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Metric space, Applications and its properties

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ABSTRACT

A **metric** or **distance** function is a function that defines a **distance** between each pair of elements of a set. The main source of **metrics** in differential geometry are **metric** tensors, bilinear forms that may be defined from the tangent vectors of a differentiable manifold onto a scalar. A **metric space** is a set X together with a function d is called a **metric** or "distance function" which is denoted by $d(x, y)$.

KEYWORDS : Metric space, quantum, convergence, distance

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1. INTRODUCTION

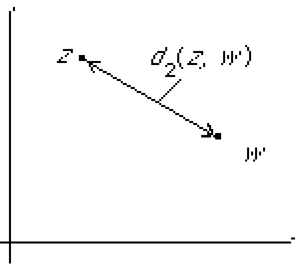
Properties of Metric

To every pair $x, y \in X$ the metric satisfying the following *axioms.*

1. $d(x, y) \geq 0$ and $d(x, y) = 0 \Leftrightarrow x = y$,
2. $d(x, y) = d(y, x)$,
3. $d(x, y) + d(y, z) \geq d(x, z)$.

Examples[1] :

1. Euclidean Space. Space R d equipped with the Euclidean distance d(x, y) = kx−yk
2. 1 2. Uniform Metric. Let X be an arbitrary non-empty set. Define a distance function d(x, y) on X by d(x, y) = 1 if x 6= y and d(x, x) = 0. The space (X, d) is called a uniform or discrete metric metric space.
3. Shortest Path Metric on Graphs. Let G = (V, E, l) be a graph with positive edge lengths l(e). Let d(u, v) be the length of the shortest path between u and v. Then (V, d) is the shortest path metric on G.
4. Tree Metrics. A very important family of graph metrics is the family oof tree metrics. A tree metric is the shortest path metric on a tree T.
5. Cut Semi-metric. Let V be a set of vertices and S ⊂ V be a proper subset of V . Cut semi-metric δS is defined by δS(x, y) = 1 if x ∈ S and y /∈ S, or x /∈ S and y ∈ S; and δS(x, y) = 0, otherwise. In general, the space (X, d) is not a metric since d(x, y) = 0 for some x
6. The line **R** with its usual distance $d(x, y) = |x - y|$.
7. The plane $\mathbf{R}^2$ with the "usual distance" (measured using Pythagoras's theorem):
   $d((x_1 , y_1), \quad (x_2 , y_2)) \quad = \sqrt{[(x_1 - x_2)^2 + \quad (y_1 - y_2)^2]}.$
   This is sometimes called the 2-metric d_2 .



8. The same picture will give metric on the complex numbers **C** interpreted as the Argand diagram. In this case the formula for the metric is now:
   $d(z, w) \quad = \quad |z - w|$
   where the | | in the formula represent the modulus of the complex number rather than the absolute value of a real number.

2. PSEUDOMETRIC [2]

A **pseudometric** on X is a function $d : X \times X \to \mathbf{R}$ which satisfies the axioms for a metric, except that instead of the second (identity of indiscernibles) only $d(x,x)=0$ for all x is required. In other words, the axioms for a pseudometric are:

1. $d(x, y) \geq 0$
2. $\mathbf{d(x, x) = 0}$ (but possibly $d(x, y) = 0$ for some distinct values $x \neq y$.)
3. $d(x, y) = d(y, x)$
4. $d(x, z) \leq d(x, y) + d(y, z)$

DEFINITION (OPEN BALL)

Given a metric space M = (X, d), a ∈ X and r > 0 we define the closed ball of center a and radius r as the set B (a, r) = {x ∈ X : d (a, x) ≤ r} For example, in B ([−1, 1] , R) when we take the center f (x) = x 3 and radius r, is easy to see that g ∈B (f, r) ⇐⇒ g (x) ∈ (f (x) − r, f (x) + r) 3.2. Open and closed sets. In our first calculus courses, we saw that an "open set" was one that did not include its "border", or more formally, its "frontier". However, in generic metric spaces this cannot be graphically checked, so we need to have the formal definition of this concept. The basic idea is that for a set to be open (and not include its border), every time we pick an element x ∈ A we must be able to find an open ball around it that it is also completely included on the same set A.

DEFINITION (INTERIOR OF A SET).

Let M = (X, d) be a metric space and A ⊆ X. We say that x ∈ A is interior of A ⇐⇒∃rx > 0 such that B (x, rx) ⊆ A. The set of all interior points of A is called the interior of A, and is written as A˚ Definition 3.4 (Open set). A set A ⊆ X is open ⇐⇒ A = A˚ The first example of open set is in fact, the open balls themselves: Proposition 3.1 (Open balls are open). Given M = (X, d) a metric space, x ∈ X and

Definition 3.4 (Open set). A set A ⊆ X is open ⇐⇒ A = A˚ The first example of open set is in fact, the open balls themselves: Proposition 3.1 (Open balls are open). Given M = (X, d) a metric space, x ∈ X and r > 0, the set A ≡ B (x, r) is an open set.

DEFINITION(CLOSURE SET).

Given a m.s M = (X, d) and a set A ⊆ X , we say that x ∈ X is a closure point of A ⇐⇒ the following rule holds: ∀r > 0 we have B (x, r) ∩ A 6= ∅ i.e. no matter how close you get to x, there is always a point in A which is even closer to x. The set of all closure points of A is called the closure of A .

3.APPLICATION OF METRIC SPACE IN QUANTUM MECHANICS[1]

Hilbert space combines the properties of two different types of mathematical spaces: vector space and metric space. While the vector-space aspects are widely used, the metric-space aspects are much less exploited. We show that conservation laws in quantum mechanics naturally lead to metric spaces for the set of related physical quantities[1]. All such metric spaces have an "onion-shell" geometry. The related metric stratifies Fock space into concentric spheres on which maximum and minimum distances between states can be defined and geometrically interpreted. Unlike the usual Hilbert-space analysis, our results apply also to the reduced space of only ground states and to that of particle densities, which are metric, but not Hilbert, spaces. The Hohenberg-Kohn mapping between densities and ground states, which is highly complex and nonlocal in coordinate description, is found, for the systems analysed, to be simple in metric space, where it becomes a monotonic and nearly linear mapping of vicinities onto vicinities[2]. Similarly, by considering metric spaces associated to many-body systems immersed in a magnetic field, we consider the mapping between wave-function and (current and particle) densities at the core of Current Density functional theory. We find, in the related metric spaces, regions of allowed and forbidden distances, a "band structure", directly arising from the conservation of the z component of the angular momentum[1]. Finally, recent results include appropriate metrics for the external potential which allow us to directly explore the 'third leg' of the Hohenberg-Kohn theorem[3].

4.APPLICATION OF METRIC IN REAL LIFE

Metric is a generalization of ordinary distance.

A metric ‘d' is a function always defined on a set X *X having some properties:-

1. d(x,y)>=0;
2. d(x,y)<=d(x,z)+d(z,y);

The set X together with a function having some properties defined above is called a metric space.

The concept of metric is not restricted to the subsets of R, it can be applied in any set.

Using this we can find the distance between any two points in any given set.for that we shall first of all define a function on that set having those two properties.

Once we define a function having such properties, we can use it in any place where the distance between two points is required.

5. APPLICATION IN CONVERGENCE[3,4]

Metric spaces are the setting in which convergence of sequences can be defined: Let $(x_n)_{n \in N}$ be a sequence and x a point in M. We say that $(x_n)_{n \in N}$ converges to x, and write $\lim_{n \to \infty} x_n = x$, if for every number $\varepsilon > 0$ there exists an integer n_0 such that for all integers $n > n_0$ the inequality $d(x_n, x) < \varepsilon$ holds. The participants had encountered about limits of functions at school and were used to the notation $\lim_{x \to \infty} f(x)$. Our definition above is the special case where the domain of f is the set of positive integers, i.e., $f : N \to R$, $f(n) = x_n$. This is, however, usually not discussed in German schools. In addition, limits of functions are not explained in a rigorous way. Nevertheless, we observed that our students had some intuitive idea about the concept of a limit. We used Application in order to point out that the notion of a metric space now allows for a formal definition. This is a clear advantage compared with an only intuitive idea. Moreover, it shows that convergence is not an intrinsic feature of the real numbers, but depends on the selection of a metric

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