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Vertex Even Mean Labeling of New Families of Graphs

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ABSTRACT

The main aim of this paper is to identify some cycle related graphs which admit vertex even mean labeling. We prove that every cycle C_n ($n \geq 6$) with parallel chords is vertex even mean graph. Using graph operations on cycles with parallel chords we have obtained new families of graphs namely chain of even cycles with parallel chords, crown with parallel chords, subdivided cycle with parallel chords, two copies of odd cycles sharing a common edge, two copies of odd cycles sharing a common vertex which are vertex even mean graphs.

KEYWORDS: Mean labeling, Vertex Even Mean labeling, Cycles with parallel chords, Subdivided Cycle with parallel chords, Chain of cycles

AMS Subject Classification:05C78

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I. INTRODUCTION

In Graph theory labeling of a graph is obtained by assigning integer values to the vertices or edges of a graph under some conditions.β-valuation introduced by Rosa[9] was the origin of graph labelling which was later renamed by Golomb[5] as graceful labeling. In this paper, a finite simple and undirected graph $G = (V, E)$ with p vertices and q edges is considered. Many kinds of labeling and its variations have been introduced by researchers over the past five decades. Mean labeling was introduced by Somasundaram.S and et al[13]. Several graphs that are proved to be mean graphs are seen in[4,13,16,17]. Various types of mean labeling have been introduced since then. N. Revathi[8] introduced a kind of mean labeling namely vertex even mean labeling and proved that Umbrella graph, $K_1 + C_n$ are vertex even mean graphs. Vertex even mean labeling of various graphs are discussed in[1,7,8]. Gallian[3] gives a detailed survey on graph labeling. Several labelings on cycles with zigzag chords, cycles with parallel chords are seen in[2,6,12,14,15]. This paper is focussed on vertex even mean labeling of cycles with parallel chords and also new classes of graphs obtained from cycles with parallel chords using graph operations.

Definition 1.1:[13]

A graph G with $|V(G)|=p$, and $|E(G)|=q$is said to be a mean graph if there is an injection $f : V(G) \rightarrow \{0,1,2,...,q\}$ such that when each edge xy is labeled with $\frac{f(x)+f(y)}{2}$ if $f(x)+f(y)$ is even, and $\frac{f(x)+f(y)+1}{2}$ if $f(x)+f(y)$ is odd, the resulting edge labels are distinct.

Definition 1.2:[8]

A Graph $G=(V, E)$ with $|V(G)|=p$, and $|E(G)|=q$ is said to be vertex even mean graph if there exist an injection$f : V(G)\rightarrow\{2,4,...2q\}$such that the induced mapping$f^* : E(G) \rightarrow I\{$set of positive integers$\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ are distinct for each edge uv. Such a function f is called a vertex even mean labeling.

Definition 1.3:[12]

A cycle with parallel chords is defined as a graph G obtained from a cycle C_n $(n \geq 6)$ with consecutive vertices $v_0,v_1, ...v_{n-1},v_0$ by adding the chords $v_1 v_{n-1}$, $v_2 v_{n-2}$,...$v_\alpha v_\beta$ where $\alpha = \left\lfloor\frac{n}{2}\right\rfloor -1$, $\beta = \left\lfloor\frac{n}{2}\right\rfloor + 1$ if n is even and $\beta = \left\lfloor\frac{n}{2}\right\rfloor + 2$ if n is odd. Then G has n vertices and M edges where $M = (3n-3) / 2$ if n is odd and $M = (3n-2) / 2$ if n is even. Here chord is an edge connecting two otherwise non adjacent vertices of the cycle C_n.

Definition 1.4:[11]

Consider n copies of cycle C_{2m}. Chain of Cycles is the graph obtained by identifying $v_{i,m}$ with $v_{i+1,m}$ for $i = 1,2,\ldots n$-1 where $v_{i,1}$, $v_{i,2}$ …$v_{i,2m}$ are the successive vertices of n copies of C_{2m}.

Definition 1.5:[10]

Subdivision of a graph is a graph obtained by subdividing each edge of the graph with a vertex exactly once.

Definition 1.6:

We defineSubdivided cycle with parallel chords as a graph obtained by subdividing each edge of cycle only with a vertex.

II. MAIN RESULTS

Theorem 2.1*:* Every cycle C_n ($n \geq 6$) with parallel chords is a vertex even mean graph

Proof: Let G be the cycle C_n with parallel chords. Then by definition 1.3G has n vertices and M edges. The n vertices of G are $v_0, v_1,\ldots,v_{n-1}$. Define the vertex labelling $f : V(G) \rightarrow \{2,4,\ldots,2M\}$ as follows for the two cases depending on n.

$f(v_i) = 4i$, $1 \leq i \leq n/2$ if n is even and $1 \leq i \leq (n-1)/2$ if n is odd

$f(v_{n-i}) = 4i + 2$, $0 \leq i \leq n/2 - 1$ if n is even and $0 \leq i \leq (n-1)/2$ if n is odd

From the above it is clear that all the vertices labeled are distinct.

$E(G) = E_1 U E_2 U E_3 U E_4$ is the edge set where

$E_1 = \{v_i v_{i+1}, 1 \leq i \leq n/2 - 1$ if n is even and $1 \leq i \leq (n-3)/2$ if n is odd$\}$

$E_2 = \{v_{n-i} v_{n-(i+1)}, 0 \leq i \leq n/2 - 2$ if n is even and $0 \leq i \leq (n-3)/2$ if n is odd$\}$

$E_3 = \{v_i v_{n-i}, 1 \leq i \leq n/2 - 1$ if n is even and $1 \leq i \leq (n-1)/2$ if n is odd$\}$

$E_4 = \{v_0 v_1, v_{n/2} v_{n/2+1}$ if n is even and $v_0 v_1$ if n is odd$\}$.

Define the induced function $f^* : E(G) \rightarrow I$ (Set of Positive integers) as

$f^*(v_i v_{i+1}) = 4i + 2$, $1 \leq i \leq n/2 - 1$ if n is even and $1 \leq i \leq (n-3)/2$ if n is odd

$f^*(v_{n-i} v_{n-(i+1)}) = 4i + 4$, $0 \leq i \leq n/2 - 2$ if n is even and $0 \leq i \leq (n-3)/2$if n is odd

$f^*(v_i v_{n-i}) = 4i + 1$, $1 \leq i \leq n/2 - 1$ if n is even and $1 \leq i \leq (n-1)/2$ if n is odd

$f^*(v_0 v_1) = 3$

$f^*(v_{n/2} v_{n/2+1}) = 2n - 1$

It is clear from the above labeling that the edge labels of E_1, E_2 are even and that of E_3 is odd. To show that the edge labels of E_1, E_2 are distinct we will assume on the contrary that they are same.

For the edges of E_1 and E_2 :

if $i \neq j$, $1 \leq i \leq n/2 - 1$ and $0 \leq j \leq n/2 - 2$

let $f^*(v_i v_{i+1}) = f^*(v_{n-j}\, v_{n-(j+1)})$ Then

$4i + 2 = 4j + 4 \Longrightarrow 2(i-j) = 1$

Here the left hand side is an even integer whereas the right hand side is an odd integer, a contradiction. Hence the edge labels of E_1 and E_2 are distinct. Thus all the edges of G have distinct labels and G is a vertex even mean graph. An illustrationis given in Figure 1 and Figure 2.

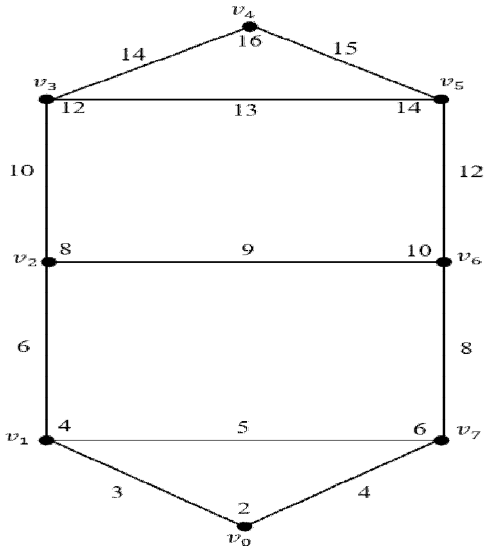

Figure 1. Vertex Even Mean Labeling of C_8 with Parallel Chords

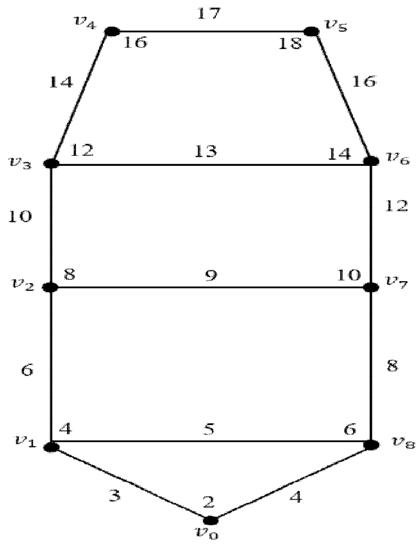

Figure 2. Vertex Even Mean Labeling of C_9 with Parallel Chords

Theorem 2.2:Chain of even cycles C_n $(n \geq 6)$ with parallel chords is a vertex even mean graph.

Proof: Consider s copies of even cycle C_n with parallel chords. By definition 1.4for j=1,2,3,……s let $v_{0,j}, v_{1,j}, \ldots v_{n-1,j}$ be the n vertices of j^{th} copy of cycle C_n with parallel chords. Chain of cycles $C_{n,s}$ is the graph obtained from s copies of cycle C_n with parallel chords by identifying $v_{n/2\,,j}$ with $v_{0,\,j+1}$ for j = 1.2,....s-1. Let this graph be denoted by G. Then G has $s(n-1) + 1$vertices and sM edges.

Define the vertex labeling $f: V(G) \rightarrow \{2,4,\ldots,2sM\}$as follows :

For $j = 1,2,\ldots s$

$f(v_{i,\,j}) = 4i + 2(n-1)(j-1), \quad 1 \leq i \leq n/2$

$f(v_{n-i,\,j}) = 4i + 2 + 2(n-1)(j-1), \quad 0 \leq i \leq n/2 - 1$

From the above it is clear that all the vertices labeled are distinct.

$E(G) = E_1 \cup E_2 \cup E_3 \cup E_4$ is the edge set where

$E_1 = \{v_{i,\,j} v_{i+1,j},\ 1 \leq i \leq n/2 - 1 \text{ and } 1 \leq j \leq s\}$

$E_2 = \{v_{n-i,j} v_{n-(i+1),\,j},\ 0 \leq i \leq n/2 - 2 \text{ and } 1 \leq j \leq s\}$

$E_3 = \{v_{i,j} v_{n-i,\,j},\ 1 \leq i \leq n/2 - 1 \text{ and } 1 \leq j \leq s\}$

$E_4 = \{v_{0,\,j} v_{1,\,j}, v_{n/2,\,j}\, v_{n/2+1,\,j},\ 1 \leq j \leq s\}$

Define the induced function $f^*: E(G) \rightarrow I$ (Set of Positive integers) as follows:

For $j = 1,2,\ldots s$

$$f^* (v_{i,j}v_{i+1,\,j}) = 4i + 2 + 2(j\text{-}1)(n\text{-}1), \qquad 1 \leq i \leq n/2 - 1$$

$$f^* (v_{n\text{-i},\,j}v_{n\text{-}(i+1),\,j}) = 4i + 4 + 2(j\text{-}1)(n\text{-}1), \qquad 0 \leq i \leq n/2 - 2$$

$$f^* (v_{i,j}v_{n\text{-}i,\,j}) = 4i + 1 + 2(j\text{-}1)(n\text{-}1), \qquad 1 \leq i \leq n/2 - 1$$

$$f^* (v_{0,\,j}v_{1,\,j}) = 3 + 2(j\text{-}1)\,(n\text{-}1)$$

$$f^* (v_{n/2,\,j}\, v_{n/2\,+1,\,j}) = 2n - 1 + 2(j\text{-}1)\,(n\text{-}1)$$

From the above labelingit is evident that the edge labels of $E_{2,}$ $E_1$and E_3 are in an increasing sequence as i increases and are distinct. Also that the edge labels of E_4 are distinct as $2n - 1 > 3$ for $n \geq 6$. Hence G is a vertex even mean graph. An illustration for this case is given in Figure3.

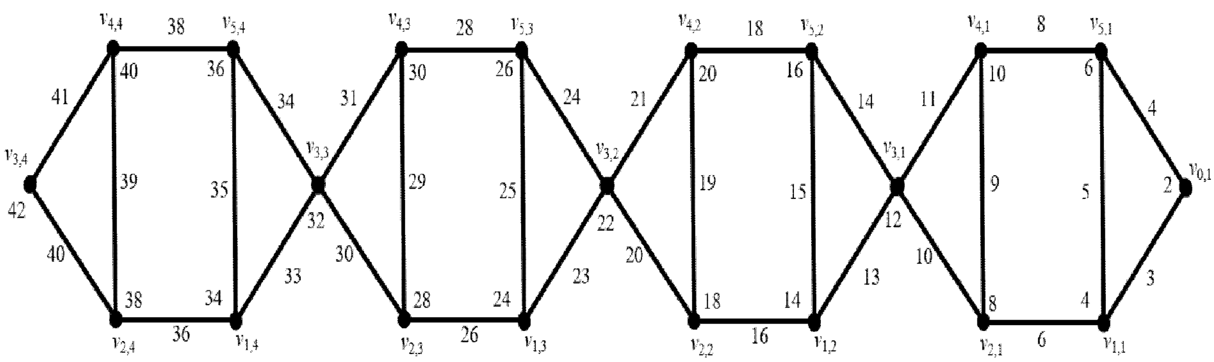

Figure 3. Vertex Even Mean Labeling of $C_{6,4}$ with Parallel Chords

Theorem 2.3: Every crown $C_n \odot K_1$ ($n \geq 6$) with parallel chords is a vertex even mean graph

Proof: A cycle C_n with parallel chords is considered. By attaching a pendant edge at each vertex of cycle C_n with parallel chords, a Crown $C_n \odot K_1$ with parallel chords is obtained. Let this graph be G'. The vertices of cycle C_n are denoted by $v_0, v_1, \ldots v_{n\text{-}1}$ and let $v_0', v_1', \ldots v_{n\text{-}1}'$ be the pendant vertices of G'. Then by definition 1.4,G' has $2n$ vertices and $M + n$ edges where $M = (3n\text{-}3) / 2$ if n is odd and $M = (3n\text{-}2) / 2$ if n is even. Define the vertex labelling $f : V(G') \rightarrow \{2,4,\ldots.2(M{+}n)\}$ as follows for the two cases depending on n.

$$f(v_i) = 8i - 2, \qquad 1 \leq i \leq n/2 \text{ if } n \text{ is even and } 1 \leq i \leq (n\text{-}1)/2 \text{ if } n \text{ is odd}$$

$$f(v_{n\text{-}i}) = 8i + 2, \qquad 0 \leq i \leq n/2 - 1 \text{ if } n \text{ is even and } 0 \leq i \leq (n\text{-}1)/2 \text{ if } n \text{ is odd}$$

$$f(v_0') = \begin{cases} 4n - 4 \text{ if } n \text{ is even} \\ 4n \text{ if } n \text{ is odd} \end{cases}$$

$$f(v_i') = 8i - 4, \qquad 1 \leq i \leq n/2 - 1 \text{ if } n \text{ is even and } 1 \leq i \leq (n\text{-}1)/2 \text{ if } n \text{ is odd}$$

$$f(v_{n\text{-}i}') = 8i, \qquad 1 \leq i \leq n/2 \text{ if } n \text{ is even and } 1 \leq i \leq (n\text{-}1)/2 \text{ if } n \text{ is odd}$$

From the above it is seen that all the vertices are labeled and are distinct.

$E(G) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_1' \cup E_2' \cup E_3'$ is the edge set where

$$E_1 = \{v_i\, v_{i+1},\ 1 \leq i \leq n/2 - 1 \text{ if } n \text{ is even and } 1 \leq i \leq (n\text{-}3)/2 \text{ if } n \text{ is odd}\}$$

$$E_2 = \{v_{n\text{-i}}v_{n\text{-}(i+1)},\ 0 \leq i \leq n/2 - 2 \text{ if } n \text{ is even and } 0 \leq i \leq (n\text{-}3)/2 \text{ if } n \text{ is odd}\}$$

$E_3 = \{v_i v_{n-i},\ 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd$\}$

$E_4 = \{ v_0 v_0', v_0 v_1\}$

$E_1' = \{v_i v_i', 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd$\}$

$E_2' = \{v_{n-i} v_{n-i}', 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd$\}$

$E_3' = \{v_{n/2} v_{n/2}',\ v_{n/2} v_{n/2+1}$ if n is even$\}$

Define the induced function $f^* : E(G') \to I$ (Set of Positive integers) as

$f^*(v_i v_{i+1}) = 8i + 2$, $\quad 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-3)/2$ if n is odd

$f^*(v_{n-i} v_{n-(i+1)}) = 8i + 6$, $\quad 0 \le i \le n/2 - 2$ if n is even and $0 \le i \le (n-3)/2$ if n is odd

$f^*(v_i v_{n-i}) = 8i$, $\quad 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

$f^*(v_0 v_0') = 2n - 1$ $\quad$ if n is even

$f^*(v_0 v_0') = 2n + 1$ $\quad$ if n is odd

$f^*(v_0 v_1) = 4$

$f^*(v_i v_i') = 8i - 3$, $\quad 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

$f^*(v_{n-i} v_{n-i}') = 8i + 1$, $\quad 1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

If n is even,

$f^*(v_{n/2} v_{n/2+1}) = 4n - 4$

$f^*(v_{n/2} v_{n/2}') = 4n - 1$.

From the above labeling it is observed that all the edge labels are distinct andG' is a vertex even mean graph. An illustration for this case is given in Figure 4& Figure5.

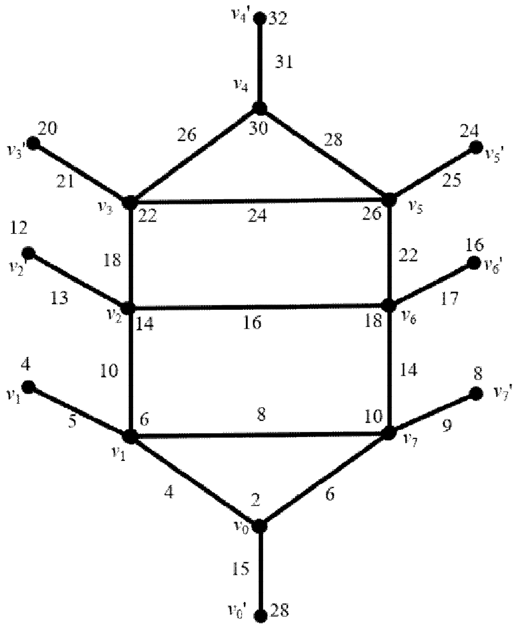

Figure 4. Vertex Even Mean Labeling of Crown $C_8 \Theta K_1$ with Parallel Chords

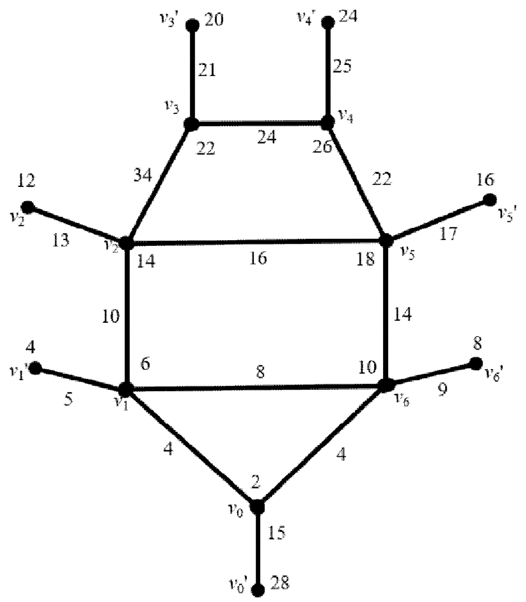

Figure 5. Vertex Even Mean Labeling of Crown $C_7 \Theta K_1$ with Parallel Chords

***Theorem 2.4*:** Subdivided Cycle C_n ($n \geq 6$) with parallel chords is a vertex even mean graph

Proof: By definition 1.6,let G be the subdivided Cycle C_n with parallel chords. The vertices of cycle C_n are denoted by $v_0, v_1, \ldots v_{n-1}$. Let the newly added vertices be $d_0, d_1, \ldots. d_{n-1}$of subdivided edges $v_0v_1, v_1v_2, \ldots v_{n-1}v_n$.Then$G$ has $2n$ vertices and $M + n$ edges.

Define the vertex labeling$f : V(G) \rightarrow \{2,4,\ldots.2(M + n)\}$ as

$f(v_i) = 8i$, $1 \leq i \leq n/2$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

$f(v_{n-i}) = 8i + 2$, $0 \leq i \leq n/2 - 1$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

$f(d_i) = 8i + 4$, $0 \leq i \leq n/2 - 1$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

$f(d_{n-i}) = 8i - 2$, $1 \leq i \leq n/2$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

It is seen from the above that all the vertices labeled are distinct.

The edge set $E(G) = E_1 U E_2 U E_3 UE_4 UE_5 UE_6UE_7$is given by

$E_1 = \{v_i d_i, 1\leq i \leq n/2 - 1$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd$\}$

$E_2 = \{v_i d_{i-1}, 1\leq i \leq n/2$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd$\}$

$E_3 = \{v_{n-(i-1)} d_{n-i}, \leq i \leq n/2$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd$\}$

$E_4 = \{v_{n-i} d_{n-i}, 1 \leq i \leq n/2 -1$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd$\}$

$E_5 = \{v_i v_{n-i}, 1 \leq i \leq n/2 -1$ if n is even and $1\leq i \leq (n-3)/2$ if n is odd$\}$

$E_6 = \{v_0 d_0\}$and$E_7 = \{v_{n/2}\, d_{n/2}$ if n is even and $v_{(n+1)/2}\, d_{(n-1)/2}$ if n is odd$\}$

Define the induced function $f^*: E(G) \rightarrow I$ (Set of Positive integers) as

$f^*(v_i d_i) = 8i + 2$, $1 \leq i \leq n/2 - 1$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

$f^*(v_i d_{i-1}) = 8i - 2$, $1\leq i \leq n/2$ if n is even and $1\leq i \leq (n-1)/2$ if n is odd

$f^*(v_{n-(i-1)}\, d_{n-i}) = 8i - 4$, $1 \le i \le n/2$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

$f^*(v_{n-i} d_{n-i}) = 8i$, $1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

$f^*(v_i v_{n-i}) = 8i + 1$, $1 \le i \le n/2 - 1$ if n is even and $1 \le i \le (n-1)/2$ if n is odd

$f^*(v_0 d_0) = 3$ and

$f^*(v_{n/2}\, d_{n/2}) = 4n - 1$ if n is even

$f^*(v_{(n+1)/2}\, d_{(n-1)/2}) = 4n - 1$if n is odd

It is clear from the above that all the edges receive distinct labels and$G$ is a vertex even mean graph. An illustration for this case is given in Figure 6&Figure 7.

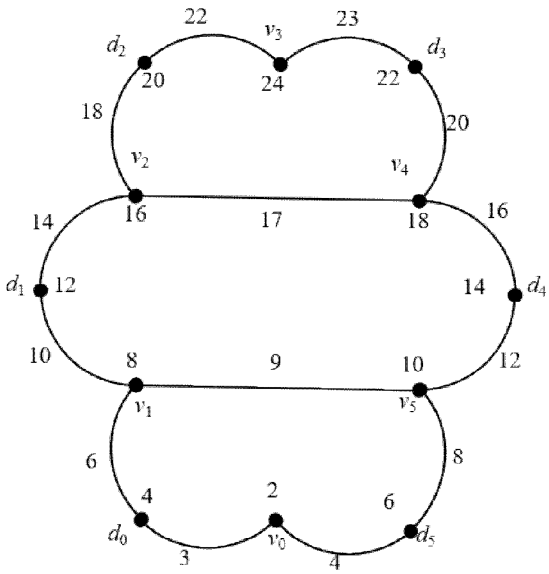

Figure 6. Vertex Even Mean Labeling of Subdivision of C_6 with Parallel Chords

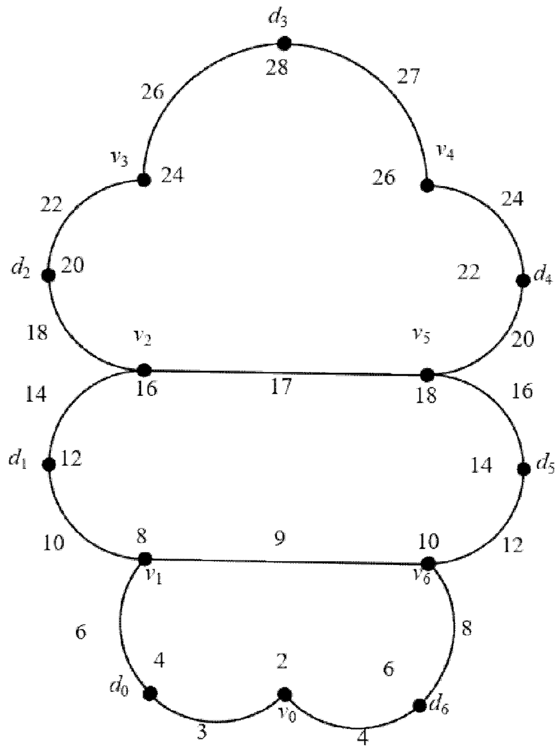

Figure 7. Vertex Even Mean Labeling of Subdivision of C_7 with Parallel Chords

Theorem 2.5: *Two copies of odd cycle C_n $(n \ge 7)$ with parallel chords sharing a common edge is a vertex even mean graph*

Proof: Consider two copies of odd cycle $C_n$with parallel chords. Let $v_0, v_1, \ldots v_{n-1}$ be the vertices of first copy of cycle C_n and let $w_0, w_1, \ldots w_{n-1}$ be the vertices of second copy of cycle C_n sharing a common edge $v_{n-1}\, v_{n-2} = w_1\, w_2$. Let this graph be G having 2n -2 vertices and 3n – 4 edges. Define the vertex labeling$f : V(G) \to \{2,4,\ldots.6n-8\}$ as follows:

$f(v_i) = 4i$, $1 \le i \le (n-1)/2$

$f(v_{n-i}) = 4i + 2$, $0 \le i \le (n-1)/2$

$f(w_{n-i}) = 4n + 2i - 6$, $0 \le i \le (n-1)/2$

$f(w_i) = 5n + 2i - 7$, $3 \le i \le (n-1)/2$

All the vertices are labelled and are distinct.

$E(G) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7 \cup E_8$ *is the edge set where*

$E_1 = \{v_i v_{i+1},\ 1 \le i \le (n-3)/2\}$

$E_2 = \{v_{n-i} v_{n-(i+1)},\ 0 \le i \le (n-3)/2\}$

$E_3 = \{v_i v_{n-i},\ 1 \le i \le (n-1)/2\}$

$E_4 = \{w_i w_{i+1},\ 3 \le i \le (n-3)/2\}$

$E_5 = \{w_{n-i} w_{n-(i+1)},\ 0 \le i \le (n-3)/2\}$

$E_6 = \{w_i w_{n-i},\ 3 \le i \le (n-1)/2\}$

$E_7 = \{w_i w_{n-i},\ i = 1,2\}$

$E_8 = \{v_0 v_1,\ w_0 w_1,\ w_2 w_3\}$

Define the induced function $f^* : E(G) \rightarrow I$ *(Set of Positive integers) as follows.*

$f^*(v_i v_{i+1}) = 4i + 2, \quad 1 \le i \le (n-3)/2$

$f^*(v_{n-i} v_{n-(i+1)}) = 4i + 4, \quad 0 \le i \le (n-3)/2$

$f^*(v_i v_{n-i}) = 4i + 1, \quad 1 \le i \le (n-1)/2$

$f^*(w_i w_{i+1}) = 5n + 2i - 6, \quad 3 \le i \le (n-3)/2$

$f^*(w_{n-i} w_{n-(i+1)}) = 4n + 2i - 5, \quad 0 \le i \le (n-3)/2$

$f^*(w_i w_{n-i}) = (9n + 4i - 13)/2, \quad 3 \le i \le (n-1)/2$

$f^*(w_i w_{n-i}) = 2n + 3i - 2, \quad i = 1,2$

$f^*(v_0 v_1) = 3,\ f^*(w_0 w_1) = 2n,\ f^*(w_2 w_3) = (5n + 9)/2$

From the above it is clearly observed that all the edges receive distinct labels. Hence G is a vertex even mean graph. An illustration for this case is given in Figure 8.

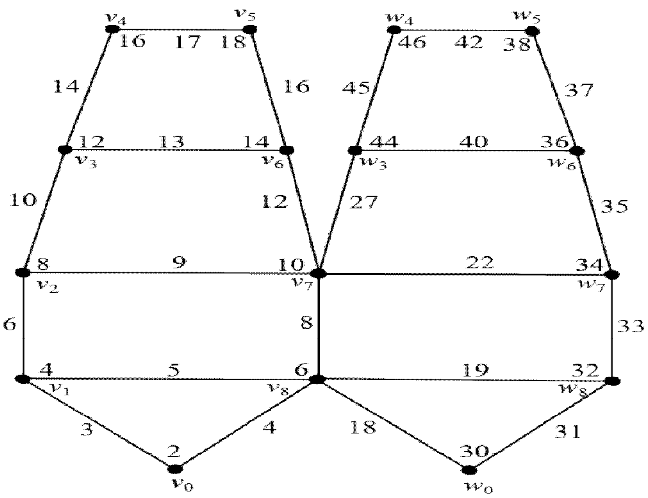

Figure 8. Vertex Even Mean Labeling of Two Copies of C_9 with Parallel Chords sharing a Common Edge

Theorem 2.6: Two copies of odd cycle C_n ($n \ge 7$) with parallel chords sharing a common vertex is a vertex even mean graph

Proof: Consider two copies of odd cycle $C_n$with parallel chords. Let v_0, $v_1,\ldots,v_{n-1}$ be the vertices of first copy of cycle C_n and let w_0, w_1, $\ldots,w_{n-1}$ be the vertices of second copy of cycle C_n sharing a common vertex v_0 which is same as w_0. Let this graph be denoted by G. Then G has $2n$ -1 vertices and $3n – 3$ edges.

Define the vertex labeling$f : V(G) \rightarrow \{2,4,\ldots.6n – 6\}$ as

$f(v_i) = 4i$, $\quad 1 \leq i \leq (n-1) / 2$

$f(v_{n-i}) = 4i + 2$, $\quad 0 \leq i \leq (n-1) / 2$

$f(w_i) = 4n + 4i - 6$, $\quad 1 \leq i \leq (n-1) / 2$

$f(w_{n-i}) = 4n + 4i - 4$, $\quad 1 \leq i \leq (n-1) / 2$

It is seen from the above that all the vertices label dare distinct.

The edge set $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7$ is given by

$E_1 = \{v_i v_{i+1}, 1 \leq i \leq (n-3)/2\}$

$E_2 = \{v_{n-i}v_{n-(i+1)}, 0 \leq i \leq (n-3)/2\}$

$E_3 = \{v_i v_{n-i}, 1 \leq i \leq (n-1)/2\}$

$E_4 = \{w_iw_{i+1}, 1 \leq i \leq (n-3)/2\}$

$E_5 = \{w_{n-i}w_{n-(i+1)}, 1 \leq i \leq (n-3)/2\}$

$E_6 = \{w_iw_{n-i}, 1 \leq i \leq (n-1)/2\}$and $E_7 = \{v_0 v_1, v_0 w_1, v_0 w_{n-1}\}$

Define the induced function $f^* : E(G) \rightarrow I$ (Set of Positive integers) as

$f^*(v_i v_{i+1}) = 4i + 2$, $\quad 1 \leq i \leq (n-3)/2$

$f^*(v_{n-i}v_{n-(i+1)}) = 4i +4$, $\quad 0 \leq i \leq (n-3)/2$

$f^*(v_i v_{n-i}) = 4i + 1$, $\quad 1 \leq i \leq (n-1)/2$

$f^*(w_iw_{i+1}) = 4n + 4i - 4$, $\quad 1 \leq i \leq (n-3)/2$

$f^*(w_{n-i}w_{n-(i+1)}) = 4n + 4i – 2$, $\quad 1 \leq i \leq (n-3)/2$

$f^* (w_iw_{n-i}) = 4n +4i - 5$, $\quad 1 \leq i \leq (n-1)/2$

$f^*(v_0 v_1) = 3$,

$f^*(v_0 w_1) = 2n$ and

$f^*(v_0 w_{n-1}) = 2n + 1$

From the above it is evident that all the edge labels are distinct. Hence G is a vertex even mean graph. An illustration is given in Figure 9.

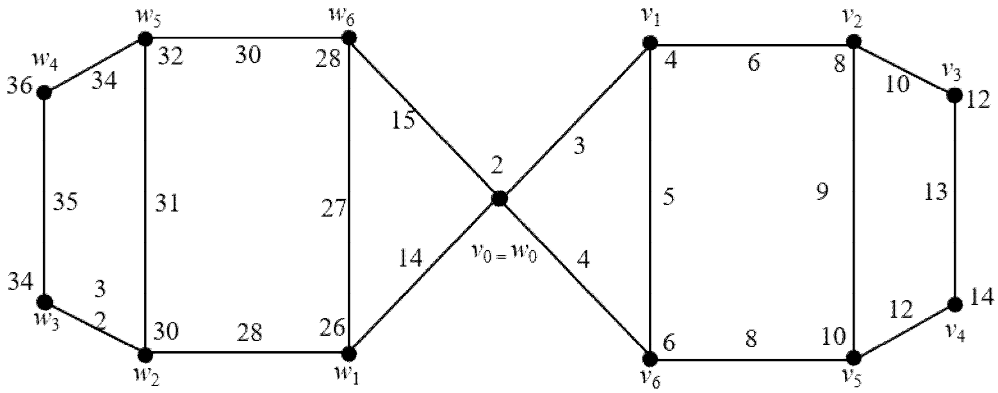

Figure 9. Vertex Even Mean Labeling of Two Copies of C_7 with Parallel Chords Sharing a Common Vertex

CONCLUSION

In this paper we have obtained new families of graphs that are vertex even mean graphs using graph operations on cycles with parallel chords. In future we prove yet another labeling on similar graph families.

REFERENCES

1. ArockiaAruldoss.J, Pushparaj.S, "Vertex odd mean and even mean labeling of fan graph, mangolian tent, and $K_2 + C_n$," International Journal of Mathematics and its Applications, 2016; 4(4):223-227.
2. Elumalai and AnandEphremnath, "Gracefulness of a cycle with zigzag chords", Global Journal of Pure and Applied Mathematics, 2015;1(5): 629-635
3. Gallian.J.A., "A dynamic survey of graph labeling", Electronics Journal of Combinatorics, 2014; l(17): #DS6,
4. Gayathri.B and Gopi.R, "Cycle related mean graphs," Elixir Applied Mathematics, 2014; 71:25116 – 25124
5. Golomb.S.W, "How to number a graph in graph theory and computing", R.C. Read ex., Academic, New York, 1972 :23–37.
6. Govindarajan.R. and Srividya.V, "Odd graceful labeling of cycle with parallel P_k chords", Annals of Pure and Applied Mathematics, 2014;8(2):123–129
7. Raval.K.K and Prajapati.U.M, "Vertex Even and Odd Mean labeling in the context of Some Cyclic Snake Graphs", Journal of Emerging Technologies and Innovative Research, 2018; 5(7):319-324
8. Revathi.N, "Vertex odd mean and even mean labeling of some graphs," IOSR Journal of Mathematics, 2015;11(2):70-74

9. Rosa.A, "On certain valuation of the vertices of a graph, Theory of graphs", Proceedings of the Symposium, Rome, Gordon and Breach, New York, 1967:349–35
10. Sankar.K and Sethuraman.G, "Graceful and Cordial labeling of subdivision of Graphs", Electronic Notes in Discrete Mathematics, September 2016; 53:123-131.
11. Sekar.C, "Studies in Graph Theory", Ph.DThesis , Madurai Kamaraj University, 2002
12. Sethuraman.G and Elumalai.A, "Gracefulness of a cycle with parallel P_k chords", Australian Journal of Combinatorics, 2005; 32:205-211
13. Somasundaram.S and Ponraj.R, "Mean labeling of graphs", National Academy Science Letter, 2003; 26:10 – 13
14. Srividya.V and Govindarajan.R, "On odd harmonious labeling of even cycles with parallel chords and dragons with parallel chords", International Journal of Computer Aided Engineering and Technology, Accepted 2018
15. Srividya.V and Govindarajan.R**,** "Strongly multiplicativelabeling of Cycle $C_n$with Parallel P_3 chords", Mathematical Sciences International Research Journal, 2018;7(spl issue 4):66-74
16. Vaidya.S.K. andKanani.K.K, "Some new mean graphs", International Journal of Information Science and Computer Mathematics,2010; 1:73-80.
17. Vaidya.S.Kand LekhaBijukumar, "Mean labeling for some new families of graphs", Journal of Pure and Applied Sciences,2010;1(18):115-116.