Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

A note on Cubic Difference Prime Labeling

Sunoj B.S.[1*] and Mathew Varkey T. K.[2]

*[1]Department of Mathematics, Government Polytechnic College, Attingal, Kerala
[2]Department of Mathematics,T K M College of Engineering , Kollam 5, Kerala
spalazhi@yahoo.com, mathewvarkeytk@gmail.com

ABSTRACT

Cubic difference prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the cubes of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits cubic difference prime labeling. Here we identify wheel graph, gear graph, fan graph, friendship graph, prism graph, key graph, helm graph and umbrella graph for cubic difference prime labeling.

KEYWORDS: Graph labeling, greatest common incidence number, cubic difference, prime labeling.

***Corresponding Author**

B.S. Sunoj

Department of Mathematics, Government Polytechnic College,

Attingal, Kerala

E Mail - spalazhi@yahoo.com,

1. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1, 2, 3, 4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated cube difference prime labeling of wheel graph, gear graph, fan graph, friendship graph, prism graph, key graph, helm graph and umbrella graph.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

Definition: 2.1 G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{cdpl}^{*} : E(G) →set of natural numbers N by $f_{cdpl}^{*}(uv) = |\{f(u)\}^3 - \{f(v)\}^3|$The induced function f_{cdpl}^{*} is said to be cubic difference prime labeling, if for each vertex of degree at least 2, the greatest common incidence number of the labels of the incident edges is 1.

Definition: 2.2 Graph which admits cubic difference prime labeling is called a cubic difference prime graph.

Theorem 2.1 Wheel graph[1] W_n admits cubic difference prime labeling.

Proof: Let G =W_n and let v_1,v_2,----------------,v_{n+1} are the vertices of G

Here |V(G)| = n+1 and |E(G)| = 2n

Define a function f : V → {0,1,2,3,----------------,n } by

$f(v_i)$ = i-1 , i = 1,2,------,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{cdpl}^{*} is defined as follows

$f^*_{cdpl}(v_i v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_{n+1} v_i)$ = n^3-$(i-1)^3$, i = 1,2,------------,n

$f^*_{cdpl}(v_1 v_n)$ = $(n-1)^3$

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = gcd^2 of $\{f^*_{cdpl}(v_i v_{i+1}), f^*_{cdpl}(v_{i+1} v_{i+2})\}$

= gcd of $\{3i^2-3i+1, 3i^2+3i+1\}$

= gcd of $\{6i^2, 3i^2-3i+1\}$

= gcd of $\{3i^2, 3i^2-3i+1\}$

= gcd of $\{3i-1, 3i^2-3i+1\}$

=gcd of $\{i, 3i-1\}$

= gcd of $\{i, i-1\}$ =1, i = 1,2,------------,n-1

gcin of (v_1) = gcd of $\{f^*_{cdpl}(v_1 v_2), f^*_{cdpl}(v_1 v_n)\}$

= gcd of $\{1, (n-1)^3\}$=1.

gcin of (v_{n+1}) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence W_n, admits cubic difference prime labeling. ■

Theorem 2.2 Gear graph[3] G_n admits cubic difference prime labeling.

Proof: Let G =G_n and let v_1,v_2,----------------,v_{2n+1} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i)$ = i-1 , i = 1,2,------,2n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,2n

$f^*_{cdpl}(v_{2n+1} v_{2i})$ = $(2n)^3$-$(2i-1)^3$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_1 v_{2n})$ = $(2n-1)^3$,

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-1

gcin of (v_1) = gcd of $\{f^*_{cdpl}(v_1 v_2), f^*_{cdpl}(v_1 v_{2n})\}$

= gcd of {1, $(2n-1)^3$}=1

gcin of (v_{2n+1}) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G_n, admits cubic difference prime labeling. ■

Theorem 2.3 Fan graph[4] F_n admits cubic difference prime labeling.

Proof: Let G =F_n and let v_1,v_2,----------------,v_{n+1} are the vertices of G

Here |V(G)| = n+1 and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,----------------,n } by

$f(v_i)$ = i-1 , i = 1,2,------,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i v_{i+1})$ = $i^3-(i-1)^3$, i = 1,2,------------,n

$f^*_{cdpl}(v_{n+1} v_i)$ = $(n)^3-(i-1)^3$, i = 1,2,------------,n-1

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-1

gcin of (v_1) = 1.

gcin of (v_{n+1}) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence F_n, admits cubic difference prime labeling. ■

Theorem 2.4 Prism graph[3] Y_n admits cubic difference prime labeling.

Proof: Let G =Y_n and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i v_{i+1})$ = $i^3-(i-1)^3$, i = 1,2,------------,2n-1

$f^*_{cdpl}(v_i v_{2n+i-1})$ = $(2n-i)^3-(i-1)^3$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_1 v_n)$ = $(n-1)^3$,

$f^*_{cdpl}(v_{n+1} v_{2n})$ = $(2n-1)^3 - n^3$.

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-2

gcin of (v_1) = 1.

gcin of (v_{2n}) = gcd of $\{f^*_{cdpl}(v_{2n-1} v_{2n}), f^*_{cdpl}(v_1 v_{2n})\}$

= gcd of $\{12n^2-18n+7, (2n-1)^3\}$

= gcd of $\{12n^2-18n+7, (2n-1)\}$

= gcd of {(2n-1(6n-6)+1, (2n-1)}=1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Y_n, admits cubic difference prime labeling. ■

Theorem 2.5 Friendship graph[3] F_n admits cubic difference prime labeling.

Proof: Let G =F_n and let u, v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i) = i$, i = 1,2,------,2n

f(u) = 0.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(u\, v_{2i-1})$ = $(2i-1)^3$, i = 1,2,------------,n

$f^*_{cdpl}(u\, v_{2i})$ = $8i^3$, i = 1,2,------------,n

$f^*_{cdpl}(v_{2i-1} v_{2i})$ = $12i^2-6i+1$, i = 1,2,------------,n

Clearly f^*_{cdpl} is an injection.

gcin of (v_{2i-1}) = gcd of $\{f^*_{cdpl}(u\, v_{2i-1}), f^*_{cdpl}(v_{2i-1} v_{2i})\}$

= gcd of $\{(2i-1)^3, 12i^2-6i+1\}$

= gcd of {2i-1 ,6i(2i-1)+1}

= 1, i = 1,2,------------,n

gcin of (v_{2i}) = gcd of $\{f^*_{cdpl}(u\, v_{2i}), f^*_{cdpl}(v_{2i-1} v_{2i})\}$

= gcd of $\{(2i)^3, 12i^2-6i+1\}$

= gcd of {2i , 2i(6i-3)+1}

= 1, i = 1,2,------------,n

gcin of (u) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence F_n, admits cubic difference prime labeling. ■

Theorem 2.6 Umbrella graph[3] U(n,m) admits cubic difference prime labeling.

Proof: Let G =U(n,m) and let $u,v_1,v_2,$----------------$,v_{m+n-1}$ are the vertices of G

Here |V(G)| = m+n and |E(G)| = 2n+m-2

Define a function f : V → {0,1,2,3,----------------,m+n-1 } by

$f(v_i) = i$, i = 1,2,------,m+n-1

f(u) = 0.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(u\, v_i)$ = i^3, i = 1,2,------------,n

$f^*_{cdpl}(u\, v_{n+1})$ = $(n+1)^3$,

$f^*_{cdpl}(v_i v_{i+1})$ = $3i^2+3i+1$, i = 1,2,------------,n-1.

$f^*_{cdpl}(v_{n+i} v_{n+i+1})$ = $(n+i+1)^3 - (n+i)^3$, i = 1,2,------------,m-2.

Clearly f^*_{cdpl} is an injection.

gcin of (v_n) = gcd of $\{f^*_{cdpl}(v_{n-1}v_n)\, ,\, f^*_{cdpl}(u\, v_n)\}$

= gcd of { $3n^2-3n+1, n^3$}

= gcd of {n ,n(3n-3)+1}

= 1,

gcin of (v_{n+1}) = gcd of $\{f^*_{cdpl}(v_{n+1}v_{n+2})\, ,\, f^*_{cdpl}(u\, v_{n+1})\}$

= gcd of { $3n^2+9n+7, (n+1)^3$}

= gcd of {n+1 , (n+1)(3n+6))+1}

= 1

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2

gcin of (v_{n+i+1}) = 1, i = 1,2,------------,m-3

gcin of (v_1) = 1.

gcin of (u) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence U(n,m), admits cubic difference prime labeling. ■

Theorem 2.7 Helm graph[3] H_n admits cubic difference prime labeling.

Proof: Let G =H_n and let v_1,v_2,----------------,v_{2n+1} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i)$ = i-1 , i = 1,2,------,2n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_iv_{i+1})$	= $i^3-(i-1)^3$,	i = 1,2,------------,n
$f^*_{cdpl}(v_{n+1}v_i)$	= $n^3-(i-1)^3$,	i = 1,2,------------,n-1
$f^*_{cdpl}(v_iv_{n+i+1})$	= $(n+i)^3-(i-1)^3$,	i = 1,2,------------,n
$f^*_{cdpl}(v_1v_n)$	= $(n-1)^3$	

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1})	= 1,	i = 1,2,------------,n-1
gcin of (v_1)	= 1.	
gcin of (v_{n+1})	= 1.	

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence H_n, admits cubic difference prime labeling. ■

Theorem 2.8 Key graph[3] K(n,m) admits cubic difference prime labeling.

Proof: Let G =K(n,m) and let v_1,v_2,----------------,v_{n+2m-2} are the vertices of G

Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2

Define a function f : V → {0,1,2,3,----------------,n+2m-3 } by

$f(v_i)$ = i-1 , i = 1,2,------,n+2m-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_iv_{i+1})$	= $i^3-(i-1)^3$,	i = 1,2,---------,m+n-1
$f^*_{cdpl}(v_{n+i}v_{n+2m-i-1})$	= $(n+2m-i-2)^3-(n+i-1)^3$,	i = 1,2,---------,m-2
$f^*_{cdpl}(v_1v_n)$	= $(n-1)^3$	

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,m+n-2

gcin of (v_1) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence K(n,m), admits cubic difference prime labeling. ■

REFERENCES

1. F Harary. Graph Theory. 2nd ed. Addison-Wesley: Massachuset: 1972.
2. Apostol. Tom M. Introduction to Analytic Number Theory. 2nd ed. Narosa: New Delhi: 1998.
3. Joseph A Gallian. A Dynamic Survey of Graph Labeling. The Electronic Journal of Combinatorics 2016; #DS6: 1 – 408.
4. Mathew Varkey T. K. Some Graph Theoretic Generations Associated with Graph Labeling. PhD Thesis. University of Kerala: Kariavattam: 2000.