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Target Excitation Dependence Levy Index Analysis In Hadron-nucleus Interaction

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ABSTRACT

This work presents an in-depth study on Levy index and its dependence on target excitation in two dimensional (η - ϕ) self affine space using the experimental data of pions obtained from $\pi^- - AgBr$ interactions at 200 GeV/c. For studying target excitation dependence the data for produced pions are divided into three sets depending on the number of grey particles (n_g).The different sets corresponds to the different degrees of target excitation. The Levy indices μ measured from the analysis fulfills the requirement of the levy stable region $0 \le \mu \le 2$. The Levy index $\mu \langle 1$ indicates that a thermal phase transition may exist in the $\pi^- - AgBr$ interactions at 200 GeV/c. Further the analysis indicates different degrees of multifractality for different target excitation. Moreover, the value of universal scaling exponent (ν) obtained from Ginzburg-Landau (GL) theory indicates that no evidence of second order phase transition has been found in the interaction.

KEY WORDS: Phase transition; Target excitation; Levy index; Ginzburg – Landau theory

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1. INTRODUCTION:

The ultimate aim of high-energy experiments is to search for the signal of QGP phase transition. Moreover, another interesting aspect to be investigated is the order of such phase transition, if it occurs at all. However the formation of QGP and as well as the order of such phase transition is under debate. Thus further studies about the signal are needed.

According to the predictions of a simple scale invariant cascade model[1], the higher order scaled factorial moments are related to the second order scaled factorial moments by a modified power law, which may provide some vital information about the underlying dynamics.The dependence of ratios of higher order anomalous fractal dimension on the order of moments can help to search for an intermittent type of fluctuations in the multiparticle production process. The Levy stable law[2,3] has been used to study such dependences, where multiplicity fluctuation is described quite successfully. The study of variations of these ratios on the order of moments has suggested the existence of self-similar cascade process and a second order phase transition. If the underlying mechanism is a self-similar cascade mechanism, then it leads to intermittent fluctuations and this type of behaviour is characterised by multifractals, whereas, if it is a second order phase transition, e.g., quark- gluon plasma, the behaviour is characterised by monofractals.

A unique anomalous fractal dimension for different order of moments suggests monofractality whereas order dependence suggests the presence of multifractality. As a self-similar fractal system, the multiparticle final state in high energy collisions can be characterised by an important parameter known as Levy stability index[2,3] μ . This parameter tells us the behaviour of elementary fluctuations at the tail of distribution. To extract its value from experimental data more reliably is an important task for the understanding of fluctuation dynamics. The value of μ lies between 0 and 2. The values of Levy stability index are considered to be a measure of degree of multifractality. When $\mu \langle 1$, there is a thermal phase transition. On the other hand, when $\mu \rangle 1$, there is a non-thermal phase transition during the cascading process[3].

According to Van Hove[4] phase space in high-energy process is anisotropic. The fluctuation pattern is also expected to be anisotropic and the scaling behavior should also be different in different directions giving rise to self-affine scaling. In self-affine scenario, the phase space should be shrunk according to the inherent self-affine parameter --- Hurst exponent H. The Levy index μ obtained only in this way is meaningful in characterizing the self-affine random cascading process.

The existence of second order phase transitions in multiparticle production process can be investigated by Ginsburg-Landau (GL) theory[5,6]. Here, the anomalous fractal dimension (d_q) follows the relation

$$\frac{d_q}{d_2} = (q-1)^{\nu-1}$$

Where scaling exponent ν = 1.304 , a universal quantity that is valid for all systems describable by the GL theory. If the measured value of ν is significantly different from the critical value, then the GL description is inappropriate and second order phase transition can most likely be ruled out. On the other hand, if it is close, then a second order quark-hadron phase transition can be expected.

It is generally believed that grey particles (medium energyknocked out protons)are supposed to carry relevant information about the hadronization mechanism.The number of collisions in nuclei can be measured by the number of grey particles (n_g). Generally, n_g together with the number of pions are used as a measure of violence of the target fragmentation[7,8]. To get more information about the inner dynamics of the particle production in high-energy interactions, the phase transition and its dependence on target excitation has to be studied thoroughly using the available tools. To do this, we have divided the data for produced pions for $\pi^- - AgBr$interactions at 200 GeVinto three sets depending upon the number of grey tracks (n_g). The different data sets correspond to different degrees of target excitation.

In this present paper phase transition study of the produced pions is performed in two dimensional $(\eta - \phi)$ phase space under self-affine scenario imposing special emphasis on Levy stability analysis. Levy stable law has been used to determine the value of μ for different target excitations (different values of n_g) in $\pi^- - AgBr$ interactions at 200 GeV/c to asses the dependence of the phase transition on target excitation. Finally using the GL theory the value of ν is determined to investigate the possibility of second order quark-hadron phase transition.

2. EXPERIMENTAL DETAILS

In this analysis hadron-nucleus interaction data of π^--AgBr at 200 GeV/c has been used. A stack of G5 nuclear emulsion plate was exposed horizontally to a π^-beam at Fermilab with 200 GeV/c.

According to nuclear emulsion terminology[9], the particles emitted in high-energy interactions are classified as:

(a) Black particles: They are target fragments with ionization greater than or equal to 10 I_0 , I_0 being the minimum ionization of a singly charged particle. Their ranges are less than 3 mm.

Their velocity is less than 0.3c and their energy is less than 30 MeV, where c is the velocity of light in free space.

(b) Grey particles: They are mainly fast target recoil protons with energy up to 400 MeV. The ionization power of gray particles lies between 1.4I_0 to 10I_0. Their ranges are greater than 3 mm and they have velocities between 0.3c to 0.7c.

(c) Shower particles: They are mainly pions with ionization ≤ 1.4I_0. These particles are generally not confined within the emulsion pellicle.

3. METHODOLOGY

To analyse the fluctuation pattern of emitted particles in two-dimensional phase space the method of scaled factorial moment is used here. Denoting the two-phase space variables as x_1 and x_2, factorial moment of order $q$may be defined as[10]

$$F_q(\delta x_1, \delta x_2) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)..........(n_m-q+1)\rangle}{\langle n_m\rangle^q} \qquad (1)$$

where $\delta x_1 \delta x_2$ is the size of a two-dimensional cell. The brackets $\langle\rangle$ denote the average over the whole ensemble of events. n_m is the multiplicity in the m^{th} cell. $M$is the number of two-dimensional cells into which the considered phase space has been divided.

Let us fix a two-dimensional region $\Delta x_1 \Delta x_2$ and divide it in to sub cells of width $\delta x_1 = \Delta x_1 / M_1$ and $\delta x_2 = \Delta x_2 / M_2$. Here M_1 is the number of bins along x_1 direction and M_2 is the number of bins along x_2 direction. Cell size dependence of factorial moment is studied by shrinking the bin widths in both directions. There are two ways of doing it. Widths may be shrinked equally ($M_1 = M_2$) or unequally ($M_1 \neq M_2$) in the two dimensions. The shrinking ratios along x_1 and x_2 directions are characterised by a parameter $H = \ln M_1 / \ln M_2$ where H ($0 < H \leq 1$) is called Hurst exponent. $H=1$ signifies that the phase space is divided isotropically and consequently fluctuations are self-similar. When $H<1$ it is clearly understood that the phase spaces along x_1 and x_2 directions are divided anisotropically consequently the fluctuations are self affine in nature.

The power law dependence of factorial moment on the cell size as cell size approaches zero is given by,

$$\langle F_q \rangle \propto M^{\alpha_q} \qquad (2)$$

The index α_q is obtained from a linear fit of the form

$$\ln\langle F_q\rangle = \alpha_q \ln M + a \qquad (3)$$

where a is a constant.

According to the cascade model[1], the higher order scaled factorial moments are related to the second order scaled factorial moments by a modified power law

$$F_q \propto F_2^{\beta_q} \qquad (4)$$

which may provide some vital information about the underlying dynamics.

Now β_q is defined by the following relation

$$\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{d_q}{d_2}(q-1) \qquad (5)$$

β_q is related to Levy index (μ) by the equation

$$\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{q^\mu - q}{2^\mu - 2} \qquad (6)$$

Here, μ , known as Levy index, is considered a measure of degree of multifractality[5].

According to GL theory for second order phase transition the anomalous fractal dimension follows the relation

$$\beta_q = (q-1)^\nu \qquad (7)$$

With ν= 1.304 as the critical exponent.

4. RESULT AND DISCUSSION

The cumulative variables X_η and X_ϕ are used instead of η and ϕ[11]. In the X_η - X_ϕ space we divided the region [0, 1] into M_η & M_ϕ bins respectively. The partitioning was taken as $M_\eta = M_\phi{}^H$. We choose M_ϕ = 2, 3, ……, 20. The (X_η - X_ϕ) space is divided into $M = M_\eta \times M_\phi$ cells and calculation is done in each bin independently.

To analyze the anisotropic nature of pions in the (X_η - X_ϕ) phase space factorial moment of different orders for different Hurst exponents starting from 0.3 to 0.7 in steps of 0.1 and for H =1 are calculated. The variation of $\ln\langle F_q\rangle$ against $\ln M$ have been studied for different orders (q =2, 3, 4 & 5) and for the considered H values. From the linear best fits intermittency exponents (α_q) are extracted. χ^2/d.o.f. values are calculated for each linear fits. We have also estimated the confidence level of fittings from the χ^2 values. The minimum value of χ^2 per degree of freedom indicates the best linear behavior. For pions the best linear fit occurs at H = 0.4 which shows that the anisotropic behavior is best revealed at H = 0.4. So the dynamical fluctuation pattern of shower particles in π^- -AgBr interaction at 200 GeV/c is not self-similar but self-affine in nature.

The values of β_q , calculated by using Eqn. (5), are listed in Table 1. The β_q versus q graph is shown in Fig. 1. It is observed that the parameter β_q increases with increasing order of moments indicating the fact that charged particle density distribution has multifractal structure. Therefore, we can infer that hadrons in the final state are produced due to self-similar cascade mechanism.

Then Levy stability index μ is calculated using Eqn. (6) and tabulated in Table 1. Here the Levy index obtained for the $\eta - \phi$ space is μ = 0.472 ± 0.005, which is within the specified limit $0 \leq \mu \leq 2$. Here $\mu \langle 1$ would have indicated a thermal phase transition of second order.

Table 1:Values of different parameters (β_q, μ and ν)for full data set

H	q	β_q	μ	ν
0.4	2	1	0.472 ± 0.005	1.121 ± 0.002
	3	2.22		
	4	3.54		
	5	4.91		

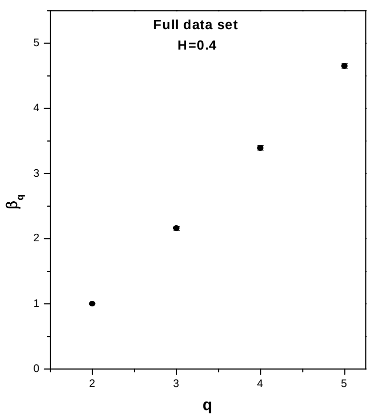

Figure 1:Variation of β_q with q for full data set

Using Ginzburg – Landau (GL) theory it has been found that the scaling exponent ν = 1.121 ± 0.002. This value of ν (considering the errors) differs significantly from the critical value, 1.304. So it is evident that no second order QGP phase transition takes place in the hadronization process.

For studying target excitation dependence the data set for pions is divided into three sets , $0 \leq n_g \leq 2$, $3 \leq n_g \leq 5$, $6 \leq n_g \leq 13$, depending on the number of grey tracks (n_g).The sets correspond to different degrees of target excitation. The division is made in such a way that each set contains reasonable number of events. The self – affine analysis is repeated for the three data sets. The fluctuation pattern is self-affine in nature in all the three sets of n_g.

The Levy index analysis is repeated for the three target excitation data sets in the self-affine space. The β_q versus q graphs for the three data sets are shown in Fig. 2. It is observed that the parameter β_q increases with increasing order of moments revealing multifractal pattern of

Table 2:Values of different parameters (β_q, μ and ν) for different n_g intervals

n_g	H	q	β_q	μ	ν
$0 \leq n_g \leq 2$	0.3	2	1	0.542 ± 0.002	1.131 ± 0.004
		3	2.17		
		4	3.46		
		5	4.79		
$3 \leq n_g \leq 5$	0.7	2	1	0.478 ± 0.012	1.112 ± 0.007
		3	2.18		
		4	3.42		
		5	4.66		
$6 \leq n_g \leq 13$	0.3	2	1	0.425 ± 0.012	1.098 ± 0.007
		3	2.16		
		4	3.37		
		5	4.57		

produced pions in different n_g intervals. The errors shown in the figures are standard errors. The values of μ are calculated following the same procedure as in the previous cases and listed in Table 2. We get $\mu \langle 1$ for three target excitation data sets indicating a second order thermal phase transition and thus probing for a possible QGP formation.

Again according to the GL theory the values of ν for three n_g intervals are calculated and are listed in Table 2. From the table it is observed that the values of ν are significantly different from the critical value of ν making the GL description inappropriate and second order phase transition can most likely be ruled out.

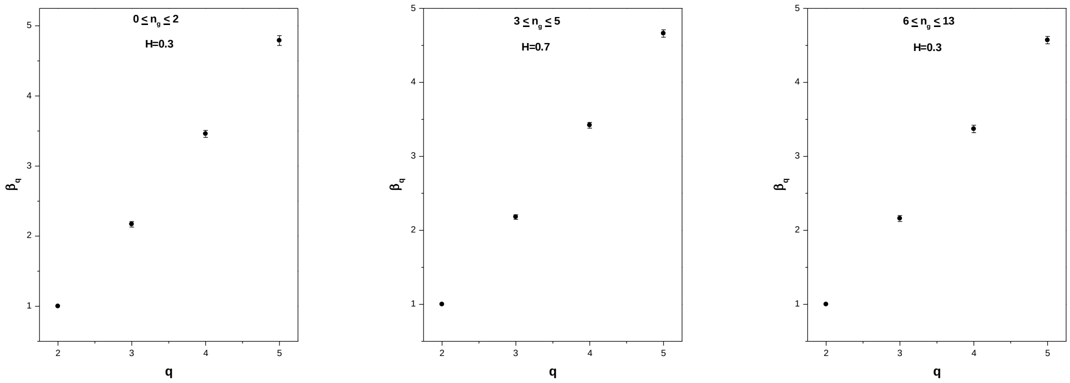

Fig. 2(a) Fig. 2(b)Fig. 2(c)

Figure 2:Variation of β_q with q for different n_g intervals

5. CONCLUSIONS

The following interesting features are revealed from the present investigation:

1. The parameter β_q increases with increasing order of moments q , which indicates that self-similar cascading to be the mechanism responsible for multiparticle production . From our analysis we find that the particle density distribution possesses multifractal structure and the degree of multifractality is different for different target excitations.
2. The values of the Levy stability index μ obtained in our study are consistent with the Levy stable region $0 \leq \mu \leq 2$.
3. We get $\mu \langle 1$ for full data set and as well as for three target excitation data sets indicating a second order thermal phase transition and thus may serve as a possible indication of QGP being formed.
4. In our analysis, no evidence for the existence of second order phase transition has been found according to the GL theory.

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