ceramic plate ($n=0$) the non-dimensional shear stress is approximately 378 whereas in metal plate ($n=\infty$), it is 400. Further the non-dimensional shear stress in P-FGM-n0.1 is approximately 423 whereas in P-FGM-n100, it is 644.

iv. The non-dimensional shear stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2, in ceramic rich region (P-FGM-n=0.1) the non-dimensional shear stress is approximately 399 whereas in metal rich region (P-FGM-n=100), it is 644.

3.2 Effect of aspect ratio (a/b) in constant thermal environment under mechanical load

In this section, the results of the analysis performed on FGM plate with varying aspect ratio subject to constant uniformly distributed load in constant thermal environment are discussed. The FGM plate is considered to be simply supported. The effect of values of various volume fraction exponent and various laws i.e. P-FGM and S-FGM are studied. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection (\bar{u}_z), non-dimensional tensile stress ($\bar{\sigma}_x$), non-dimensional shear stress ($\bar{\sigma}_{xy}$), strain (e_x) and shear strain (e_{xy}).

a. Non-Dimensional Deflection (\bar{u}_z)

Fig.7 and Fig. 8 show the effect of variation of aspect ratio (a/b) on non-dimensional deflection (\bar{u}_z) for simply supported FGM plate under uniformly distributed load in constant thermal environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent ‘n’ for P-FGM and S-FGM has been presented.

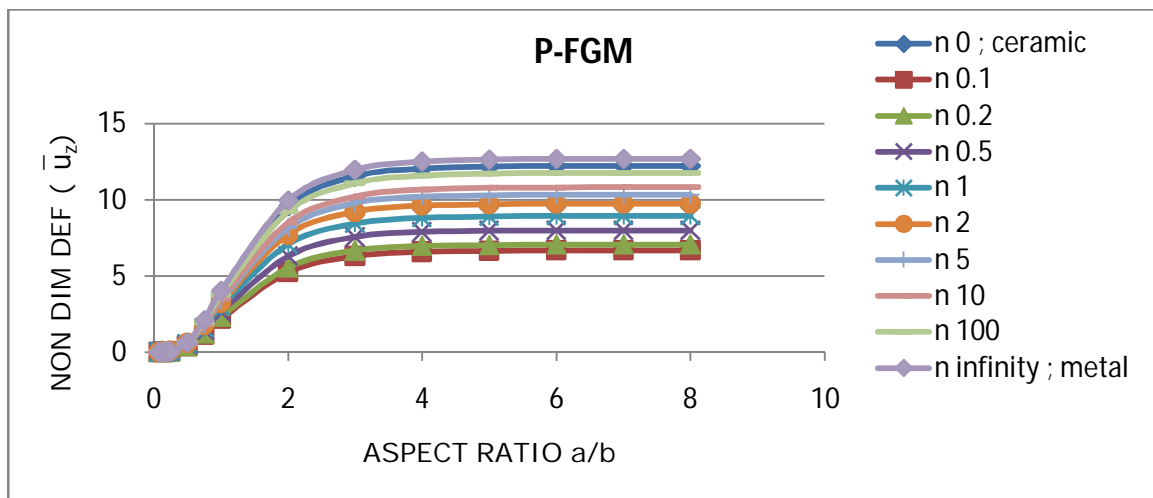


Fig.7: Effect of aspect ratio (a/b) on non-dim. def. (\bar{u}_z) under constant thermo-mechanical load (P-FGM)

The effect of variation of aspect ratio (a/b) of a FGM plate in a constant thermal environment and mechanical load reveals the following information:

- i. In both P-FGM and S-FGM initially upto aspect ratio equals to 4, the deflection increases and thereafter its value is almost constant.
- ii. In case of pure metallic plate (i.e. $n = \infty$) the deflection is maximum ($\bar{u}_z = 12.8$) while in case of pure ceramic plate, its value is about 12.1.

- iii. The non-dimensional deflection values of FGM plates (i.e. $0 < n < \infty$) are much lower than that of fully metal plate.
- iv. The non-dimensional deflection in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 1 (square plate), in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional deflection is approximately 2.1 whereas in metal rich region (P-FGM- $n=100$), it is 3.76.
- v. The value of non-dimensional deflection under thermo-mechanical load is less than that of under pure mechanical load. For example at aspect ratio 1 (square plate), the non-dimensional deflection under pure mechanical load, for P-FGM- $n=100$, is 3.9 whereas under thermo-mechanical load it is 3.76.
- vi. As the volume fraction exponent 'n' increases the non-dimensional deflection increases. This is due to the fact that the coefficient of thermal expansion increases as 'n' increases.

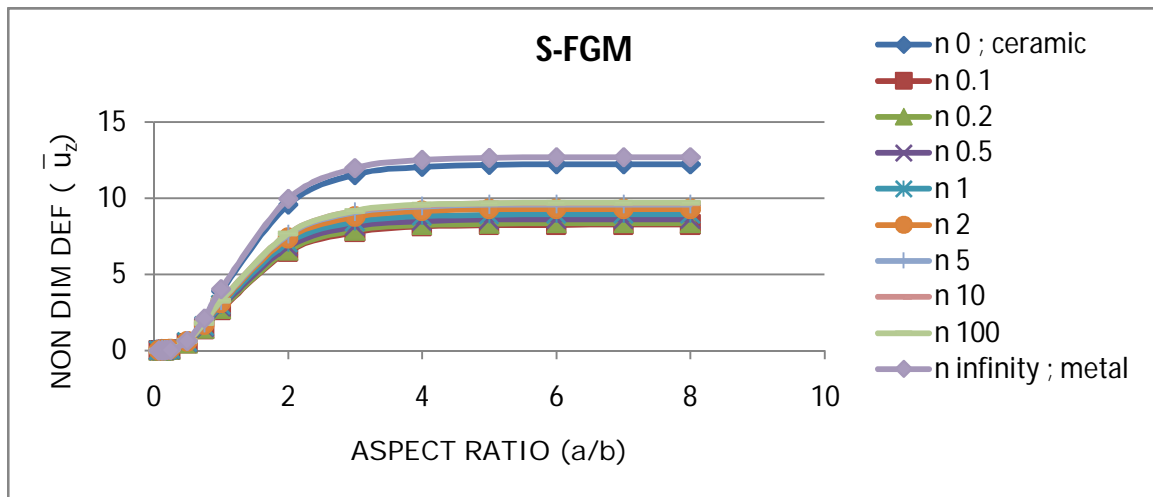


Fig.8: Effect of aspect ratio (a/b) on non-dim. def. (\bar{u}_z) under constant thermo-mechanical load for (S-FGM)

b. Non-Dimensional Tensile Stress ($\bar{\sigma}_x$)

The numerical results for variation of non-dimensional tensile stress ($\bar{\sigma}_x$) with aspect ratio (a/b) for simply supported plate under uniformly distributed load in constant thermal environment for P-FGM and S-FGM are shown in Fig. 9 and Fig. 10 respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.

The following observations are made while studying the effect of aspect ratio on direct tensile stress:

- i. The non-dimensional tensile stress increases, reaches a maximum value for aspect ratio 1. It shows that the maximum non-dimensional tensile stress occurs for square plate.
- ii. As the aspect ratio increase beyond 1 the non-dimensional tensile stress reduces moderately upto aspect ratio 2. Beyond the aspect ratio 2, non-dimensional tensile stress becomes constant. Its peak value is about 610 and 540 in case of P-FGM and S-FGM respectively.

- iii. The non-dimensional tensile stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 1 (square plate), in ceramic rich region (P-FGM-n=0.1) the non-dimensional tensile stress is approximately 418 whereas in metal rich region (P-FGM-n=100), it is 613.
- iv. The value of non-dimensional tensile stress under thermo-mechanical load is less than that of under pure mechanical load. For example at aspect ratio 1 (square plate), the non-dimensional tensile stress under pure mechanical load, for P-FGM-n100, is 1042 whereas under thermo-mechanical load it is 613.

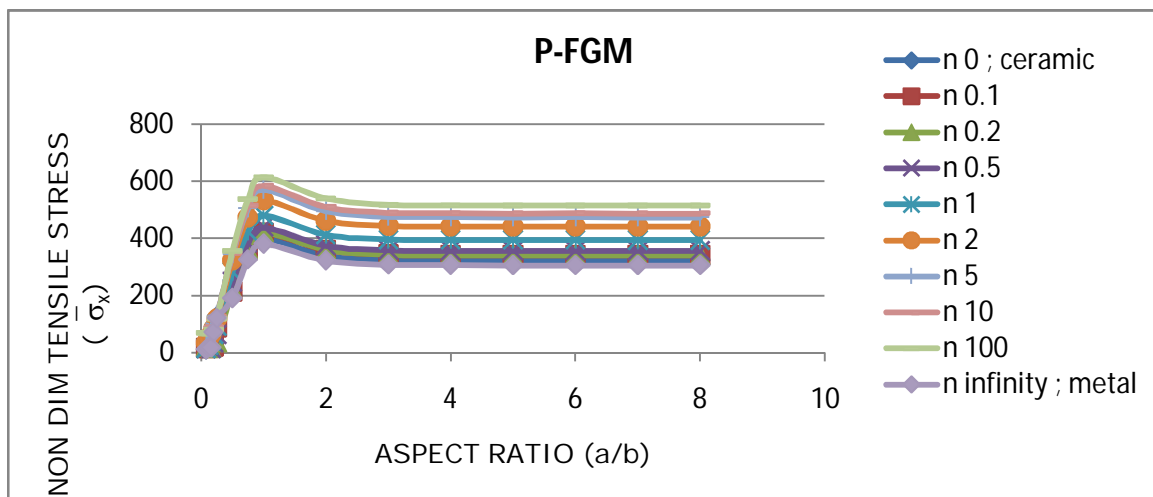


Fig.9: Effect of aspect ratio (a/b) on non-dim. tensile stress ($\overline{\sigma_x}$) under constant thermo-mechanical load (P-FGM)

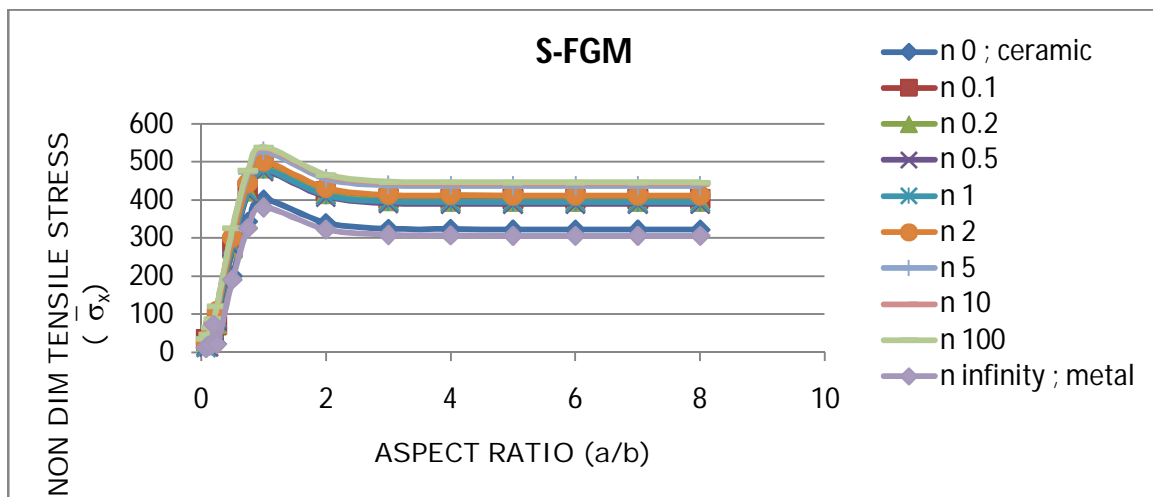


Fig.10: Effect of aspect ratio (a/b) on non-dim. tensile stress ($\overline{\sigma_x}$) under constant thermo-mechanical load (S-FGM)

c. *Non-dimensional Shear Stress ($\overline{\sigma_{xy}}$)*

Fig. 11 and Fig. 12 show the effect of variation of aspect ratio (a/b) on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under uniformly distributed load in constant thermal

environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent ‘n’ for P-FGM and S-FGM has been presented.

The following observations are made by comparing the non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various values of aspect ratio and types of FGM:

- i. In both P-FGM and S-FGM initially upto aspect ratio equals to 2, the non-dimensional shear stress ($\overline{\sigma_{xy}}$) increases rapidly and thereafter its value is almost constant.
- ii. In case of pure metallic plate (i.e. $n=\infty$), for square plate, the non-dimensional shear stress ($\overline{\sigma_{xy}}$) is minimum ($\overline{\sigma_{xy}} = 517$) while in case of pure ceramic plate, its value is about 544 which is near to the pure metal plate and it is lower than that of FGM of different configuration ($0 < n < \infty$).

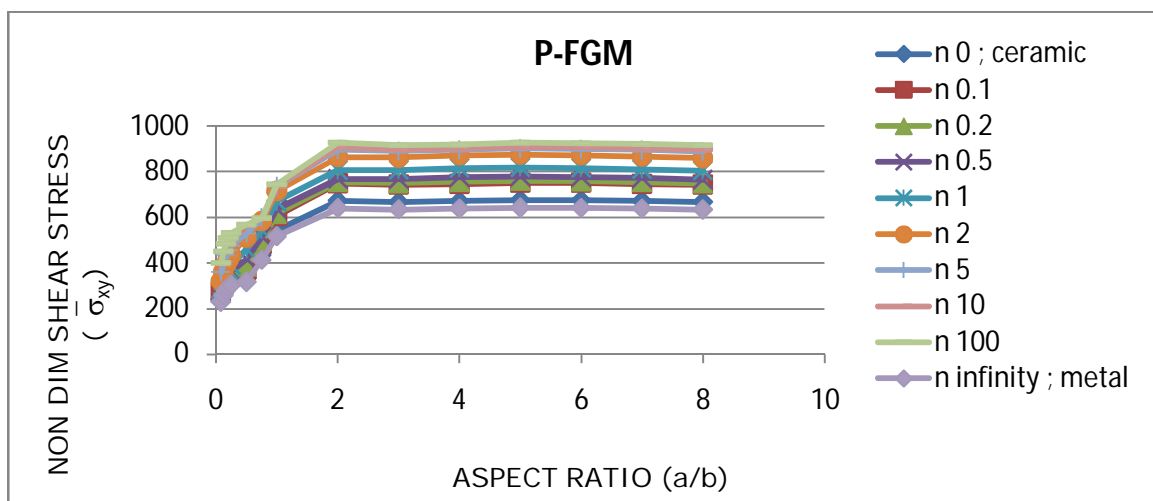


Fig.11: Effect of aspect ratio (a/b) on non-dim. shear stress ($\overline{\sigma_{xy}}$) under constant thermo-mechanical load (P-FGM)

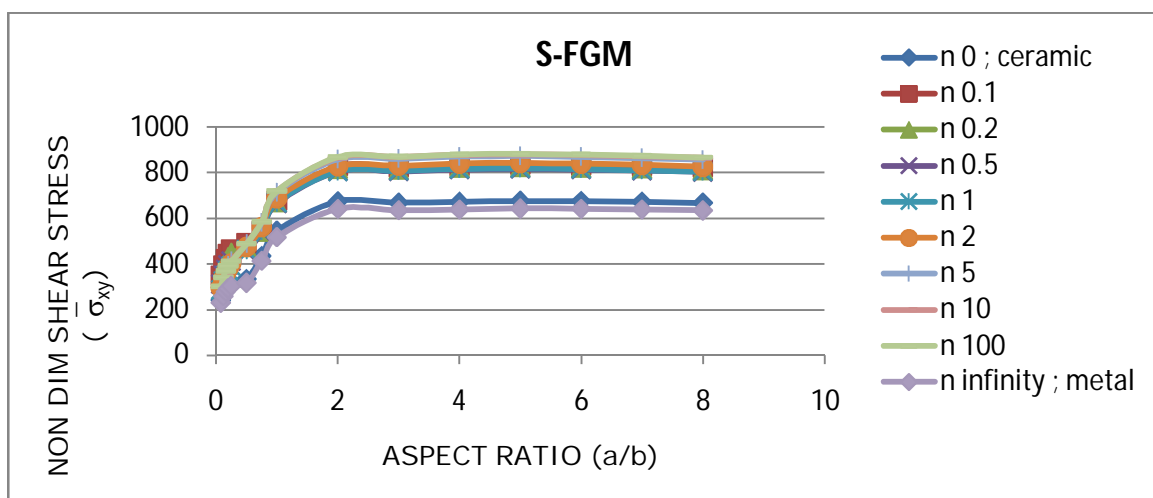


Fig.12: Effect of aspect ratio (a/b) on non-dim. shear stress ($\overline{\sigma_{xy}}$) under constant thermo-mechanical load (S-FGM)

iii. The non-dimensional shear stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2, in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional shear stress is approximately 749 whereas in metal rich region (P-FGM- $n=100$), it is 927.

iv. The value of non-dimensional shear stress under thermo-mechanical load is more than that of under pure mechanical load. For example at aspect ratio 1 (square plate), the non-dimensional shear stress under pure mechanical load, for P-FGM- $n=100$, is 683 whereas under thermo-mechanical load it is 745.

4. CONCLUSION

Thermal and thermo mechanical responses of functionally graded ceramic-metal plates with varying aspect ratio are analyzed. The non-dimensional deflection in the ceramic rich portion may be comparable to that in the metal rich region. The non-dimensional deflection is maximum for the case of pure metal ($n = \infty$) and pure ceramic ($n = 0$). The non-dimensional deflection of both the metallic and the ceramic plates is higher in magnitude than the graded plates. In the presence of the temperature field, compression occurs at the top surface while tension is at the bottom surface. Except fully ceramic or fully metal plates, the stress distribution of FGM plates has a similar trend. In both thermal and thermo mechanical the maximum tensile stress occurs for square plate. The tensile stress keeps on reducing with increase in aspect ratio in case of thermal environment. The non-dimensional shear stress reaches a constant value in thermo mechanical environment but it reduces continuously in pure thermal environment. The non-dimensional shear stress for S-FGM remains closer for various values of 'n' as compared to that of the P-FGM. The work can be extended for variation in load, loading pattern, boundary conditions and other ceramic metal combinations.

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