

## *International Journal of Scientific Research and Reviews*

### **Equilibrium Dynamics of one dimensional Plasma**

**Grima Dhingra\***

Department of Physics, Maharshi Dayanand University, Rohtak, Haryana-124001, India,  
Email: [grimadhingra@gmail.com](mailto:grimadhingra@gmail.com)

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#### **ABSTRACT**

The detailed dynamical structure factors for weakly coupled one dimensional plasma have been computed through involved space-time dependant correlation functions. Theoretical investigations have been performed for a wide range of wave-vectors: 2.0 to 10.0  $\text{cm}^{-1}$ . Dynamical modes for plasma have been obtained as singularities of dynamical structure factor and yield dispersion relation. The dynamical modes have also been investigated for their temperature and density dependence and are observed to be altered significantly with change in number density of constituent particles.

**KEYWORDS:** dynamical structure factor, dispersion relation, correlation functions, degenerate plasma

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#### **\*Corresponding Author:**

**Grima Dhingra,**

Assistant Professor

Department of Physics,

Maharshi Dayanand University,

Rohtak-124001, India.

Mob.-7404220854, Email: [grimadhingra@gmail.com](mailto:grimadhingra@gmail.com)

## INTRODUCTION

Collective modes in any condensed matter are representatives of the degree of correlations present between its constituent particles. They are capable of describing the whole dynamical picture of the concerned matter. The experimental measurement of such modes, however, suffers from a number of kinematical restrictions and is hard to measure, specifically for one dimensional plasma systems which are apparently unrealizable. Such kind of plasma systems however, can now be practically realized through modern fabrication techniques like, lithography, photolithography, x-ray lithography etc<sup>1-10</sup>. In such a fabrication process of (e.g. MOSFET) hetero-structures on semiconductor surfaces, motion of electrons can be restricted in two directions where one dimension is negligible as compared to another dimension and thus creates a quasi single dimensional system.

A one dimensional system comprised of two kinds of mobile constituents, positive and negative charge carriers, where, the charges on two components are of opposite polarity but are equal in quantity so as to maintain the neutrality of the system as a whole. Masses of two components may differ, with negative component carrying electronic charge & electronic mass whereas positive component carries a unit positive charge & some finite mass. Coupling between the mobile components in such a system can be described through coupling parameter  $\Gamma$  ( $=ne^2/k_B T$ ), which is the ratio of electronic to kinetic energies. Here, 'T' is temperature, 'n' is number density per unit length and 'e' is electronic charge. Depending upon the value of coupling parameter, any plasma can be classified as weakly coupled plasma ( $\Gamma \leq 1$ ) or a strongly coupled plasma ( $\Gamma > 1$ ).

In the present study, theoretically predicted detailed dynamical structure factors and current correlation functions of such a weakly coupled one dimensional plasma have been reported. Collective modes obtained from dynamical structure factors have also been investigated.

## MATHEMATICAL FORMALISM

Dynamical structure factor is related to complex dielectric function by fluctuation-dissipation theorem as follows:

$$S(\kappa, \omega) = \frac{1}{\pi\omega} \frac{\kappa^2}{\kappa^2} \left[ \text{Im} 4\pi\alpha(\kappa, \omega) \left\{ \frac{1 + 4\pi\alpha(\kappa, \omega)}{\epsilon(\kappa, \omega)} \right\} \right] \quad (1)$$

Where,  $\alpha(\kappa, \omega)$  is polarizability and

$$4\pi\alpha(\kappa, \omega) = \text{Re} 4\pi\alpha(\kappa, \omega) + i \text{Im} 4\pi\alpha(\kappa, \omega) \quad (2)$$

Expression (1) can be re-written as follows:

$$S(\kappa, \omega) = \frac{1}{\pi \omega} \frac{\kappa^2}{\kappa_-^2} \left[ \frac{1}{\varepsilon_1(\kappa, \omega)^2 + \varepsilon_2(\kappa, \omega)^2} \right] P(\kappa, \omega) \tag{3}$$

Where,

$$P(\kappa, \omega) = \text{Im} 4\pi\alpha(\kappa, \omega) \left[ (1 + \text{Re} 4\pi\alpha(\kappa, \omega)) \varepsilon_1(\kappa, \omega) + \varepsilon_2(\kappa, \omega) \text{Im} 4\pi\alpha(\kappa, \omega) \right] + \text{Re} 4\pi\alpha(\kappa, \omega) \left[ \varepsilon_1(\kappa, \omega) \text{Im} 4\pi\alpha(\kappa, \omega) - \varepsilon_2(\kappa, \omega) (1 + \text{Re} 4\pi\alpha(\kappa, \omega)) \right] \tag{4}$$

$$\text{And, } \varepsilon(\kappa, \omega) = \varepsilon_1(\kappa, \omega) + i\varepsilon_2(\kappa, \omega) \tag{5}$$

is complex dielectric function<sup>12-16</sup> given by expression:

$$\varepsilon_{\pm}^{\omega}(\kappa, \omega) = \sqrt{\frac{\pi}{2}} \frac{\omega_{p\pm}^2}{\kappa^2 v_{\pm}^2} \frac{m_{\pm} v_{\pm}}{h\kappa} \left[ e^{-\left(\frac{\omega}{\sqrt{2}\kappa v_{\pm}} - \frac{h\kappa}{2\sqrt{2}m_{\pm}v_{\pm}}\right)^2} - e^{-\left(\frac{\omega}{\sqrt{2}\kappa v_{\pm}} + \frac{h\kappa}{2\sqrt{2}m_{\pm}v_{\pm}}\right)^2} \right] \tag{6}$$

In expression (4),

$$\kappa_-^2 = \frac{\omega_{p-}^2}{v_-^2}, \text{ where, } \omega_{p-} = \sqrt{\pi n_- e^2 \kappa^2 (\ln \kappa) / m_-} \text{ is plasma frequency of electron}^{17-23} \text{ and}$$

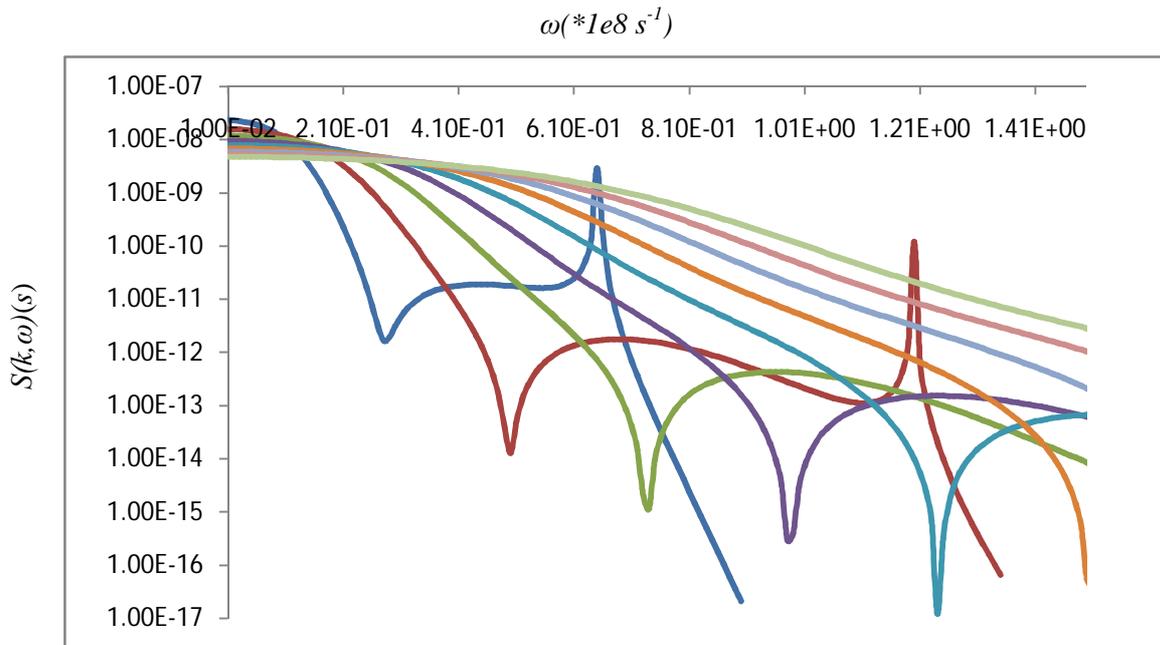
$$v_- = \sqrt{k_B T_- / m_-} \text{ is thermal velocity of negative component.}$$

Current2 correlation function is related to dynamical structure factor by expression:

$$C(k, \omega) = \omega^2 S(k, \omega) \tag{7}$$

## RESULTS AND DISCUSSION

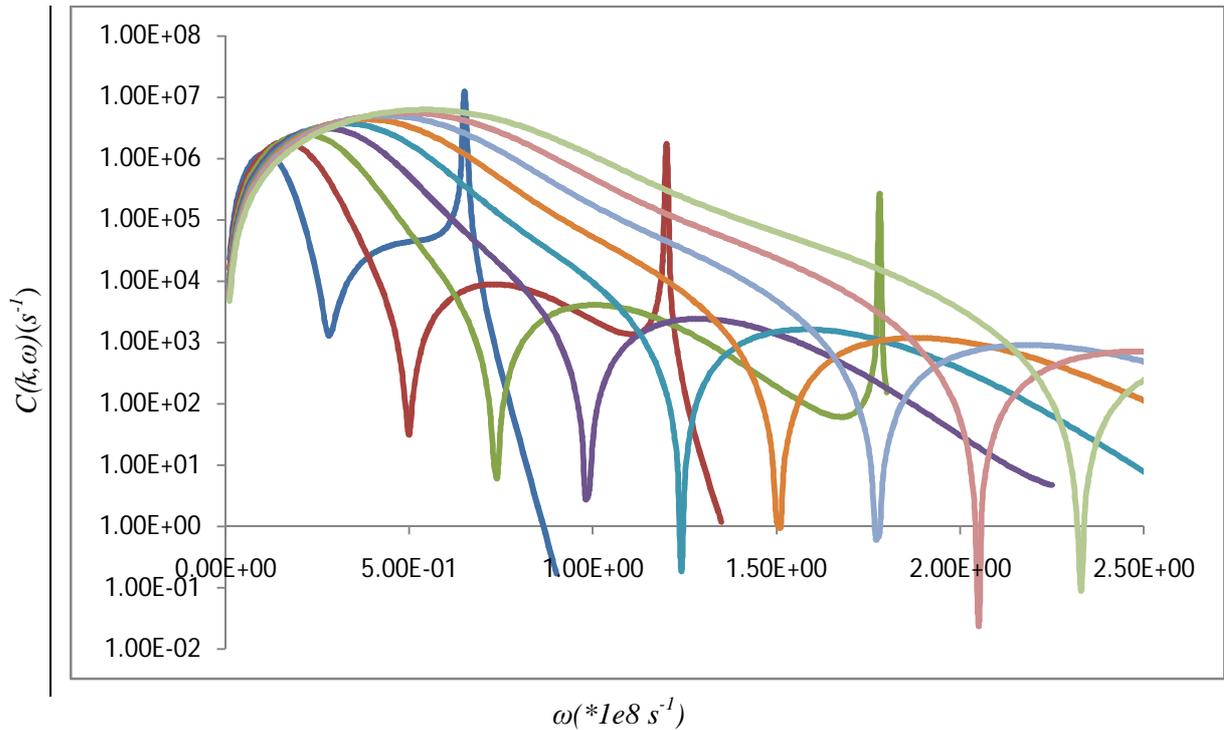
Dynamical structure factor is the physical quantity which is capable of yielding the complete information about the dynamics of a given condensed system, particularly in their fluid phase. These structure factors are Fourier transforms of space-time correlation functions which account for space and time dependant correlations present between the movements of constituent particles. Detailed dynamical structure factors for one dimensional weakly degenerate plasma have been calculated using expression (3) and expression (5). The computations have been performed for a huge wave-vector range, 2.0 to 10.0 cm<sup>-1</sup> where, each of the oppositely charged 1.4x10<sup>6</sup> particles are assumed to occupy per centimetre of the plasma, while it is assumed to be at a temperature of 300 K. The generated theoretical results are shown in figure 1 as their variation against frequency, ω, for nine different wave-vector values, κ = 2.0 cm<sup>-1</sup>, 3.0 cm<sup>-1</sup>, 4.0 cm<sup>-1</sup>, 5.0 cm<sup>-1</sup>, 6.0 cm<sup>-1</sup>, 7.0 cm<sup>-1</sup>, 8.0 cm<sup>-1</sup>, 9.0 cm<sup>-1</sup> and 10.0 cm<sup>-1</sup>, with (—), (—), (—), (—), (—), (—), (—), (—), (—) respectively.



**Figure1:** Variation of dynamical structure factor,  $S(k,\omega)$  with frequency  $\omega$  for different values of  $k$ : (—)  $2.0 \text{ cm}^{-1}$ , (—)  $3.0 \text{ cm}^{-1}$ , (—)  $4.0 \text{ cm}^{-1}$ , (—)  $5.0 \text{ cm}^{-1}$ , (—)  $6.0 \text{ cm}^{-1}$ , (—)  $7.0 \text{ cm}^{-1}$ , (—)  $8.0 \text{ cm}^{-1}$ , (—)  $9.0 \text{ cm}^{-1}$ , (—)  $10.0 \text{ cm}^{-1}$ .

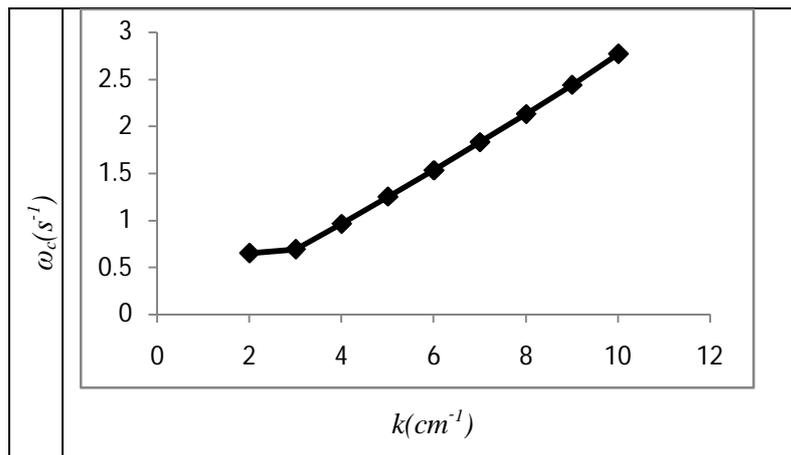
This can be observed from the figure that the dynamical structure factors for a whole  $\kappa$ -range are doubly peaked structures (for  $\omega \geq 0$ ), where first peak lies at  $\omega=0$  and second peak is at finite frequency value ( $\omega=\omega_c \neq 0$ ), called the Rayleigh peak. However, Rayleigh peaks for smaller  $\kappa$ -values,  $\kappa < 5.0 \text{ cm}^{-1}$  are much more sharp, whereas with increase in  $\kappa$ , for  $\kappa \geq 5.0 \text{ cm}^{-1}$ , the peak at  $\omega=\omega_c$  becomes more and more wider having maximum FWHM(full width at half maximum) for  $\kappa = 10.0 \text{ cm}^{-1}$ . The first peak i.e. the Brillouin peak at  $S(\kappa, \omega \rightarrow 0)$ , on the other hand decreases in magnitude with increase in wave-vector  $\kappa$ . The frequency  $\omega_c$ , here, is the peak position of Rayleigh peak and represents the frequency of collective mode at particular wave-vector.

In Figure2, current2 correlation functions corresponding to dynamical structure factors as obtained from expression (7) has been calculated and are plotted as their variation against frequency,  $\omega$ , for different values of  $\kappa$ : (—)  $2.0 \text{ cm}^{-1}$ , (—)  $3.0 \text{ cm}^{-1}$ , (—)  $4.0 \text{ cm}^{-1}$ , (—)  $5.0 \text{ cm}^{-1}$ , (—)  $6.0 \text{ cm}^{-1}$ , (—)  $7.0 \text{ cm}^{-1}$ , (—)  $8.0 \text{ cm}^{-1}$ , (—)  $9.0 \text{ cm}^{-1}$ , (—)  $10.0 \text{ cm}^{-1}$ . The current2 correlation functions are well peaked structures and exhibit peaks corresponding to collective mode frequencies (i.e.  $\omega=\omega_c$ ), for  $C(\kappa, \omega)$ . The corresponding peaks for larger  $\kappa$  values are more wide and damped and are shifted towards larger values of  $\omega$ .



**Figure2:** Variation of current correlation function,  $C(k,\omega)$  with frequency  $\omega$  for different values of  $k$ : (—)2.0  $\text{cm}^{-1}$ , (—) 3.0  $\text{cm}^{-1}$ , (—) 4.0  $\text{cm}^{-1}$ , (—) 5.0  $\text{cm}^{-1}$ , (—) 6.0  $\text{cm}^{-1}$ , (—) 7.0  $\text{cm}^{-1}$ , (—) 8.0  $\text{cm}^{-1}$ , (—) 9.0  $\text{cm}^{-1}$ , (—) 10.0  $\text{cm}^{-1}$ .

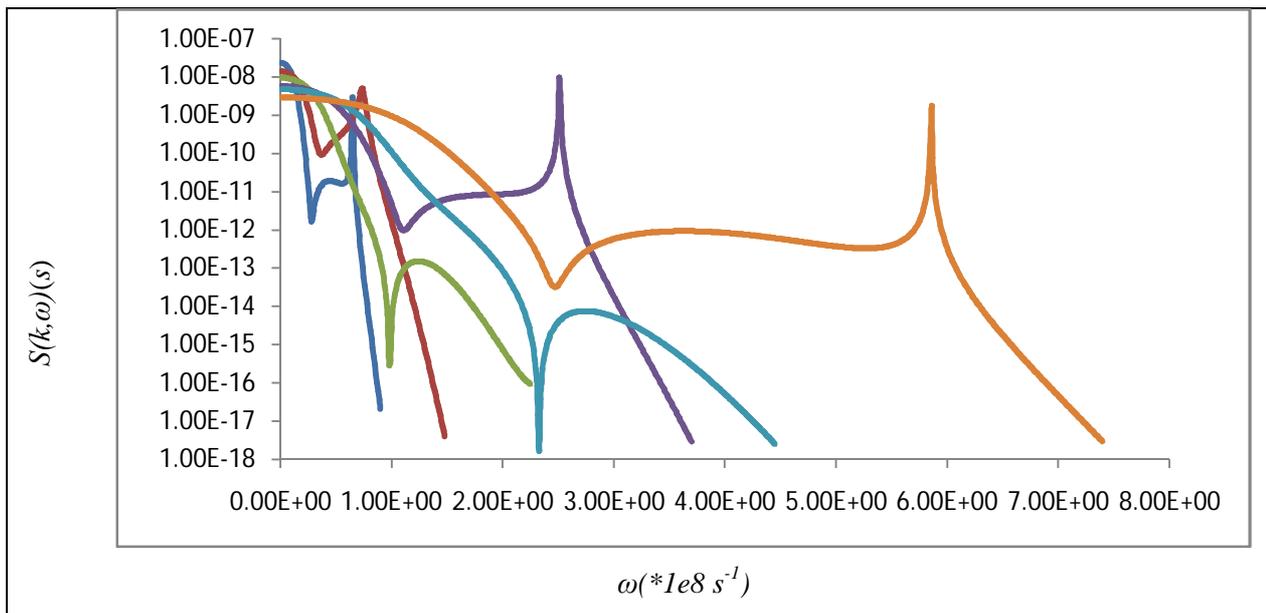
Figure3, shows the variation of collective mode frequencies with wave-vector  $\kappa$  with solid curve. The collective mode frequencies are as obtained from current correlation functions. The collective frequencies are found to increase with increase in  $\kappa$  and for  $\kappa > 4.0 \text{ cm}^{-1}$ , increases linearly with wave-vector. Thus, it may be concluded that the two component plasma follow linear dispersion relation for larger  $\kappa$  values.



**Figure3:** Variation of collective mode frequencies,  $\omega_c$  with wave-vector,  $k$ .

Temperature and densities are critical parameters which determine the interactions present between its particles, their kinetic energies and hence, the real physical picture of any condensed matter. Dynamics of such a constituency of mobile particles, hence, is also expected to be

dependent upon its temperature and number density. Such a temperature and density dependence has been investigated in the present study and the results have been shown in figure4 and figure5. Figure4, explores the temperature dependant variation of dynamical structure factors for three  $\kappa$ -values,  $2.0 \text{ cm}^{-1}$  (—) & (—),  $5.0 \text{ cm}^{-1}$  (—) & (—) and  $10.0 \text{ cm}^{-1}$  (—) & (—) at temperatures,  $T=300\text{K}$  &  $800\text{K}$  respectively. The figure clearly indicates that with increase in temperature, positions of Rayleigh peak for all  $\kappa$ -values shift towards right, i.e. higher  $\omega$ -values. One may conclude from the graph that collective modes frequencies are temperature dependant and increases numerically with increase in temperature.



**Figure4:** Variation of dynamical structure factor,  $S(k,\omega)$  with frequency  $\omega$  at temperatures  $T_1=300 \text{ K}$  &  $T_2=800 \text{ K}$ , different values of  $k$ :  $k=2.0 \text{ cm}^{-1}$  (—)  $T_1$  & (—) $T_2$ ;  $k=5.0 \text{ cm}^{-1}$  (—) $T_1$  & (—) $T_2$ ;  $k=10.0 \text{ cm}^{-1}$  (—) $T_1$  & (—) $T_2$

Similarly, variation of dynamical structure factors with frequency  $\omega$  are plotted in Figure5 for two values of wave-vectors,  $\kappa = 5.0 \text{ cm}^{-1}$  with (—) & (—) and  $\kappa = 9.0 \text{ cm}^{-1}$  with (—) & (—) and are compared with the corresponding results at  $n_2=2*n=2.8 \times 10^6 \text{ cm}^{-1}$ . This can be observed from the figure5 that increase in number density of particles results in increased frequencies of collective modes, as the peak in the  $S(\kappa,\omega)$  at  $\omega=\omega_c$  shifts towards the larger  $\omega$ -values for higher number density of particles.

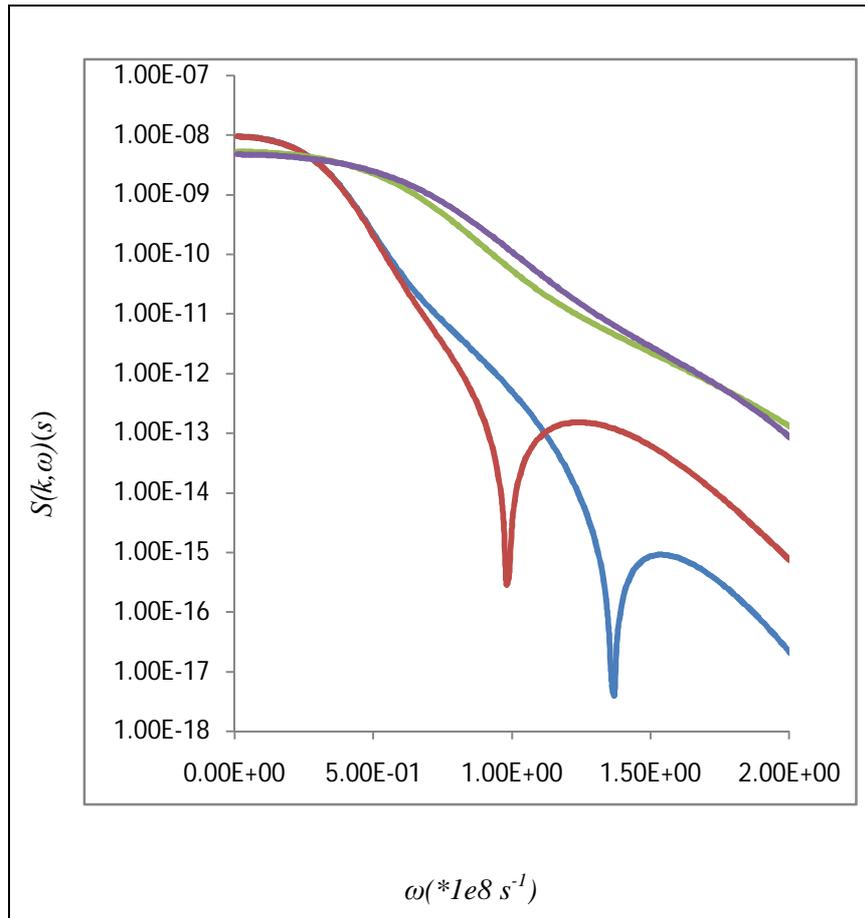


Figure5: Variation of dynamical structure factor,  $S(k, \omega)$  with frequency  $\omega$  for number densities  $n=1.4 \times 10^6 \text{ cm}^{-3}$  &  $n_2=2 \times n=2.8 \times 10^6 \text{ cm}^{-3}$ , different values of  $k: k=5.0 \text{ cm}^{-1}$  (—)  $n_1$  & (—)  $n_2$ ;  $k=9.0 \text{ cm}^{-1}$  (—)  $n_1$  & (—)  $n_2$

## CONCLUSION

It may be concluded that the dynamical structure factors of one dimensional two component plasma can be obtained from its dielectric functions. The obtained detailed dynamical structure factors can yield corresponding current correlation functions and the associated collective mode frequencies of the plasma. The dynamical structure factors as well as collective modes exhibit dependence upon temperature and number density.

## REFERENCE

1. Illing M., Bacher G., Kummel T. and Forchel A., Hommel D., Jobst B. and Landwehr G.,
2. Fabrication of CdZnSe/ZnSe quantum dots and quantum wires by electron beam lithography and wet chemical etching, Journal of vacuum science & technology B 1995;13(6):2792-2796.
3. Simmonds P.J., Holmes S.N., Beere H.E., Farrer I., Sfigapis F., Ritchie D.A. and Pepper M., Molecular beam epitaxy of high

- mobility In<sub>0.75</sub>Ga<sub>0.25</sub>AsIn<sub>0.75</sub>Ga<sub>0.25</sub>As for electron spin transport applications, Journal of vacuum science & technology B 2009;27(4):2066-2078.
4. Hansen W., Horst M., Kotthaus J.P., Merkt U., Sikorski Ch. and Ploog K., Intersubband resonance in quasi one-dimensional inversion channels, Phys. Rev Lett 1987;58(24):2586-2589
  5. Scott-Thomas J.H.F., Field S. B., Kastner M. A., Smith H. I. and Antoniaddis D. A., Conductance Oscillations Periodic in the Density of a One-Dimensional Electron Gas Phys Rev Lett 1989;62(5):583-586.
  6. Pepper M. and Uren M.J., The Wigner glass and conductance oscillations in silicon inversion layers, J Phys C 1982;15(20):L617-L626.
  7. Demel T., Heitmann D., Grambow P. and Ploog K., One-dimensional electronic systems in ultrafine mesa-etched single and multiple quantum well wires, Appl phys Lett 1988;53(22):2176-2178.
  8. Fowler A.B., Hartstein A. and Webb R.A., Conductance in Restricted-Dimensionality Accumulation Layers, Phys Rev Lett 1982;48(3):196-198.
  9. Das Sharma S. and Wu-yan Lai, Screening and elementary excitations in narrow-channel semiconductor microstructures, Phys Rev B 1985;32(2):1401-1404.
  10. Hartstein A., Webb R.A., Fowler A.B. and Wainer J.J., One-dimensional conductance in silicon mosfet's, Surf Sci 1984;142:1-13.
  11. Skocpol W.J., Jackel L.D., Hu E.L., Howard R.E. and Fetter L.A. 1982, One-Dimensional Localization and Interaction Effects in Narrow (0.1- $\mu\text{m}$ ) Silicon Inversion Layers, Phys Rev Lett;49(13):951-956.
  12. Ichimaru S., Theory of fluctuations in a plasma, Ann Phys 1962;20(1):78-118.
  13. Ichimaru S., Statistical Plasma Physics, Addison Wesley Publishing Company: Boston, United states;1992.
  14. Dendy R.O., Plasma Dynamics, Oxford: Clarendon;1990.
  15. Tewari S.P. & Sood J., complex dielectric function and collective dynamics of one-dimensional weakly coupled quantum and classical hot plasmas, Indian J of Pure & Appl Phys 2004;42(7):518-523.
  16. Tewari S.P. & Sood J., Reflectivity of low energy photons from one and two dimensional rare hot quantum and classical plasmas Indian J of Pure & Appl Phys 2006;44(12):927-929.

17. Tewari S.P., Sood J. & Dhingra G., Temperature dependent positron annihilation in one dimensional weakly coupled one component plasma Indian J of Pure & Appl Phys 2007;45(9):738-740.
18. Li Q.P. and S. Das Sharma, Elementary excitation spectrum of one-dimensional electron systems in confined semiconductor structures: Zero magnetic field, Phys Rev B 1991;43(14):11768-11786.
19. Tewari S.P., Joshi H. & Bera K., Wave-vector and frequency-dependent collective modes in one-component rare hot quantum and classical plasmas, J Phys: Cond Matter 1995;7: 8045.
20. Trubnikov B.A. and Elesin V.F., Quantum Correlation Functions in a Maxwellian Plasma, Sov Phys JETP 1965;20(4):866-872.
21. Burns G., Solid State Physics, Academic Press: New York and London; 1985; 460.
22. Tewari S.P., Joshi H. & Bera K., Semiconductor Devices, Narosa Publications: New Delhi 1996; 298.
23. Sjolander A. and Stott M.J., Electron distribution around positrons in metals, Solid State Commun 1970; 8(22):1881-1884.
24. Sjolander A. and Stott M.J., Electron Distribution around Mobile and Fixed Point Charges in Metals Phys Rev B 1972;5(6):2109-2117.