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Parametric study of different operators on m-polar fuzzy matrices: Ring sum, Ring product and Ring difference

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ABSTRACT

The concept of m-polar fuzzy set is well established with lots of applications. In real world problems many times we have to deal with multi-polar information, for which this concept is not enough. In an attempt to overcome this problem, we introduce the concept of m-polar fuzzy matrix in this article. We extend the binary operations like maximum, minimum etc. to these matrices and also introduce the new binary operations, ring sum ($\oplus$) and ring product ($\odot$) in m-polar fuzzy matrices. We study some properties of these operations.

KEYWORDS: m-polar fuzzy matrix, m-polar fuzzy operators.

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INTRODUCTION

In many situations, the problems in social sciences, cryptography, computer sciences etc. always involve vague boundaries. So, Contour set theory was generalized to sets of objects with vague boundaries by Zadeh[1-3] in 1965 which spelled the origin of fuzzy set theory. Further, Zhang[4, 5]presented the idea of bipolar fuzzy sets in 1994. The idea which lies behind such illustration is associated with the bipolar information (i.e., affirming information and negating information) about the given set. In addition to the growth of social networking internet and other technologies have given rise to uncertainty alongside certainty of information. It is in these cases where we have multiple attributes that are uncertain, we need fuzzy sets that can go beyond the domains of bipolar fuzzy sets and the concept of m- polar fuzzy sets has been introduced. m-Polar fuzzy sets are theorized by Chen et. al.[6] in 2014 to include relationships among several individual elements of a set. In this set, each element has a value between 0 and 1 indicating the degree of absence or presence of a particular predicate. They extended this concept to show that cryptographic mathematical notions can be concisely obtain for m-polar fuzzy sets. This theory has applications in solving real world problems even when the number of predicates (parameters or measurements) are more than two.

Many problems of practical importance have been solved using matrices as a mathematical modeling tool. In computer based applications, matrices play a major role in the projection of three dimensional image into a two dimensional screen, creating the realistic screening motions. Several works on classical matrices are available in different journals as well as in text books. Now-a-days probability, fuzzy sets, intuitionistic fuzzy sets, vague sets, rough sets are used as mathematical tools for dealing uncertainties. Fuzzy matrices arise in many applications, one of which is as adjacency matrices of fuzzy relations. Fuzzy relational equations have important applications in pattern classification and in handing fuzziness in knowledge based systems[7]. Lot of literature is available, by several authors, on fuzzy matrices. For example, Hashimoto[8] introduced the concept of fuzzy matrices and studied the canonical form of a transitive matrix. Kim et. al.[9] studied the canonical form of an idempotent matrix. Kolodziejczyk[10] presented the canonical form of a strongly transitive fuzzy matrix. Xin[11,12] studied controllable fuzzy matrices. Hemasinhaet. al.[13] investigated iterations of fuzzy circulants matrices. Ragabet. al.[14] presented some properties of the min-max compositionof fuzzy matrices. Thomason[15] and Kim[16] defined the adjoint of a square fuzzy matrix. Tianet. al.[17] studied power sequence of a fuzzy matrices. Kim et. al.[18] studied determinant of square fuzzy matrices. Ragabet. al.[19] presented some properties on determinant and adjointof a square fuzzy matrix. Pal [20] defined intuitionistic fuzzy determinant. Khanet. al.[21] introduced intuitionistic fuzzy matrices.

In the present work, we introduce the concept of m-polar fuzzy matrix and some binary operations on them including ring sum and ring product. Further, some properties of m-polar fuzzy matrices with respect to the new operations as well as pre-defined operations are presented. These results strengthen decision-making in critical situations.

2. PRELIMINARIES

Some basic operators on m-polar fuzzy matrices are defined and their notations are introducedbelow.

Definition 1.(See[4])Let X be a non-empty set. Then a bipolar fuzzy set W on X is of the form $W=\{(s,\ \mu^{+}(s),\ \mu^{-}(s))|s\in X\}$ where $\mu^{+}:X\rightarrow[0,1]$ and $\mu^{-}:X\rightarrow[-1,0]$ are functions. If $\mu^{+}(s)$ is non-zero and $\mu^{-}(s)=0$, then the element s is regarded as having only positive satisfaction for W. If $\mu^{+}(s)=0$ and $\mu^{-}(s)$ is non-zero, s does not satisfy the property of W but satisfies the counter property of W. It is possible for an element s to be such that $\mu^{+}(s)$ is non-zero and $\mu^{-}(s)$ is non-zero when membership function of the property overlaps that of its counter property over some portion of X. For simplicity, we shall use the symbol $W=(\mu_w^{+},\ \mu_w^{-})$ for the bipolar fuzzy set $W=\{(s,\ \mu^{+}(s),\ \mu^{-}(s))|s\in X\}$.

Definition 2.(See[6]) In our study, $[0,1]^m$ ($m\in N$ copies of the closed interval [0, 1]) is considered to be a poset with the point-wise order $\leq$, is given by $w\leq f \Leftrightarrow$ for all $i=1,2,\cdots,m$, $p_i(w)\leq p_i(f)$ where $w,\ f\in[0,1]^m$ and $p_i:[0,1]^m\rightarrow[0,1]$ is the i^{th} projection mapping.

An m-polar fuzzy set on X is a mapping $W:X\rightarrow[0,1]^m$.

*Definition 3.*Let W be an m-polar fuzzy set on X and $l=\langle l_1,l_2,\cdots,l_m\rangle$, $n=\langle n_1,n_2,\cdots,n_m\rangle$ be two elements of W where $l_1,l_2,\cdots,l_m$ and $n_1,n_2,\cdots,n_m\in[0,1]$.

Then for any $\alpha\in[0,1]$, we define

i) Maximum of $\{l,n\}=l\vee n=\langle l_1,l_2,\cdots,l_m\rangle\vee\langle n_1,n_2,\cdots,n_m\rangle=\langle l_1\vee n_1,l_2\vee n_2,\cdots,l_m\vee n_m\rangle$

ii) Minimum of $\{l,n\}=l\wedge n=\langle l_1,l_2,\cdots,l_m\rangle\wedge\langle n_1,n_2,\cdots,n_m\rangle=\langle l_1\wedge n_1,l_2\wedge n_2,\cdots,l_m\wedge n_m\rangle$

iii) Ring subtraction of $\{l,n\}=l\,!\;n=\langle l_1,l_2,\cdots,l_m\rangle\,!\;\langle n_1,n_2,\cdots,n_m\rangle$

$$=\begin{cases}\langle l_1,l_2,\cdots,l_m\rangle, & \text{if } \langle l_1,l_2,\cdots,l_m\rangle>\langle n_1,n_2,\cdots,n_m\rangle\\ 0, & \text{if } \langle l_1,l_2,\cdots,l_m\rangle\leq\langle n_1,n_2,\cdots,n_m\rangle\end{cases}$$

iv) Upper α-cut of $l = l^{(\alpha)} = \begin{cases} \langle 1, 1, \cdots, 1 \rangle, & \text{if } \langle l_1, l_2, \cdots, l_m \rangle \geq \langle \alpha, \alpha, \cdots, \alpha \rangle \\ \langle 0, 0, \cdots, 0 \rangle, & \text{if } \langle l_1, l_2, \cdots, l_m \rangle < \langle \alpha, \alpha, \cdots, \alpha \rangle \end{cases}$

v) Lower α-cut of $l = l_{(\alpha)} = \begin{cases} \langle l_1, l_2, \cdots, l_m \rangle, & \text{if } \langle l_1, l_2, \cdots, l_m \rangle \geq \langle \alpha, \alpha, \cdots, \alpha \rangle \\ \langle 0, 0, \cdots, 0 \rangle, & \text{if } \langle l_1, l_2, \cdots, l_m \rangle < \langle \alpha, \alpha, \cdots, \alpha \rangle \end{cases}$

vi) Complement of $l = l^c = \langle 1, 1, \cdots, 1 \rangle - \langle l_1, l_2, \cdots, l_m \rangle = \langle 1-l_1, 1-l_2, \cdots, 1-l_m \rangle$.

Now, we introduce two more new operators on W, $\oplus$ and $\odot$ as follows:

vii) Ring sum of $\{l, n\} = l \oplus n = \langle l_1, l_2, \cdots, l_m \rangle \oplus \langle n_1, n_2, \cdots, n_m \rangle$

$$= \langle l_1 + n_1 - l_1.n_1,\ l_2 + n_2 - l_2.n_2,\ \cdots,\ l_m + n_m - l_m.n_m \rangle$$

viii) Ring product of $\{l, n\} = l \odot n = \langle l_1, l_2, \cdots, l_m \rangle \odot \langle n_1, n_2, \cdots, n_m \rangle = \langle l_1.n_1, l_2.n_2, \cdots, l_m.n_m \rangle$ where the operators '.', '−' and '+' are ordinary multiplication, subtraction and addition on real numbers respectively.

Immediately, we can observe that

i) $\langle 1, 1, \cdots, 1 \rangle \oplus \langle l_1, l_2, \cdots, l_m \rangle = \langle 1, 1, \cdots, 1 \rangle$,

ii) $\langle 1, 1, \cdots, 1 \rangle \odot \langle l_1, l_2, \cdots, l_m \rangle = \langle l_1, l_2, \cdots, l_m \rangle$,

iii) $\langle 0, 0, \cdots, 0 \rangle \oplus \langle l_1, l_2, \cdots, l_m \rangle = \langle l_1, l_2, \cdots, l_m \rangle$,

iv) $\langle 0, 0, \cdots, 0 \rangle \odot \langle l_1, l_2, \cdots, l_m \rangle = \langle 0, 0, \cdots, 0 \rangle$.

Definition 4. [m-polar fuzzy matrix] An m-polar fuzzy matrix $X = \left[\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \right]$ is a matrix on fuzzy algebra. The zero matrix O_r of order $r \times r$ is the matrix where all the elements are $O_m = \langle 0, 0, \cdots, 0 \rangle$ and the identity matrix I_r of order $r \times r$ is the matrix where all the diagonal elements are $I_m = \langle 1, 1, \cdots, 1 \rangle$ and other entries are $O_m = \langle 0, 0, \cdots, 0 \rangle$.

The set of all rectangular m-polar fuzzy matrices of order $r \times k$ is denoted by M_{rk} and that of square matrices of order $r \times r$ is denoted by M_r.

Next, we introduce some operations on m-polar fuzzy matrices.

Let $W = \left[\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle \right]$ and $F = \left[\left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle \right]$ be two m-polar fuzzy matrices of order $r \times s$. Then

i) Ring sum of $\{W, F\} = W \oplus F = \left[\left\langle w_{1_{lk}} + f_{1_{lk}} - w_{1_{lk}}.\ f_{1_{lk}},\ w_{2_{lk}} + f_{2_{lk}} - w_{2_{lk}}.\ f_{2_{lk}}, \cdots, w_{m_{lk}} + f_{m_{lk}} - w_{m_{lk}}.\ f_{m_{lk}} \right\rangle \right]$

ii) Ring product of $\{W, F\} = W \odot F = \left[\left\langle w_{1_{lk}}.\ f_{1_{lk}},\ w_{2_{lk}}.\ f_{2_{lk}}, \cdots, w_{m_{lk}}.\ f_{m_{lk}} \right\rangle \right]$

iii) Maximum of $\{W, F\} = W \vee F = \left[\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle \vee \left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle \right]$

iv) Minimum of $\{W, F\} = W \wedge F = \left[\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle \wedge \left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle\right]$

v) Ring subtraction of $\{W, F\} = W \, ! \, F = \left[\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle \, ! \, \left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle\right]$

vi) Upper α - cut m - polar fuzzy matrix of $W = W^{(\alpha)} = \left[\left\langle w_{1_{lk}}{}^{(\alpha)}, w_{2_{lk}}{}^{(\alpha)}, \cdots, w_{m_{lk}}{}^{(\alpha)} \right\rangle\right]$

vii) Lower α - cut m - polar fuzzy matrix of $W = W_{(\alpha)} = \left[\left\langle w_{1_{lk}(\alpha)}, w_{2_{lk}(\alpha)}, \cdots, w_{m_{lk}(\alpha)} \right\rangle\right]$

viii) The transpose m - polar fuzzy matrix of $W = W^{T} = \left[\left\langle w_{1_{kl}}, w_{2_{kl}}, \cdots, w_{m_{kl}} \right\rangle\right]$

ix) The complement m - polar fuzzy matrix of $W = W^{c} = \left[\left\langle 1 - w_{1_{lk}}, 1 - w_{2_{lk}}, \cdots, 1 - w_{m_{lk}} \right\rangle\right]$

x) $W \le F$ if and only if $w_{1_{lk}} \le f_{1_{lk}}, w_{2_{lk}} \le f_{2_{lk}}, \cdots, w_{m_{lk}} \le f_{m_{lk}}$ for all l, k.

xi) For any two m-polar fuzzy matrices W and F, $W \wedge F = \min\{W, F\}$.

Notations 5. If W is any m-polar fuzzy matrix, then we denote $W \oplus W$ as $[2]W$ and in general we have $[h+1]W = [h]W \oplus W$. Similarly, $W \square W = W^{[2]}$ and $W^{[h+1]} = W^{[h]} \square W$ for all h.

Further, we define some types of m-polar fuzzy matrices.

Let $Q = \left[\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle\right]$ be a matrix of order n. Then we say

a. Q is reflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 1, 1, \cdots, 1 \rangle$ for all $l = 1, 2, \cdots, n$.

b. Q isirreflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 0, 0, \cdots, 0 \rangle$ for all $l = 1, 2, \cdots, n$.

c. Q is nearly irreflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \le \left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle$ for all $l, k = 1, 2, \cdots, n$.

d. Q is symmetric if $Q^{T} = Q$.

e. Q is constant if $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \left\langle q_{1_{ik}}, q_{2_{ik}}, \cdots, q_{m_{ik}} \right\rangle$ for all l, $k, i = 1, 2, \cdots, n$.

f. Q is identity if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 1, 1, \cdots, 1 \rangle$ and $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \langle 0, 0, \cdots, 0 \rangle$ $(l \ne k)$ for all l, k.

g. Q is weakly reflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \ge \left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle$ for all l, k.

h. Q is diagonal if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \ge \langle 0, 0, \cdots, 0 \rangle$ and $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \langle 0, 0, \cdots, 0 \rangle$ $(l \ne k)$ for all l, k.

Notations 6. If all the elements of a matrix are $\langle 0, 0, \cdots, 0\rangle$ then we denote it by O and all the elements of a matrix are $\langle 1, 1, \cdots, 1\rangle$ then we denote it by U. Generally, the identity matrix of order $m \times m$ is denoted by I_m.

3. GEOMETRICAL REPRESENTATION OF m-POLAR FUZZY MATRICES

The 3D representation of the m-polar fuzzy matrices X and Y are shown in Figures 1 and 2. But this depiction is not possible for classical matrices. The 3D representation of the m-polar fuzzy matrices $X \oplus Y$, $X \odot Y$, $X^{(0.3)}$, $X_{(0.3)}$, $X \,!\, Y$, X^c, $X \vee Y$ and $X \wedge Y$ are shown in Figures 3-10.

$$X = \begin{bmatrix} \langle 0.3, 0.8, 0.4\rangle & \langle 0.8, 0.4, 0.3\rangle & \langle 0.1, 0.2, 0.2\rangle \\ \langle 0.7, 0.9, 0.3\rangle & \langle 0.3, 0.9, 0.7\rangle & \langle 0.2, 0.2, 0.1\rangle \\ \langle 0.4, 0.8, 0.7\rangle & \langle 0.4, 0.8, 1.0\rangle & \langle 0.1, 0.2, 0.2\rangle \end{bmatrix}$$

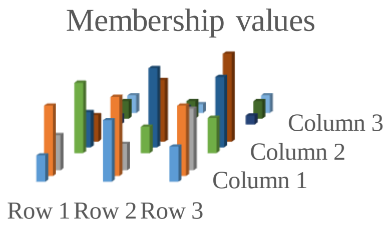

Figure 1: Geometrical representation of the matrix X

$$Y = \begin{bmatrix} \langle 0.8, 0.9, 0.5\rangle & \langle 0.1, 0.3, 0.2\rangle & \langle 0.5, 0.6, 0.7\rangle \\ \langle 0.6, 0.1, 0.2\rangle & \langle 0.8, 1.0, 0.8\rangle & \langle 0.8, 0.9, 0.7\rangle \\ \langle 0.3, 0.4, 0.6\rangle & \langle 0.3, 0.6, 0.1\rangle & \langle 0.5, 0.5, 0.3\rangle \end{bmatrix}$$

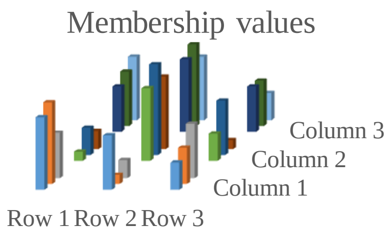

Figure 2: Geometrical representation of the matrix Y

$$X \oplus Y = \begin{bmatrix} \langle 0.86, 0.98, 0.7\rangle & \langle 0.82, 0.58, 0.44\rangle & \langle 0.55, 0.68, 0.76\rangle \\ \langle 0.88, 0.91, 0.44\rangle & \langle 0.86, 1.0, 0.94\rangle & \langle 0.84, 0.92, 0.73\rangle \\ \langle 0.58, 0.88, 0.88\rangle & \langle 0.58, 0.92, 1.0\rangle & \langle 0.55, 0.6, 0.44\rangle \end{bmatrix}$$

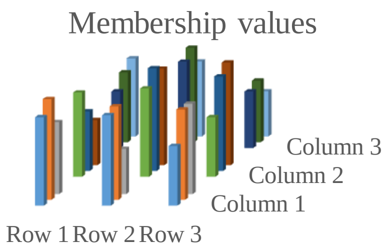

Figure 3: Representation of the matrix $X \oplus Y$

$$X \odot Y = \begin{bmatrix} \langle 0.24, 0.72, 0.2\rangle & \langle 0.08, 0.12, 0.06\rangle & \langle 0.05, 0.12, 0.14\rangle \\ \langle 0.42, 0.09, 0.06\rangle & \langle 0.24, 0.9, 0.56\rangle & \langle 0.16, 0.18, 0.07\rangle \\ \langle 0.12, 0.32, 0.42\rangle & \langle 0.12, 0.48, 0.1\rangle & \langle 0.05, 0.1, 0.06\rangle \end{bmatrix}$$

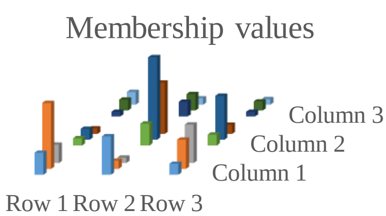

Figure 4: Representation of the matrix $X \odot Y$

$$X^{(0.3)} = \begin{bmatrix} \langle 1.0, 1.0, 1.0 \rangle & \langle 1.0, 1.0, 1.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 1.0, 1.0, 1.0 \rangle & \langle 1.0, 1.0, 1.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 1.0, 1.0, 1.0 \rangle & \langle 1.0, 1.0, 1.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \end{bmatrix}$$

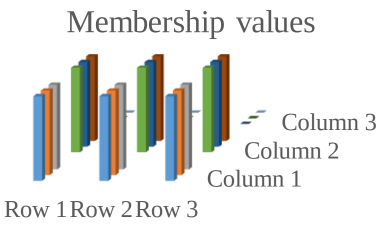

Figure 5: Representation of the matrix $X^{(0.3)}$

$$X_{(0.3)} = \begin{bmatrix} \langle 0.3, 0.8, 0.4 \rangle & \langle 0.8, 0.4, 0.3 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 0.7, 0.9, 0.3 \rangle & \langle 0.3, 0.9, 0.7 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 0.4, 0.8, 0.7 \rangle & \langle 0.4, 0.8, 1.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \end{bmatrix}$$

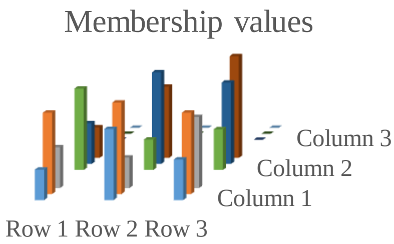

Figure 6: Representation of the matrix $X_{(0.3)}$

$$X\,!\,Y = \begin{bmatrix} \langle 0.0, 0.0, 0.0 \rangle & \langle 0.8, 0.4, 0.3 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 0.7, 0.9, 0.3 \rangle & \langle 0.0, 0.0, 0.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \\ \langle 0.4, 0.8, 0.7 \rangle & \langle 0.4, 0.8, 1.0 \rangle & \langle 0.0, 0.0, 0.0 \rangle \end{bmatrix}$$

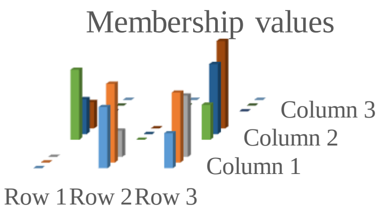

Figure 7: Representation of the matrix $X\,!\,Y$

$$X^{c} = \begin{bmatrix} \langle 0.7, 0.2, 0.6 \rangle & \langle 0.2, 0.6, 0.7 \rangle & \langle 0.9, 0.8, 0.8 \rangle \\ \langle 0.3, 0.1, 0.7 \rangle & \langle 0.7, 0.1, 0.3 \rangle & \langle 0.8, 0.8, 0.9 \rangle \\ \langle 0.6, 0.2, 0.3 \rangle & \langle 0.6, 0.2, 0.0 \rangle & \langle 0.9\ 0.8, 0.8 \rangle \end{bmatrix}$$

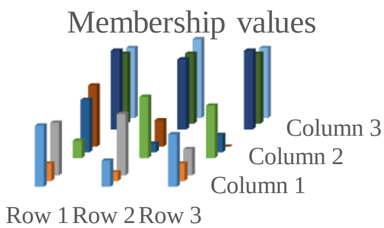

Figure 8: Representation of the matrix X^{c}

$$X \vee Y = \begin{bmatrix} \langle 0.8, 0.9, 0.5 \rangle & \langle 0.8, 0.4, 0.3 \rangle & \langle 0.5, 0.6, 0.7 \rangle \\ \langle 0.7, 0.9, 0.3 \rangle & \langle 0.8, 1.0, 0.8 \rangle & \langle 0.8, 0.9, 0.7 \rangle \\ \langle 0.4, 0.8, 0.7 \rangle & \langle 0.4, 0.8, 1.0 \rangle & \langle 0.5, 0.5, 0.3 \rangle \end{bmatrix}$$

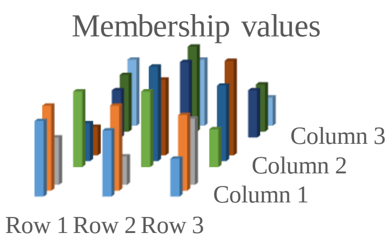

Figure 9: Representation of the matrix $X \vee Y$

$$X \wedge Y = \begin{bmatrix} \langle 0.3, 0.8, 0.4 \rangle & \langle 0.1, 0.3, 0.2 \rangle & \langle 0.1, 0.2, 0.2 \rangle \\ \langle 0.6, 0.1, 0.2 \rangle & \langle 0.3, 0.9, 0.7 \rangle & \langle 0.2, 0.2, 0.1 \rangle \\ \langle 0.3, 0.4, 0.6 \rangle & \langle 0.3, 0.6, 0.1 \rangle & \langle 0.1, 0.2, 0.2 \rangle \end{bmatrix}$$

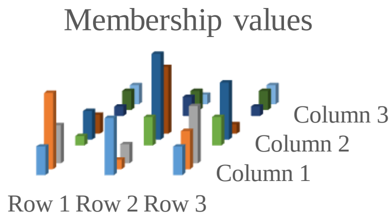

Figure 10: Representation of the matrix $X \wedge Y$

4. THEORETICAL RESULTS ON m-POLAR FUZZY MATRICES

Throughout the paper, we use the notation $X = [x_{lk}]$, $Y = [y_{lk}]$, $Z = [z_{lk}]$ and $T = [t_{lk}]$ to denote m-polar fuzzy matrices.

Next, we prove some properties of m-polar fuzzy matrices.

Property 7. *Let* $X = [x_{lk}]$ *be an m-polar fuzzy matrix of order* n. *Then*

a. If $\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle < \langle 1, 1, \cdots, 1 \rangle$ *for all* l, k, *then* $\lim_{p\to\infty} X^{[p]} = O$,

b. If $\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \langle 0, 0, \cdots, 0 \rangle$ *for all* l, k, *then* $\lim_{p\to\infty} [p]X = U$.

Proof. a. Let $X = [x_{lk}]$ and $Y = [y_{lk}]$ be two m-polar fuzzy matrices. Then we have $X \odot Y = \left[\langle x_{1_{lk}}.y_{1_{lk}}, x_{2_{lk}}.y_{2_{lk}}, \cdots, x_{m_{lk}}.y_{m_{lk}} \rangle\right]$.

Therefore $X \odot X = X^{[2]} = \left[\langle x^2_{1_{lk}}, x^2_{2_{lk}}, \cdots, x^2_{m_{lk}} \rangle\right]$, $X^{[3]} = X^{[2]} \odot X = \left[\langle x^3_{1_{lk}}, x^3_{2_{lk}}, \cdots, x^3_{m_{lk}} \rangle\right]$, etc.

Taking limit as $p \to \infty$, we get that $\lim_{p\to\infty} X^{[p]} = \langle 0, 0, \cdots, 0 \rangle = O$.

*b.*Again, $[2]X = X \oplus X = \left[\langle 2x^2_{1_{lk}} - x^2_{1_{lk}}, 2x^2_{2_{lk}} - x^2_{2_{lk}}, \cdots, 2x^2_{m_{lk}} - x^2_{m_{lk}} \rangle\right]$

$$= \left[\left\langle \left[1-\left(1-x_{1_{lk}}\right)^2\right], \left[1-\left(1-x_{2_{lk}}\right)^2\right], \cdots, \left[1-\left(1-x_{m_{lk}}\right)^2\right] \right\rangle\right].$$

Also, $\langle 1-x_{1_{lk}}, 1-x_{2_{lk}}, \cdots, 1-x_{m_{lk}} \rangle \le \langle 1, 1, \cdots, 1 \rangle$.

Therefore, for positive integer p, we have

$$[p]X = \left[\left\langle \left[1-\left(1-x_{1_{lk}}\right)^p\right], \left[1-\left(1-x_{2_{lk}}\right)^p\right], \cdots, \left[1-\left(1-x_{m_{lk}}\right)^p\right] \right\rangle\right]$$

andtaking limit as $p \to \infty$, we get $\lim_{p\to\infty} [p]X = \langle 1, 1, \cdots, 1 \rangle = U$. $\square$

For $0 \le x, y \le 1$, $x.y \le x$ and $x.y \le y \Rightarrow x.y \le \min\{x, y\}$.

Therefore, $X \odot Y \le \min\{X, Y\}$.

Property 8. *Let X and Y be two m-polar fuzzy matrices, then*

a. $X \oplus Y \geq X \odot Y$,

b.If X and Y are symmetric, then $X \oplus Y$ and $X \odot Y$ are symmetric,

c.If X and Y are nearly irreflexive, then $X \oplus Y$ and $X \odot Y$ are nearly irreflexive.

Proof.a. The lk th element of $X \oplus Y$ is $\left\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \right\rangle$ and that of $X \odot Y$ is $\left\langle x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} . y_{m_{lk}} \right\rangle$. Assume that $\left\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \right\rangle \geq \left\langle x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} . y_{m_{lk}} \right\rangle$.

i.e., $\left\langle x_{1_{lk}} .\left(1 - y_{1_{lk}}\right) + y_{1_{lk}} .\left(1 - x_{1_{lk}}\right), x_{2_{lk}} .\left(1 - y_{2_{lk}}\right) + y_{2_{lk}} .\left(1 - x_{2_{lk}}\right), \cdots, x_{m_{lk}} .\left(1 - y_{m_{lk}}\right) + y_{m_{lk}} .\left(1 - x_{m_{lk}}\right) \right\rangle \geq$ $\langle 0, 0, \cdots, 0 \rangle$, which is true as $\langle 0, 0, \cdots, 0 \rangle \leq \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \leq \langle 1, 1, \cdots, 1 \rangle$ and $\langle 0, 0, \cdots, 0 \rangle \leq \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \leq \langle 1, 1, \cdots, 1 \rangle$. Hence, $X \oplus Y \geq X \odot Y$.

*b.*Let $X = [x_{lk}]$ and $Y = [y_{lk}]$ be two symmetric m-polar fuzzy matrices such that $X \oplus Y$ and $X \odot Y$ are defined. Therefore $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle = \left\langle x_{1_{kl}}, x_{2_{kl}}, \cdots, x_{m_{kl}} \right\rangle$ and $\left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle = \left\langle y_{1_{kl}}, y_{2_{kl}}, \cdots, y_{m_{kl}} \right\rangle$.

Let $\left\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \right\rangle$ be the lk th element of $X \oplus Y$. Then

$\left\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \right\rangle = \left\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \right\rangle =$

$\left\langle x_{1_{kl}} + y_{1_{kl}} - x_{1_{kl}} . y_{1_{kl}}, x_{2_{kl}} + y_{2_{kl}} - x_{2_{kl}} . y_{2_{kl}}, \cdots, x_{m_{kl}} + y_{m_{kl}} - x_{m_{kl}} . y_{m_{kl}} \right\rangle = \left\langle z_{1_{kl}}, z_{2_{kl}}, \cdots, z_{m_{kl}} \right\rangle$.

Hence $X \oplus Y$ is symmetric.

Again, let $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle$ be the lk th element of $X \odot Y$. Then,

$\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle = \left\langle x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} . y_{m_{lk}} \right\rangle = \left\langle x_{1_{kl}} . y_{1_{kl}}, x_{2_{kl}} . y_{2_{kl}}, \cdots, x_{m_{kl}} . y_{m_{kl}} \right\rangle$ $= \left\langle t_{1_{kl}}, t_{2_{kl}}, \cdots, t_{m_{kl}} \right\rangle$. Hence $X \odot Y$ is symmetric.

c. Since X and Y are nearly irreflexive, $\left\langle x_{1_{ll}}, x_{2_{ll}}, \cdots, x_{m_{ll}} \right\rangle \leq \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle$ and $\left\langle y_{1_{ll}}, y_{2_{ll}}, \cdots, y_{m_{ll}} \right\rangle \leq \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$ for all l, k. Let $\left\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \right\rangle$ and $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle$ be the lk th elements of $X \oplus Y$ and $X \odot Y$ respectively. Then

$$\left\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \right\rangle - \left\langle z_{1_{ll}}, z_{2_{ll}}, \cdots, z_{m_{ll}} \right\rangle =$$

$$\left\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \right\rangle -$$

$$\left\langle x_{1_{ll}} + y_{1_{ll}} - x_{1_{ll}} . y_{1_{ll}}, x_{2_{ll}} + y_{2_{ll}} - x_{2_{ll}} . y_{2_{ll}}, \cdots, x_{m_{ll}} + y_{m_{ll}} - x_{m_{ll}} . y_{m_{ll}} \right\rangle$$

$$= \left\langle \left(1 - x_{1_{ll}}\right).\left(1 - y_{1_{ll}}\right) - \left(1 - x_{1_{lk}}\right).\left(1 - y_{1_{lk}}\right), \left(1 - x_{2_{ll}}\right).\left(1 - y_{2_{ll}}\right) - \left(1 - x_{2_{lk}}\right).\left(1 - y_{2_{lk}}\right), \cdots,\right.$$

$$\left. \left(1 - x_{m_{ll}}\right).\left(1 - y_{m_{ll}}\right) - \left(1 - x_{m_{lk}}\right).\left(1 - y_{m_{lk}}\right) \right\rangle \geq \langle 0, 0, \cdots, 0 \rangle$$

as $\left\langle 1 - x_{1_{ll}}, 1 - x_{2_{ll}}, \cdots, 1 - x_{m_{ll}} \right\rangle \geq \left\langle 1 - x_{1_{lk}}, 1 - x_{2_{lk}}, \cdots, 1 - x_{m_{lk}} \right\rangle$ and

$\left\langle 1 - y_{1_{ll}}, 1 - y_{2_{ll}}, \cdots, 1 - y_{m_{ll}} \right\rangle \geq \left\langle 1 - y_{1_{lk}}, 1 - y_{2_{lk}}, \cdots, 1 - y_{m_{lk}} \right\rangle$,

i.e., $\left\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \right\rangle \geq \left\langle z_{1_{ll}}, z_{2_{ll}}, \cdots, z_{m_{ll}} \right\rangle$. Hence $X \oplus Y$ is irreflexive.

Now $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle - \left\langle t_{1_{ll}}, t_{2_{ll}}, \cdots, t_{m_{ll}} \right\rangle =$

$$\left\langle x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} . y_{m_{lk}} \right\rangle - \left\langle x_{1_{ll}} . y_{1_{ll}}, x_{2_{ll}} . y_{2_{ll}}, \cdots, x_{m_{ll}} . y_{m_{ll}} \right\rangle \geq \langle 0, 0, \cdots, 0 \rangle$$

i.e., $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle \geq \left\langle t_{1_{ll}}, t_{2_{ll}}, \cdots, t_{m_{ll}} \right\rangle$. Hence $X \square Y$ is irreflexive.

Property 9. *For any m-polar fuzzy matrix X, we have*

a. $X \oplus X \geq X$

b. $X \square X \leq X$.

*Proof.a.*The lk th element of $X \oplus X$ is $\left\langle 2x_{1_{lk}} - x^2_{1_{lk}}, 2x_{2_{lk}} - x^2_{2_{lk}}, \cdots, 2x_{m_{lk}} - x^2_{m_{lk}} \right\rangle =$

$$\left\langle x_{1_{lk}} + x_{1_{lk}} . \left(1 - x_{1_{lk}}\right), x_{2_{lk}} + x_{2_{lk}} . \left(1 - x_{2_{lk}}\right), \cdots, x_{m_{lk}} + x_{m_{lk}} . \left(1 - x_{m_{lk}}\right) \right\rangle \geq \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle.$$

Therefore, $X \oplus X \geq X$.

b. The lk th element $\left\langle x^2_{1_{lk}}, x^2_{2_{lk}}, \cdots, x^2_{m_{lk}} \right\rangle$ of $X \square X$ is less than $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle$.

Therefore, $X \square X \leq X$.

Property 10.*Let X, Y and Z be any three m-polar fuzzy matrices. Then the operations $\oplus$ and $\square$ are commutative and associative,*

i.e., a. $X \oplus Y = Y \oplus X$,

b. $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$,

c. $X \square Y = Y \square X$,

d. $(X \square Y) \square Z = X \square (Y \square Z)$.

Proof. The proof is clear from the above definitions.

Property 11. *Let X, Y and Z be any three m-polar fuzzy matrices. Then*

a. $(X \oplus Y)^T = X^T \oplus Y^T$,

b. $(X \odot Y)^T = X^T \odot Y^T$,

c. If $X \le Y$, *then* $X \oplus Z \le Y \oplus Z$ *and* $X \odot Z \le Y \odot Z$.

Proof. a. Let $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \rangle$ and $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$ be the lkth element of $X \oplus Y$ and $X^T \oplus Y^T$ respectively. Therefore, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle w_{1_{kl}}, w_{2_{kl}}, \cdots, w_{m_{kl}} \rangle$ is the lkth element of $(X \oplus Y)^T$. Then

$$\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \rangle = \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle,$$

$$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle x_{1_{kl}} + y_{1_{kl}} - x_{1_{kl}}.\, y_{1_{kl}}, x_{2_{kl}} + y_{2_{kl}} - x_{2_{kl}}.\, y_{2_{kl}}, \cdots, x_{m_{kl}} + y_{m_{kl}} - x_{m_{kl}}.\, y_{m_{kl}} \rangle$$

$$\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle x_{1_{kl}} + y_{1_{kl}} - x_{1_{kl}}.\, y_{1_{kl}}, x_{2_{kl}} + y_{2_{kl}} - x_{2_{kl}}.\, y_{2_{kl}}, \cdots, x_{m_{kl}} + y_{m_{kl}} - x_{m_{kl}}.\, y_{m_{kl}} \rangle$$

$$= \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle.$$

Therefore, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$ for all l, k. Hence, $(X \oplus Y)^T = X^T \oplus Y^T$.

b. Let $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \rangle$ and $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$ be the lkth element of $X \odot Y$ and $X^T \odot Y^T$ respectively. Therefore, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle w_{1_{kl}}, w_{2_{kl}}, \cdots, w_{m_{kl}} \rangle$ is the lkth element of $(X \odot Y)^T$. Then $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \rangle = \langle x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}}.\, y_{m_{lk}} \rangle$.

Thus, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle x_{1_{kl}}.\, y_{1_{kl}}, x_{2_{kl}}.\, y_{2_{kl}}, \cdots, x_{m_{kl}}.\, y_{m_{kl}} \rangle$ and

$$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle x_{1_{kl}}.\, y_{1_{kl}}, x_{2_{kl}}.\, y_{2_{kl}}, \cdots, x_{m_{kl}}.\, y_{m_{kl}} \rangle.$$

Therefore, $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle$, for all l, k. Hence, $(X \odot Y)^T = X^T \odot Y^T$.

c. Let $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle$, $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle$ and $\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$ be the lkth elements of $X \oplus Z, Y \oplus Z, X \odot Z$ and $Y \odot Z$ respectively. Then

$$\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle,$$

$$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle y_{1_{lk}} + z_{1_{lk}} - y_{1_{lk}}.\, z_{1_{lk}}, y_{2_{lk}} + z_{2_{lk}} - y_{2_{lk}}.\, z_{2_{lk}}, \cdots, y_{m_{lk}} + z_{m_{lk}} - y_{m_{lk}}.\, z_{m_{lk}} \rangle,$$

$\left\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \right\rangle = \left\langle x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} . z_{m_{lk}} \right\rangle,$

$\left\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \right\rangle = \left\langle y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} . z_{m_{lk}} \right\rangle.$

Since $X \leq Y, \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \leq \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle.$ Then $\left\langle x_{1_{lk}}.\left(1-z_{1_{lk}}\right), x_{2_{lk}}.\left(1-z_{2_{lk}}\right), \cdots, x_{m_{lk}}.\left(1-z_{m_{lk}}\right) \right\rangle \leq \left\langle y_{1_{lk}}.\left(1-z_{1_{lk}}\right), y_{2_{lk}}.\left(1-z_{2_{lk}}\right), \cdots, y_{m_{lk}}.\left(1-z_{m_{lk}}\right) \right\rangle$ or, $\left\langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}} . z_{m_{lk}} \right\rangle \leq$ $\left\langle y_{1_{lk}} + z_{1_{lk}} - y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} + z_{2_{lk}} - y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} + z_{m_{lk}} - y_{m_{lk}} . z_{m_{lk}} \right\rangle.$ That is, $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle \leq \left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle$ for all l, k. Hence, $X \oplus Z \leq Y \oplus Z$.

Again, $\left\langle x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} . z_{m_{lk}} \right\rangle \leq \left\langle y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} . z_{m_{lk}} \right\rangle,$ i.e., $\left\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \right\rangle \leq \left\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \right\rangle$ for all l, k. Hence, $X \square Z \leq Y \square Z$.

CONCLUSIONS

It is well known that m-polar fuzzy sets are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological and social systems. In this paper, we introduced m-polar fuzzy matrices and two operations are defined between two m-polar fuzzy matrices. Some properties of an m-polar fuzzy matrices are obtained.

CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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