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Connected Domination Number of Fuzzy Digraphs

Sarala N[*] and Janaki P[**]

[*] (Department of Mathematics, A.D.M.College for Women, Nagapattinam, Tamilnadu, India)
[**] (Department of Mathematics, Rabiammal Ahamed Maideen College for women,Thiruvarur Tamilnadu, India)

ABSTRACT:

In this paper we introduce Connected dominating set and connected Domination number of a fuzzy digraph and denoted as $\gamma_c(\vec{\xi})$. The relation between the connected domination number of fuzzy digraph and its vertices is also discussed.

KEYWORDS

Fuzzy digraph, Spanning fuzzy sub digraph, Connected dominating set , Connected domination number .

***Corresponding author:**

Sarala N

Department of Mathematics,

A.D.M.College for Women,

Nagapattinam, Tamilnadu, India

I INTRODUCTION

The concept of fuzzy graph was introduced by Rosenfeld[1] in 1975. Fuzzy graph theory has a vast area of applications. It is used in evaluation of human cardiac function, fuzzy neural networks, etc. Fuzzy graphs can be used to solve traffic light problem, time table scheduling, etc. In fuzzy set theory, there are different types of fuzzy graphs which may be a graph with crisp vertex set and fuzzy edge set or fuzzy vertex set and crisp edge set or fuzzy vertex set and fuzzy edge set or crisp vertices and edges with fuzzy connectivity, etc. A lot of works have been done on fuzzy graphs [3,4,5]

The Fuzzy Directed Graph, Fuzzy competition digraphs are well known topic.The concept of domination in fuzzy graphs are introduced by A. Somasundaram and S. Somasundaram [] in 1998. In this paper we analyze bounds on connected dominating set of fuzzy digraph and proves some results based on connected dominating fuzzy digraph.

II PRELIMINARIES

Definition 2.1:

Fuzzy digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$ is a non-empty set V together with a pair of functions$\sigma : V \to [0,1]$ and $\vec{\mu} : V \times V \to [0,1]$ such that for all x, y ∈ V , $\vec{\mu}(x,y) \leq \sigma(x) \wedge \sigma(y)$.Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}(u,v)$ is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}(x,x) \neq 0$. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}(x,y)$ and $\vec{\mu}(y,x)$ may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges.

Definition 2.2:

The fuzzy sub digraph $\vec{\zeta}\ (V_1, \sigma', \vec{\mu}')$ is said to be spanning fuzzy sub digraph of $\vec{\xi}(V,\sigma,\vec{\mu})$ if $\sigma(u) = \sigma'(u)$ for all u $\in V_1$ and $\vec{\mu}'(u,v) \leq \vec{\mu}(u,v)$ for all u, v ∈ V.

Definition2.3:

The Strength of the connectedness between two vertices (u,v) in a fuzzy Dgraph $\vec{\xi}$ is $\vec{\mu}^{\infty}(u,v)=\text{Sup}\{\vec{\mu}^{k}(u,v);k=1,2,3\ldots.\}$ where $\vec{\mu}^{k}(u,v)=\text{Sup}\{\vec{\mu}(u,u_1) \wedge \vec{\mu}(u_2,u_3) \wedge \ldots\ \vec{\mu}(u_{k-1},v)\}$. An directed arc (u,v) is said to be strong arc if $\vec{\mu}(u,v)=\vec{\mu}^{\infty}(u,v)$.If $\vec{\mu}(u,v)=0$ for every v ∈V then u is called isolated vertices.

Definition2.4:

An directed arc (u,v) is called a fuzzy bridge in $\vec{\xi}$, if the removal of (u,v) reduces the strength of connectedness between some pair of nodes in $\vec{\xi}$.

Definition2.4:

Let $\vec{\xi} = (V,\sigma,\vec{\mu})$ be a fuzzy digraph and D $\subseteq$V .D is a Dominating set if for every u ϵV-D there exist vϵ D such that (u,v) is strong directed arc and σ(u) $\leq$σ(v).

Definition2.5:

A Dominating set D of a fuzzy Digraph with minimum number of vertices is called a minimum dominating set. The domination number of $\vec{\xi}$ is denoted by $\gamma(\vec{\xi})$.Domination number of a fuzzy graph is the sum of the membership values of the vertices of a minimum dominating set.

III Connected Dominating set in Fuzzy Digraphs.

Definition 3.1:

A dominating set D of a Fuzzy Digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$ is connected dominating set if the induced fuzzy subdigraph $\vec{\zeta}$=<D> is connected. The minimum cardinality of a connected dominating set of $\vec{\xi}$ is called the connected domination number of $\vec{\xi}$ and is denoted by $\gamma_c(\vec{\xi})$.

Theorem 3.2:

Let $\vec{\xi} = (V,\sigma,\vec{\mu})$ be a connected fuzzy digraph of order n and let $\gamma_c(\vec{\xi})$=d, let S $\subseteq$V be such that <S> is connected then $|N(S) - S| \leq$n-d.(Since N(S) = $N^+(S) \cup N^-(S)$)

Proof:The result is clear if d$\leq |S|$ so assume d$> |S|$.If V-S-N(S)= Φ then $\gamma_c(\vec{\xi})$< d a contradiction.

Let $\vec{\zeta}$=< V-S-N(S)> and let $\vec{\zeta}_1\vec{\zeta}_2\ldots\vec{\zeta}_l$ be the connected components of $\vec{\zeta}$Because $\vec{\xi}$ is connected for each i ,1$\leq i \leq l$,there exists $u_i \epsilon$ N(S) and v_i ϵV($\vec{\zeta}_i$) such that u_i is adjacent v_i is where $\mu(u_i,v_i)$>0 $\forall$ i. The subgraph induced by X is connected and that X dominates $\vec{\xi}$. Hence d $|X| \leq |V| - |N(S) - S|$ that is $|N(S) - S| \leq |V|$-d.

Corollary 3.3:

If is $\vec{\xi}$ connected and has n vertices and $\gamma_c(\vec{\xi})$=d then every vertex $\vec{\xi}$ in has degree at most n-d.
Proof: Apply the proceeding theorem with S consisting of just one vertex.

Theorem 3.4:

If $\vec{\xi}$ is connected fuzzy digraph and n≥ 3 then $\gamma_c(\vec{\xi}) \leq$n-2.

Proof:Let T($V_1,\tau,\vec{\rho}$) be a spanning fuzzy directed tree of $\vec{\xi}$ such that $\tau(u) = \sigma(u)$ for all uϵV_1 and $\vec{\rho}$(u,v) $\leq \vec{\mu}$(u,v) with $\varepsilon_T(\vec{\xi})$ pendant vertices and let L denote the set of pendant vertices then T-L is connected dominating set having n-$\varepsilon_T(\vec{\xi})$ vertices , that is $\gamma_c(\vec{\xi}) \leq \varepsilon_T(\vec{\xi})$. Conversely , Let D be connected dominating set .Since < D> is connected . < d> has a spanning tree T_s. A spanning tree T of $\vec{\xi}$ is formed by adding the remaining n- $\gamma_c(\vec{\xi})$ vertices of V-D to T_s. and adding edges of $\vec{\xi}$ such that the vertex in V-D is adjacent to exactly one vertex in D . Now T has atleast n-$\gamma_c(\vec{\xi})$ pendant vertices . Thus $\varepsilon_T(\vec{\xi}) \geq n - \gamma_c(\vec{\xi})\ or\ \gamma_c(\vec{\xi}) \geq n - \varepsilon_T(\vec{\xi})$. Hence $\gamma_c(\vec{\xi}) = n - \varepsilon_T(\vec{\xi})$ and since $\varepsilon_T(\vec{\xi}) \geq 2$ $ie.,\gamma_c(\vec{\xi}) \leq$ n-2.

Theorem 3.5:

Let G be a connected fuzzy digraph and have n vertices m edges if n $\geq$4, m $\geq$n and G is not a circuit then $\gamma_c(\vec{\xi}) \leq$n-3.

Proof: If $\vec{\xi}$ is not a circuit then T is a spanning fuzzy directed tree of $\vec{\xi}$. $\vec{\xi}$ must have at least one vertex V with degree atleast 3. By theorem 3.2 $\gamma_c(\vec{\xi}) \leq$n-3.

Theorem 3.6:

For any connected fuzzy digraph $\vec{\xi}$, $\gamma_c(\vec{\xi}) \leq$n-$\Delta(\vec{\xi})$.

Proof :Let u be a vertices of $\vec{\xi}$ such that the strong neighborhood of u is N_s(u)= { vϵV :(u,v) is an strong arc} equal to the maximum cardinality of a strong neighborhood $\Delta_s(\vec{\xi})$ then V- N_s(u) is a dominating set . Therefore $\gamma(\vec{\xi}) \leq |V - N_s(u)| \leq n - \Delta_s(\vec{\xi})$.The maximum cardinality of a strong neighborhood is maximum degree of u and $\gamma(\vec{\xi}) \leq \gamma_c(\vec{\xi})$ that is $\gamma_c(\vec{\xi}) \leq$n-$\Delta(\vec{\xi})$.

***Corollary 3*.7:**For any fuzzy directed tree T, $\gamma_c(T)$ =n-$\Delta(T)$ then T has atmost one vertex of degree three or more.

Theorem 3.8:

If $\vec{\zeta}$ is connected fuzzy spanning subdigraph of G then $\gamma_c(\vec{\xi}) \leq \gamma_c(\vec{\zeta})$.

Proof:If $\vec{\zeta}$ is connected spanning fuzzy sub digraph of a connected fuzzy graph then every connected dominating set of $\vec{\zeta}$ is also connected dominating set of $\vec{\zeta}$ and V = V_1 . The spanning subgraph of $\vec{\zeta}$ has connected dominating set but need not be a minimum fuzzy dominating set. ie, $\gamma_c(\vec{\xi}) \leq \gamma_c(\vec{\zeta})$.

V CONCLUSION

The connected dominating number of fuzzy digraph is defined. Theorem related to this concept are derived and the relation between the dominating number of fuzzy digraph and dominating number of spanning fuzzy sub digraph are established.

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