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Vertex Coloring In Graph Theory

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ABSTRACT

In this paper first, we give a brief history and introduction about graph theory,vertexcoloring and graph coloring.second we give a basic definitions in graph theory.Third we discussed some theorems in vertex coloring and added some references.

KEYWORDS: Graph coloring, vertex coloring, Achromaticnumber,pseudoachromatic number,vertex,edge

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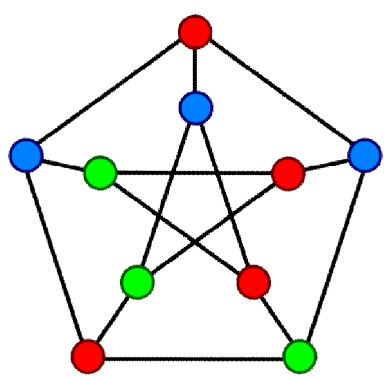
1.INTRODUCTION

The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island—but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially he proved the first theorem in graph theory.

Graph theory in mathematics is the study of graphs. Graphs are one of the prime objects of study in discrete mathematics. In general, a graph is represented as set of vertices connected by edges. Graphs are mathematical structures used to model the relations between the objects. These are found on road maps, constellations, when constructing schemes and drawings. Graphs may underlie many computer programs that make modern communication and technological processes. They contribute to the development of thinking, both logical and abstract.

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pair wise relations between objects. A graph in this context is made up of vertices which are connected by edges, arcs, or lines.

In the graph theory, graph coloring is a special case of graph labelling it is an assignment of labels called colors to the elements of a graph subject to certain constraints. In simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color this is called a vertex coloring. Similarly in edge coloring assigns a color to each edge so that no two adjacent edges are of the same color, and face coloring of a planar graph assigns a color to each face or region so that no two faces share the boundary have the same color.



A vertex coloring is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices. The most common type of vertex coloring seeks to minimize the number of colors for a given graph. Such a coloring is called minimum vertex coloring.

whenused without any qualification, a coloring of a graph is almost always a proper vertex coloring, namely a labeling of the graph vertices with colors such that no two vertices sharing the same edge have the same color. Since a vertex with a loop could never be properly colored.

A coloring using at most k colors is called a (proper) k-coloring. The smallest number of colors needed to color the graph G is called chromatic number, and is often denoted $\chi(G)$. Sometimes $\gamma(G)$ is used, since $\chi(G)$ is also used to denote the Euler characteristic of a graph. A graph that can be assigned a (proper) k-coloring is k-colorable, and it is k-chromatic if its chromatic number is exactly k. A subset of vertices assigned to the same color is called a colorclass, every such class forms an independent set. Thus, a k-coloring is the same as a partition of the vertex set into k independent sets, and the terms k-partite and k-colorable have the same meaning.

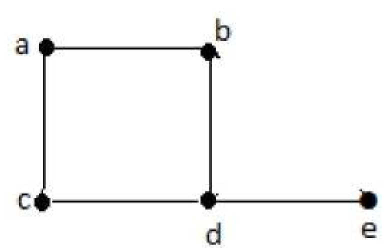
2.EXPERIMENTAL SECTION

Definition 2.1

GRAPH

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges

Example



In the above graph,

V = {a, b, c, d, e}

E = {ab, ac, bd, cd, de}

Definition 2.2

Vertex

A vertex is a point where multiple lines meet. It is also called a node. Similar to points, a vertex is also denoted by an alphabet.

Definition 2.3

DEGREE OF VERTEX

In a directed graph, each vertex has an indegree and an outdegree.

INDEGREE

- Indegree of vertex V is the number of edges which are coming into the vertex V.
- Notation − deg−(V).

OUTDEGREE

- Outdegree of vertex V is the number of edges which are going out from the vertex V.
- Notation − deg+(V).

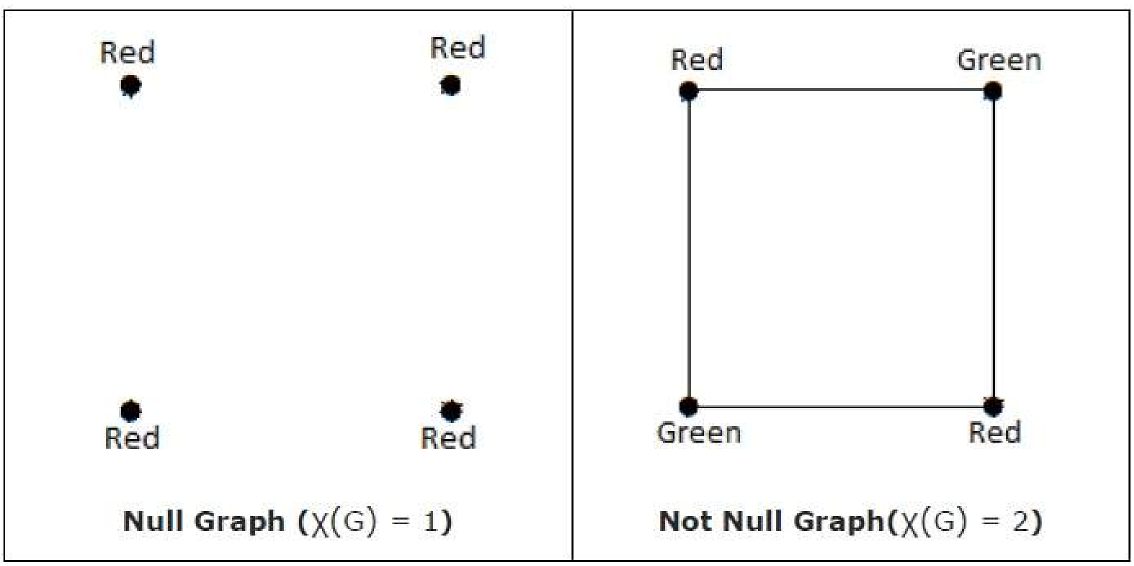
Definition 2.4

CHROMATIC NUMBER

The minimum number of colors required for vertex coloring of graph 'G' is called as the chromatic number of G, denoted by X(G).

$\chi(G) = 1$ if and only if 'G ' is a null graph. If 'G ' is not a null graph, then $\chi(G) \geq 2$

Example



Definition 2.5

PSEUDOACHROMATIC NUMBER

The pseudoachromatic number of a graph G is the maximum size of a vertex partition of G, such that between any two distinct parts there is atleast one edge G.

Definition 2.6

CHROMATIC NUMBER

The chromatic number χ(G) of a graph G is the minimum number of colors needed to color G.A graph G is called n-colorable if χ(G)≤n**.**

Definition 2.

BIPARTITE GRAPH

A simple graph G = (V, E) with vertex partition V = {V1, V2} is called a bipartite graph if every edge of E joins a vertex in V1 to a vertex in V2.

3.MAIN RESULTS

Theorem 3.1

χ(p(n,k))=α(p(n,k))=3 and ψ(p(n.k))≤3n+1

if n is even and k is odd

proof

p(n,k) is bipartite if n is even and k is odd [5,13].In view of this the chromatic and achromatic number of p(n,k) follows immediately.Further more as p(n,k)Ck_(3n,3n) and ψ is monotone it follows from theorem that the pseudoachromatic number of k_(m,n) is min(m,n)+1 (ie),ψ(p(n,k))≤ψ(k_(3n,3n))=3n+1

Theorem 3.2

The bi-star $B_{m,n}$ obtained from $K_{1,m}$ and $K_{1,n}$ by joining their centres by an edge has pseudoachromatic number 3.

Proof

Let V ($B_{m,n}$) = {u;$u_{1,...}u_n$; v;$v_{1....}v_n$} and E($B_{m,n}$) = {(u, ui), (u, v), (v, vi) : 1 ≤ i ≤ m,1≤i≤n}. Define ψ : V ($B_{m,n}$)→ {1, 2, 3} as follows: ψ(u) = 1, ψ(v) = 2, ψ(u_1) = 3, ψ(v_1) = 3 and assign to the remaining vertices any one of these three colors. It is easy to see that ψ is a pseudoachromaticcoloring of$B_{m,n}$ and ψ($B_{m,n}$)≥ 3. We claim that ψ($B_{m,n}$) ≤ 3. Suppose not and ψ($B_{m,n}$) = 4. Since ψ is an optimal pseudoachromatic coloring and deg(u) = deg(v) =m, n. ψ should assign distinct colors to u an v. Again as deg(u_i) = deg(v_i) = 1, the remaining two colors that appear among them has no edge between them, a contradiction.hence ψ($B_{m.n}$)=3.

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