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Heat and Mass Transfer in an Unsteady Free Convective Flow Through A Channel Bounded by A Long Wavy Wall with Cosinusoidally Varying Temperature and Parallel Flat Plate

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ABSTRACT

In this work we have studied the effect of cosinusoidally varying temperature at the wavy wall on the two dimensional free convective MHD viscous, incompressible flow confined between a long wavy wall and a flat wall. Introduction of cosinusodally varying temperature distribution makes the flow unsteady. Magnetic field is applied perpendicular to the flat wall. Governing equations of the fluid flow and its heat and mass transfer are formulated with relevant boundary conditions. It is assumed that the solution consists of two parts, a mean part and perturbed part. The long wavy wall approximation is used to obtain the solution of perturbed part. Analytical expressions for velocity, temperature, concentration, skin friction and the rate of heat and mass transfer at the walls are obtained. The effect of various physical parameters on the important flow characteristics are discussed numerically.

INTRODUCTION

The study of unsteady free convective flow through a channel bounded by a long wavy wall with cosinusoidally varying temperature and parallel flat wall has attracted the attention of number of researchers, because of its possible application in transpiration cooling of re-entry vehicles, rocket boosters and film vaporization in combustion chambers. In view of these applications, a series of investigations have been done by different researchers to study the problem of free convective flow through a channel bounded by a long wavy wall under the influence of magnetic field.

Lukodis *et al* [5] investigated compressible boundary layer over a wavy wall. Shankar and Sinha[10] studied the Rayleigh problem for wavy wall. They arrived at the interesting conclusion that at low Reynolds numbers the waviness of the wall quickly ceases to be of importance as the liquid is dragged along by the wall, while at large Reynolds numbers the effects of viscosity are confined to a thin layer close to the wall and a known potential solution emerges in time.

Lessen and Gangwani[6] analyzed the effect of small-amplitude wall waviness upon the stability of the laminar boundary layer. Vajravelu and Sastri[13] analysed the free convective heat transfer in a viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall. Furthermore, they extended their work for vertical wavy channels.

Rao and Sastri[8] extended the work of Vajravelu and Sastri[14] for viscous heating effects when the fluid properties are constants and variables. Laminar natural convection flow and heat transfer of a viscous incompressible fluid confined between two long vertical wavy walls has been analyzed by taking the fluid properties constant and variable. In particular, attention was restricted to estimate the effects of viscous dissipation and wall waviness on the flow and heat transfer characteristics using Galerikn's method. The solutions obtained for the velocity and the temperature-fields hold good for all values of the Grashof number and wave number of the wavy walls.

Das and Ahmed[2] studied the free convective magneto hydrodynamic flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. In the above mentioned works, the diffusion-thermo (Dufour) and the thermal-diffusion (Soret) effects were not taken into account in the energy and concentration equations respectively.

Hence several authors Dursunkaya and Worek[3], Kafoussias and Williams[4], Sattar and Alam[9], Alam *et al*[1], and Srinivasacharya and RamReddy [11, 12] studied the Soret and Dufour effects on the steady, laminar mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with micropolar fluid.

Due to the importance of the thermal-diffusion and diffusion-thermo effects on the heat and mass transfer-related problems, we propose in this paper to study the Soret and Dufour effects in free convective MHD flow of a viscous incompressible fluid through a channel bounded by a long vertical wavy wall and a parallel flat wall.

In this paper, we have made an attempt to study the heat and mass transfer effect of electrically conducting, viscous, incompressible fluid along a vertical channel bounded by a long wavy wall and a parallel flat wall. A uniform magnetic field is applied in the y direction. The temperature and species concentration profiles are assumed to be varying cosinusoidally with respect to time at the wavy wall. This makes the flow unsteady. Analytical expressions are obtained to calculate various flow characteristics and their behaviors with respect to different non dimensional parameters are discussed with the help of numerical values.

MATHEMATICAL FORMULATION

We have considered two dimensional unsteady laminar free convective MHD flow along the vertical channel bounded by long wavy wall and a parallel flat wall. The x-axis is taken vertically upwards and parallel to the flat wall and the y-axis is taken perpendicular to it. The wavy wall and the parallel flat wall are represented respectively by $\bar{y} = \bar{\varepsilon} \cos k\bar{x}$ and$\bar{y} = d$. $\bar{T}_{\omega}$, $\bar{C}_{\omega}$ and T_1, C_1 are the temperature and species concentration of the wavy wall and the flat wall respectively.

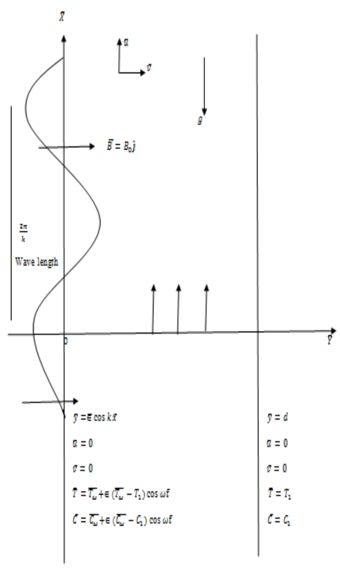

Figure 1. Flow configuration

Our investigation is restricted to the following assumptions:

- All the fluid properties, except the density in the buoyancy force term are constants.
- The viscous and magnetic dissipation of energy are negligible.
- The volumetric heat source/sink term in the energy equation is absent.
- The magnetic Reynolds number is small enough to neglect induced Magnetic field.

- The diffusion thermo (Dufour) and the thermal diffusion (Soret) terms are not taken into account in the energy and concentration equations respectively.
- The wave length of the wavy wall, which is proportional to $\frac{1}{k}$ is assumed as large.
- Temperature and concentration distributions at the wavy wall are assumed to be varying cosinusoidally which makes the flow unsteady.

Under the foregoing assumptions, the equations that govern the two dimensional unsteady laminar free convective MHD flow and heat transfer in a viscous incompressible fluid occupying the channel are given below.

The momentum equations

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}}+\mu\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)-\rho g_x-\sigma{B_0}^2\bar{u} \quad (1)$$

$$\rho\left(\frac{\partial\bar{v}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}}+\mu\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right) \quad (2)$$

The continuity equation:

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}} = 0 \quad (3)$$

The energy equation:

$$\rho C_p\left(\frac{\partial\bar{T}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right) = \kappa\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \quad (4)$$

The species concentration equation:

$$\bar{u}\frac{\partial\bar{C}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}} = \kappa\left(\frac{\partial^2\bar{C}}{\partial\bar{x}^2}+\frac{\partial^2\bar{C}}{\partial\bar{y}^2}\right) \quad (5)$$

In static conditions, Equation (1) takes the following form

$$-\frac{\partial\bar{p}_0}{\partial\bar{x}}-\rho_o = 0 \quad (6)$$

Now Equations (1) to (6) yield the following

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial}{\partial\bar{x}}(\bar{p}-\bar{p}_0)+g(\rho_0-\rho)+ \mu\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)-\sigma{B_0}^2\bar{u} \quad (7a)$$

The equation of state is given by Boussinesq approximation:

$$\rho = \rho_0[1-\beta(T-\bar{T}_0)-\beta'(C-\bar{C}_0)] \quad (7b)$$

Equations (7a) and (7b) together give:

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial}{\partial\bar{x}}(\bar{p}-\bar{p}_0)+\rho_0 g[\beta(T-\bar{T}_0)+\beta'(C-\bar{C}_0)]+ \mu\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)-\sigma B_0^2\bar{u} \quad (8)$$

The relevant boundary conditions are:

$$\bar{y}=\bar{\varepsilon}\cos k\bar{x}:\bar{u}=0,\bar{v}=0,\bar{T}=\bar{T}_\omega+\varepsilon(\bar{T}_\omega-T_1)\cos\omega\bar{t},\bar{C}=\bar{C}_\omega+\varepsilon(\bar{C}_\omega-C_1)\cos\omega\bar{t} \quad (9)$$

$$\bar{y}=d:\bar{u}=0,\bar{v}=0,\bar{T}=T_1,\bar{C}=C_1 \quad (10)$$

We introduce the following non-dimensional quantities:

$x = \frac{\bar{x}}{d}$; $y = \frac{\bar{y}}{d}$; $u = \frac{\bar{u}d}{\upsilon}$; $v = \frac{\bar{v}d}{\upsilon}$; $p = \frac{\bar{p}d^2}{\rho\upsilon^2}$;

$p_s = \frac{\bar{p}_s d^2}{\rho\upsilon^2}$ $t = \frac{\bar{t}d}{\upsilon}$; $\lambda = kd$; $\varepsilon = \frac{\bar{\varepsilon}}{d}$; $Pr = \frac{\mu C_p}{k}$; $Sc = \frac{\upsilon}{k}$;

$Gr = \frac{d^3 g\beta(\bar{T}_\omega - \bar{T}_0)}{\upsilon^2}$; $Gm = \frac{d^3 g\beta(\bar{C}_\omega - \bar{C}_0)}{\upsilon^2}$; $n = \frac{\bar{C}_1 - \bar{C}_s}{\bar{C}_\omega - \bar{C}_s}$;

$M = \frac{\sigma {B_0}^2 \bar{u}}{\rho\upsilon}$; $m = \frac{\bar{T}_1 - \bar{T}_s}{\bar{T}_\omega - \bar{T}_s}$; $C = \frac{\bar{C} - \bar{C}_s}{\bar{C}_\omega - \bar{C}_s}$; $T = \frac{\bar{T} - \bar{T}_s}{\bar{T}_\omega - \bar{T}_s}$.

The governing equations (2) - (5) & (8) - (10) become,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial}{\partial x}(p - p_x) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + GrT + GmC - Mu \quad (11)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \quad (12)$$

$$Pr\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \alpha \quad (13)$$

$$Sc\left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}\right) = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \quad (14)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (15)$$

The corresponding boundary conditions are:

$u = 0,\ v = 0, = 1 + \varepsilon\cos t,\ C = 1 + \varepsilon\cos t$ at $y = \varepsilon\cos\lambda x$ (16)

$u = 0,\ v = 0, = m,\ C = n$ at $y = 1$ (17)

SOLUTION OF THE PROBLEM

We assume the solution of the following form

$$u(x, y, t) = u_0(y) + \varepsilon e^{it} u_1(y) + \cdots \quad (18a)$$

$$v(x, y, t) = v_0(y) + \varepsilon e^{it} v_1(y) + \cdots \quad (18b)$$

$$p(x, y, t) = p_0(y) + \varepsilon e^{it} p_1(y) + \cdots \quad (18c)$$

$$T(x, y, t) = T_0(y) + \varepsilon e^{it} T_1(y) + \cdots \quad (18d)$$

$$C(x, y, t) = C_0(y) + \varepsilon e^{it} C_1(y) + \cdots \quad (18e)$$

Here the subscripts 0 and 1 denote respectively the corresponding steady and unsteady state quantities. By substituting the transformations from Equations (18a) - (18e) into Equations (11) - (15), and by equating the coefficients of ε^0, ε and neglecting the higher powers of ε and assuming $\frac{\partial}{\partial x}(p - p_x) = 0$, we derive the following set of ordinary differential equations:

$$\frac{d^2 u_0}{dy^2} - Mu_0 = -GrT_0 - GmC_0 \quad (19)$$

$$\frac{d^2 T_0}{dy^2} = -\alpha \quad (20)$$

$$\frac{d^2 C_0}{dy^2} = 0 \quad (21)$$

$$iu_1 + u_0\frac{\partial u_1}{\partial x} + v_1 u_0^{'} \quad = \quad -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial x^2} - GrT_1 + GmC_1 - Mu_1 \qquad (22)$$

$$iv_1 + u_0\frac{\partial v_1}{\partial x} \quad = \quad -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \qquad (23)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \quad = \quad 0 \qquad (24)$$

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} \quad = \quad Pr\left(iT_1 + u_0\frac{\partial T_1}{\partial x} + v_1 T_0^{'}\right) \qquad (25)$$

$$\frac{1}{Sc}\left(\frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial y^2}\right) \quad = \quad u_0\frac{\partial C_1}{\partial x} + v_1 C_0^{'} \qquad (26)$$

These are subject to the following boundary conditions:

$$u_0 = 0, T_0 = 1, C_0 = 1 \quad \text{at} \quad y = 0$$

$$u_0 = 0, T_0 = m, C_0 = n \quad \text{at} \quad y = 1 \qquad (27)$$

$$u_1 = -Re\left(u_0^{'}(0)e^{i\lambda x}\right), v_1 = 0, T_1 = \left(i - ReT_0^{'}(0)\right)e^{i\lambda x},$$

$$C_1 = \left(i - ReC_0^{'}(0)\right)e^{i\lambda x} \text{at} y = 0$$

$$u_1 = 0, v_1 = 0, T_1 = 0, C_1 = 0 \quad \text{at} \quad y = 1 \qquad (28)$$

The solutions of the equations (20), (21) and (19) subject to the boundary conditions of equation (28) are:

$$T_0 \quad = \quad 1 + \left(m - 1 + \frac{\alpha}{2}\right)y - \frac{\alpha y^2}{2} \qquad (29)$$

$$C_0 \quad = \quad 1 + (n-1)y \qquad (30)$$

$$u_0 \quad = C_1 e^{\sqrt{M}y} + C_2 e^{-\sqrt{M}y} + A_1 + A_2 y + A_3 y^2 \qquad (31)$$

Consider the transformations,

$$u_1 = u_{11}e^{i\lambda x}, v_1 = v_{11}e^{i\lambda x}, T_1 = T_{11}e^{i\lambda x}, C_1 = C_{11}e^{i\lambda x}$$

Equations (22) to (26) reduce to

$$iu_{11} + i\lambda u_0 u_{11} + v_{11}u_0^{'} = -i\lambda p_{11} - \lambda^2 u_{11} + \frac{\partial^2 u_{11}}{\partial y^2} + GrT_{11} + GmC_{11} - Mu_{11} \qquad (32)$$

$$iv_{11} + i\lambda u_0 v_{11} \quad = \quad -\frac{\partial p_{11}}{\partial y} - \lambda^2 v_{11} + \frac{\partial^2 v_{11}}{\partial y^2} \qquad (33)$$

$$i\lambda u_{11} + \frac{\partial v_{11}}{\partial y} \quad = \quad 0 \qquad (34)$$

$$-\lambda^2 T_{11} + \frac{\partial^2 T_{11}}{\partial y^2} \quad = \quad Pr\left(iT_{11} + i\lambda u_0 T_{11} + v_{11}T_0^{'}\right) \qquad (35)$$

$$\frac{1}{Sc}\left(-\lambda^2 C_{11} + \frac{\partial^2 C_{11}}{\partial y^2}\right) \quad = \quad i\lambda u_0 C_{11} + v_{11}C_0^{'} \qquad (36)$$

The relevant boundary conditions are

$$u_{11} = -Reu_0^{'}(0), v_{11} = 0, T_{11} = 1 - ReT_0^{'}(0),$$

$$C_{11} = 1 - ReC_0^{'}(0) \quad \text{at} \quad y = 0 \qquad (37)$$

$u_{11} = 0, v_{11} = 0, T_{11} = 0, C_{11} = 0$ at $y = 1$ (38)

Eliminating u_{11} and p_{11} ,

$$\frac{\partial^4 v_{11}}{\partial y^4} + (-2\lambda^2 - M - i - i\lambda u_0)\frac{\partial^2 v_{11}}{\partial y^2} + (\lambda^4 + i\lambda^3 u_0 + i\lambda^2 + i\lambda u_0'')v_{11} = i\lambda\, Gr\frac{\partial T_{11}}{\partial y} + i\lambda\, Gm\frac{\partial C_{11}}{\partial y} \quad (39)$$

$$\frac{\partial^2 T_{11}}{\partial y^2} - iPrT_{11} - i\lambda u_0 T_{11} - Prv_{11}T_0' - \lambda^2 T_{11} = 0 \quad (40)$$

$$\frac{\partial^2 C_{11}}{\partial y^2} - iSc\lambda u_0 C_{11} - Scv_{11}C_0' - \lambda^2 C_{11} = 0 \quad (41)$$

With boundary conditions,

$v_{11}' = -i\lambda Reu_0'(0)$, $v_{11} = 0$, $T_{11} = 1 - ReT_0'(0)$,

$C_{11} = 1 - ReC_0'(0)$ at $y = 0$ (42)

$v_{11}' = 0, v_{11} = 0, T_{11} = 0, C_{11} = 0$ at $y = 1$ (43)

Assuming the series expansion for v_{11} , C_{11} and T_{11} we get

$$v_{11} = \lambda v_{111} + \lambda^2 v_{112} + \cdots \quad (44)$$

$$C_{11} = C_{110} + \lambda C_{111} + \lambda^2 C_{112} + \cdots \quad (45)$$

$$T_{11} = T_{110} + \lambda T_{111} + \lambda^2 T_{112} + \cdots \quad (46)$$

Substituting the equations (44), (45) and (46) into equations (39), (40) and (41) then by equating the coefficients of $\lambda^0, \lambda, \lambda^2$ and neglecting the terms of order greater than or equal to O(λ^3) , the following solutions are obtained,

$$v_{111} = C_7 + C_8 y + C_9 e^{\sqrt{M+i}y} + C_{10}e^{-\sqrt{M+i}y} + C_{11}e^{\sqrt{iPr}} + C_{12}e^{-\sqrt{iPr}} + C_{13}y^2 \quad (47)$$

$$T_{110} = C_3 e^{\sqrt{iPr}\,y} + C_4 e^{-\sqrt{iPr}\,y} \quad (48)$$

$$C_{110} = C_5 + C_6 y \quad (49)$$

For the sake of brevity, the constants are given in appendix. Skin friction, Nusselt number and Sherwood number are calculated using appropriate formulas.

RESULTS AND DISCUSSION

The unsteady free convective flow through a channel bounded by a long wavy wall with cosinusoidally varying temperature and parallel flat wall with heat and mass transfer was investigated in the presence of heat source and mass concentration. Analytical solutions for velocity, temperature and concentration were presented in the previous section. The skin friction co-efficient, heat transfer coefficient and mass transfer co-efficient were found. From the available analytical solutions the numerical values for the distributions of velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number are calculated by fixing various values of the non-dimensionless parameters involved in the problem.

In order to get physical insight of the problem, we have carried out numerical calculations for non dimensional velocity field, temperature field, species concentration field and skin friction at the walls by assigning some specific values to the parameters entering into the problem and the effects of these values on the above fields is demonstrated graphically. In our investigation the values of the parameter λ (frequency parameter) and ε (amplitude parameter) are kept fixed at 0.001 and 0.01 respectively and the values of the other parameters are chosen arbitrarily.

Figures 2 to 7, represents the variation of velocity field u as a function of y for fixed values of Hartmann number M, Grashoff number Gr, Modified Grashof number Gm, heat source parameter α, ratio of wall temperature m and ratio of wall concentration n. From these figures we observe that the velocity field increases as Gr, Gm, α, m and n increase and decreases as M increases.

Figures 8 to17, exhibit the behavior of the temperature field against y for varying non dimensional parameters α, m, Re, Pr, M, Gm and Gr. It is inferred from these figures that the temperature field increases as α, m, Pr, M and Gm increases and decreases as Re and Gr increase. Figure 18 represents the variation of species concentration C versus y under the influence of ratio of wall concentration. It can be observed from these figures that increase in wall concentration ratio increases the specific concentration.

Figures 19 to 22 represent the nature of skin friction τ at both the wavy wall and flat wall, wall shear stress increases with increasing wall temperature ratio and modified Grashof number. Heat flux decreases with increasing modified Grashof number and wall temperature ratio. Mass flux at the flat wall is found to be increasing due to the increase in the modified Grashof number and heat source parameter.

CONCLUSION

We have considered two dimensional unsteady laminar free convective MHD flow along the vertical channel bounded by long wavy wall and a parallel flat wall. The $\bar{X}$ - axis is taken vertically upwards and parallel to the flat wall and the $\bar{Y}$ - axis is perpendicular to it. The wavy wall is represented by $\bar{y} = \bar{\varepsilon}\cos k\bar{x}$ and the flat wall is represented by $\bar{y} = d$. We have assumed a cosinusoidally varying temperature and concentration distribution at the wavy wall which makes the flow unsteady. Boussinesq approximation is considered in the momentum equation. The non-dimensional governing equations of the flow are solved by perturbation method. The effects of various physical parameters on velocity, temperature and concentration distribution, skin friction co-efficient, Nusselt number and Sherwood number for various values of physical parameters have been studied numerically. We have found qualitative agreement with the results obtained by Nazibuddin Ahmed, Kalpana Sarma & Hiren Deka[7] for steady flow.

Some important findings are given below.

- The fluid motion is retarded due to the presence of transverse magnetic field and accelerated due to thermal - diffusion and diffusion thermo effects.
- An increase in heat generating source and the ratio of wall temperature leads the fluid temperature to increases.
- An increase in the values of the heat source parameter causes the fluid temperature to rise slowly and steadily.
- The wall shear stress at the wavy wall increases with increasing wall temperature ratio and Grashof number and wall temperature ratio. Heat flux decreases with increasing Grashof number and wall temperature ratio. Mass flux at the flat wall is found to be increasing due to increase in modified Grashof number and heat source parameter.

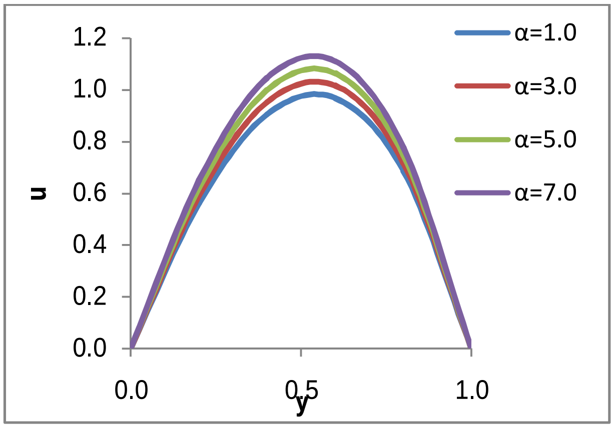

Figure 2. Velocity versus y for varying α for Pr = 0.71, Gr = 2, Gm = 2,Sc = 0.6, M = 0.5, m = 5 , n = 1, λ = 0.001, ε = 0.01,Re = 1.

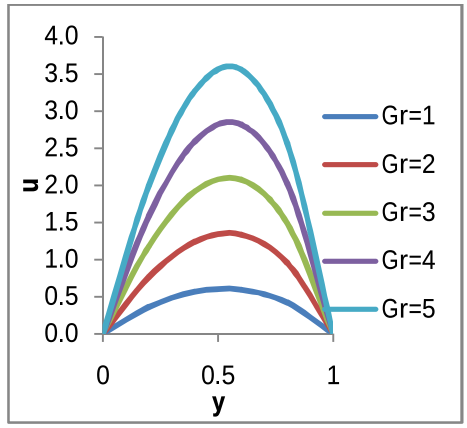

Figure 5. Velocity versus y for varying *Gr* for Pr = 0.71, α = 1, Gm = 2,Sc = 0.6, M = 0.5, m = 5 , n = 1, λ = 0.001,ε = 0.01,Re = 1

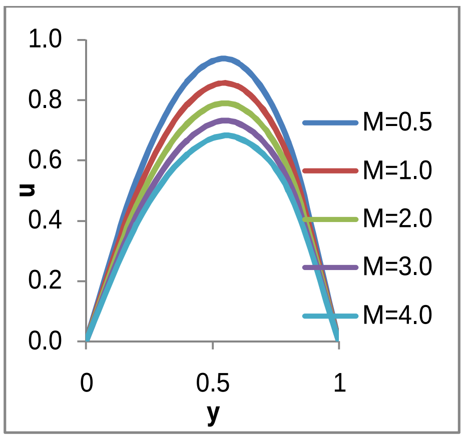

Figure 3. Velocity versus y for varying M for Pr = 0.71, α = 1, Gr = 2, Gm = 2, Sc = 0.6, m = 5, n = 1, λ = 0.001,ε = 0.01, Re = 1

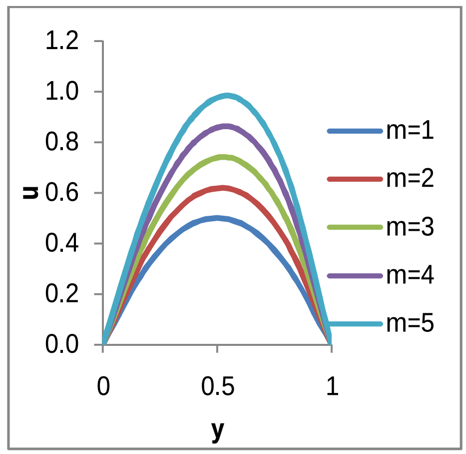

Figure 6.Velocity versus y for varying *m* for Pr = 0.71, α = 1, Gm = 2, Gr = 2, Sc = 0.6, M = 0.5, n = 1, λ = 0.001, ε = 0.01, Re = 1.

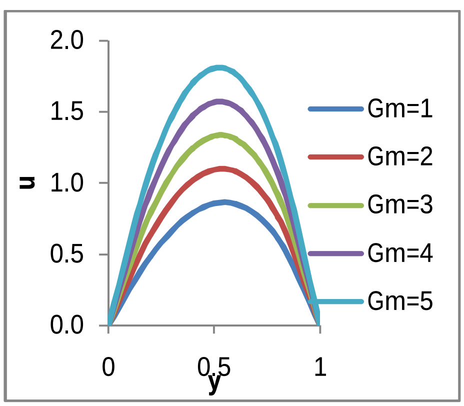

Figure 4. Velocity versus y for varying *Gm* for Pr = 0.71, α = 1, Gr = 2, Sc = 0.6, M = 0.5, m = 5, n = 1, λ = 0.001, ε = 0.01, Re = 1

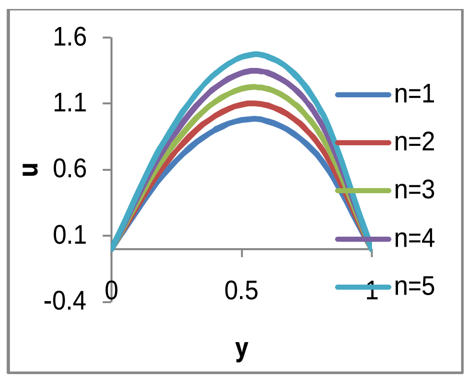

Figure 7. Velocity versus y for varying *n* for Pr = 0.71, α = 1, Gm = 2, Gr = 2, Sc = 0.6, M = 0.5, m = 5, λ = 0.001, ε = 0.01, Re = 1

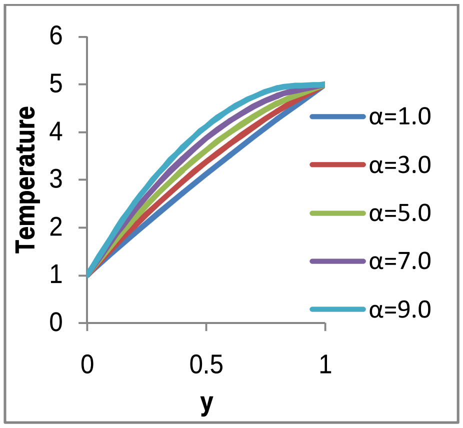

Figure 8. Temperature versus y for varying α for Pr = 0.71, Gr = 2, Gm = 2, Sc = 0.6, M = 0.5, m = 5 , n = 1, λ = 0.001, ε = 0.01, Re = 1

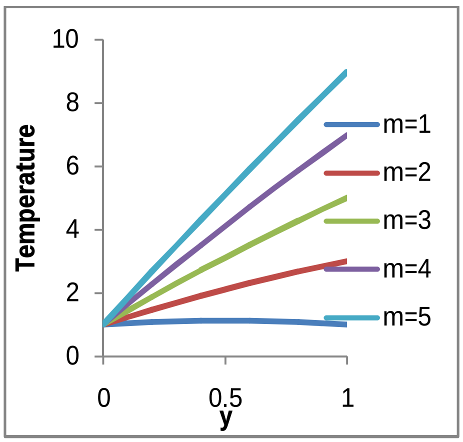

Figure 9. Temperature versus y for varying m for Pr = 0.71, Gr = 2, Gm = 2,Sc = 0.6, M = 0.5 , n = 1, λ = 0.001, ε = 0.01, Re = 1.

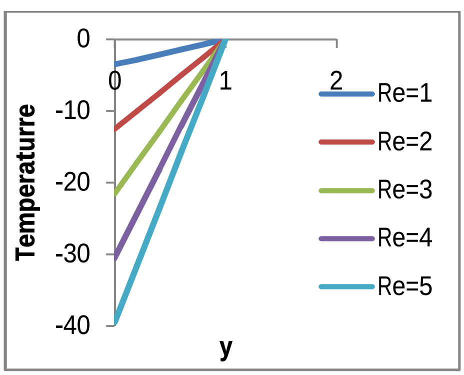

Figure 10. Temperature versus y for varying Re for Pr = 0.71, Gr = 2, Gm = 2, Sc = 0.6, M = 0.5, n = 1, λ = 0.001, ε = 0.01, m = 5.

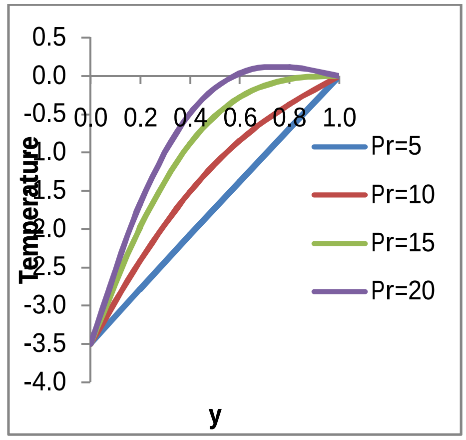

Figure 11. Temperature versus y for varying Pr for m = 5, Gr = 2, Gm = 2, Sc = 0.6, M = 0.5 , n = 1, λ = 0.001, ε = 0.01, Re = 1.

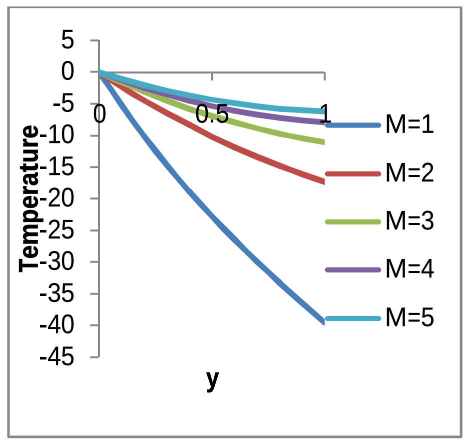

Figure 12. Temperature versus y for varying M for Pr = 0.71, Gr = 2, Gm = 2, Sc = 0.6, m =5, n = 1, λ = 0.001, ε = 0.01, Re = 1.

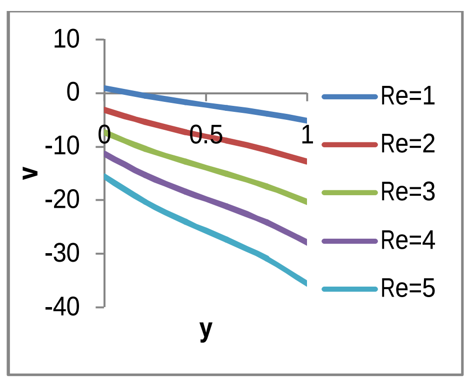

Figure 13. Temperature versus y for varying Re for Pr = 0.71, Gr = 2, Gm = 2, Sc = 0.6, m = 5, n = 1, λ = 0.001, ε = 0.01, M = 0.5.

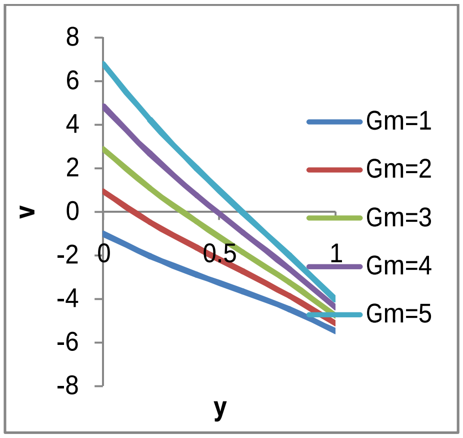

Figure 14. Temperature versus y for varying *Gm* for Pr = 0.71, Gr = 2, Sc=0.6, m=5, M=0.5, n = 1, λ = 0.001,ε = 0.01,Re=1.

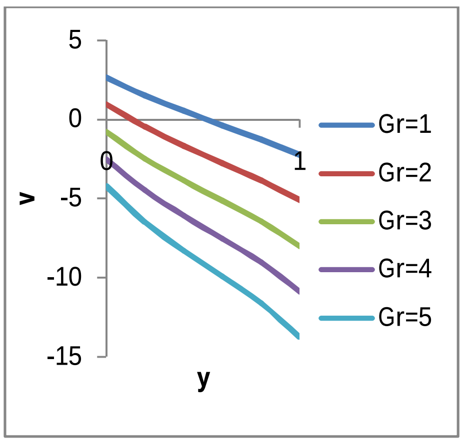

Figure 15. Temperature versus y for varying *Gr* for Pr = 0.71, Gm = 2, Sc = 0.6, m = 5, n = 1, M = 0.5, λ = 0.001, ε = 0.01, Re = 1.

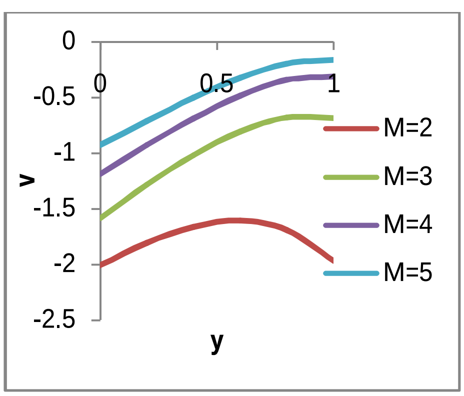

Figure 16. Temperature versus y for varying *M* for Pr = 0.71, Gr = 2, Gm = 2, Sc = 0.6, m = 5, n = 1, λ = 0.001, ε = 0.01, Re = 1

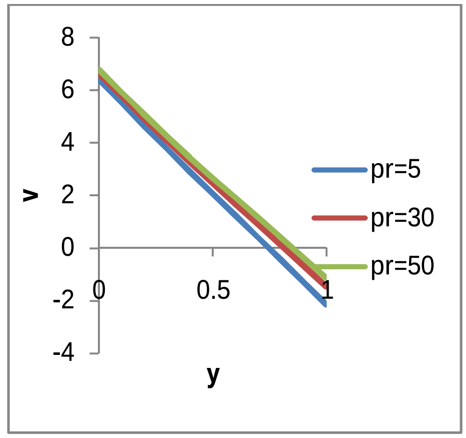

Figure 17. Temperature versus y for varying *Pr* for M = 0.5, Gr = 2, Gm = 2,Sc=0.6, m=5 , n = 1, λ = 0.001,ε = 0.01,Re=1

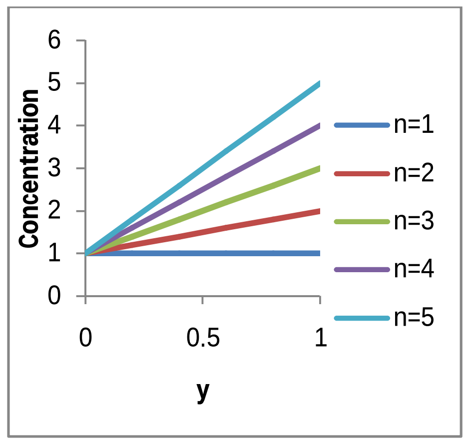

Figure 18. Concentration versus y for varying *n* for Pr = 0.71, Gr = 2, Gm = 2, Sc=0.6, M = 0.5, m = 5, λ = 0.001, ε = 0.01, Re = 1

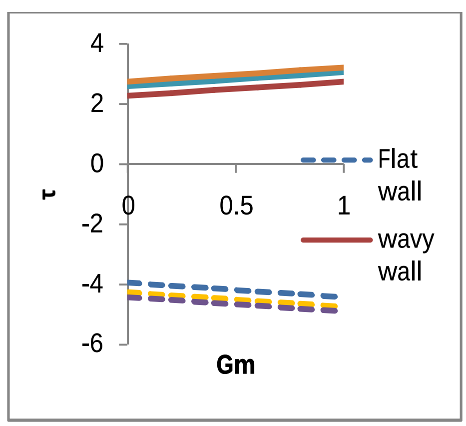

Figure 19. Skin friction versus *Gm* for varying α for Pr = 0.71, Gr = 2, n = 1, Sc = 0.6, M = 0.5, m = 5, λ = 0.001, ε = 0.01, Re = 1

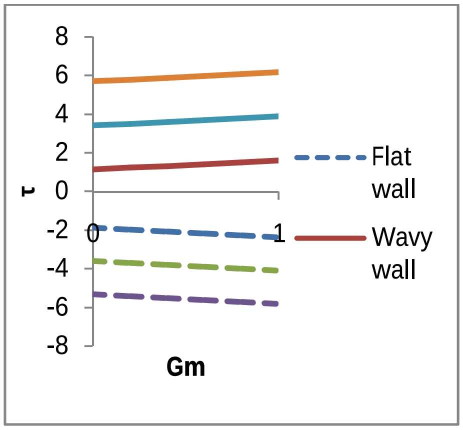

Figure 20. Skin friction versus *Gm* for varying *Gr* for Pr = 0.71, α = 1, n = 1, Sc = 0.6, M = 0.5, m = 5, λ = 0.001, ε = 0.01, Re = 1

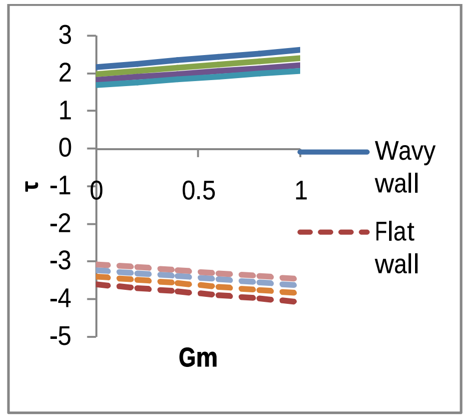

Figure 21. Skin friction versus *Gm* for varying *M* for Pr = 0.71, α = 1, n = 1, Sc = 0.6, Gr = 2, m = 5, λ = 0.001, ε = 0.01, Re = 1

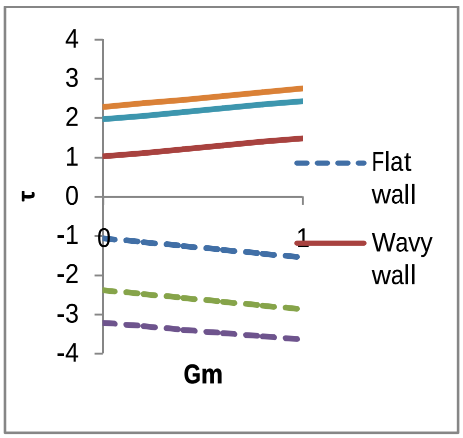

Figure 22. Skin friction versus *Gm* for varying *m* for Pr = 0.71, α = 1, n = 1, Sc = 0.6, Gr = 2, M = 0.5, λ = 0.001, ε = 0.01, Re = 1

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APPENDIX

$$v_0 = 0$$

$$A_1 = \frac{Gr}{M} - \frac{Gr\alpha}{2M^2} + \frac{Gm}{M}$$

$$A_2 = \frac{Gr}{M}\left(m - 1 + \frac{\alpha}{2}\right) + \frac{Gm}{M}(n-1)$$

$$A_3 = -\frac{Gr\alpha}{4M}$$

$$C_1 = \frac{-A_1 e^{-\sqrt{M}} + (A_1 + A_2 + A_3)}{e^{-\sqrt{M}} - e^{\sqrt{M}}}$$

$$C_2 = \frac{-(A_1 + A_2 + A_3) + A_1 e^{\sqrt{M}}}{e^{-\sqrt{M}} - e^{\sqrt{M}}}$$

$$C_3 = \frac{\left(1 - Re(m - 1 - \frac{\alpha}{2})\right)e^{-\sqrt{iPr}}}{e^{-\sqrt{iPr}} - e^{\sqrt{iPr}}}$$

$$C_4 = \frac{-\left(1 - Re(m - 1 + \frac{\alpha}{2})\right)e^{\sqrt{iPr}}}{e^{-\sqrt{iPr}} - e^{\sqrt{iPr}}}$$

$$C_5 = 1 - Re(n-1)$$

$$C_6 = Re(n-1) - 1$$

$$C_7 = C_{14} - (C_9 + C_{10})$$

$$d_1 = -C_{19}(M+i)\left(e^{-\sqrt{M+i}} - e^{\sqrt{M+i}}\right) - \left(e^{\sqrt{M+i}} - 1\right)\left(\sqrt{M+i}\right)(C_{18}e^{-\sqrt{M+i}} - C_{17}) + \left(e^{-\sqrt{M+i}} - 1\right)(C_{18}\left(\sqrt{M+i}\right)e^{\sqrt{M+i}} - C_{17}\left(\sqrt{M+i}\right))$$

$$d_2 = -(M+i)\left(e^{-\sqrt{M+i}} - e^{\sqrt{M+i}}\right) + \left(e^{\sqrt{M+i}} - 1\right)\left(\sqrt{M+i}\right)\left(e^{-\sqrt{M+i}} - 1\right) + \left(e^{-\sqrt{M+i}} - 1\right)\left(\sqrt{M+i}\right)\left(e^{\sqrt{M+i}} - 1\right)$$

$$C_8 = \frac{d_2}{d_1}$$

$$d_3 = (-\left(\sqrt{M+i}\right)C_{18}e^{-\sqrt{M+i}} + C_{17}\left(\sqrt{M+i}\right) - (C_{19}\left(-\sqrt{M+i}\right)e^{-\sqrt{M+i}} + \left(\sqrt{M+i}\right)) + \left(e^{-\sqrt{M+i}} - 1\right)(C_{17} - C_{18})$$

$$C_9 = \frac{d_3}{d_1}$$

$$d_3 = (C_{17}\left(\sqrt{M+i}\right) - \left(\sqrt{M+i}\right)C_{18}e^{-\sqrt{M+i}}) - \left(e^{\sqrt{M+i}} - 1\right)(C_{18} - C_{17}) + C_9(\left(\sqrt{M+i}\right)\left(e^{\sqrt{M+i}} - 1\right))$$

$$C_{10} = \frac{d_4}{d_1}$$

$$C_{11} = \frac{i\,Gr\,C_3\sqrt{iPr}}{(iPr)^2 - (M+i)(iPr)}$$

$$C_{12} = \frac{-i\,Gr\,C_4\sqrt{iPr}}{(iPr)^2 - (M+i)(iPr)}$$

$C_{13} \quad = \quad \frac{i\ Gm\ C_6}{2(-M-i)}$

$C_{14} \quad = \quad -(C_{11} + C_{12} + C_{13})$

$C_{15} \quad = \quad -iRe\left(C_1\sqrt{M} - C_2\sqrt{M} + A_2\right) - C_{11}\sqrt{iPr} + C_{12}\sqrt{iPr}$

$C_{16} \quad = \quad -C_{11}e^{\sqrt{iPr}} - C_{12}e^{-\sqrt{iPr}} - C_{13}$

$C_{17} \quad = \quad -C_{11}\sqrt{iPr}\, e^{\sqrt{iPr}} - C_{12}\sqrt{iPr}e^{-\sqrt{iPr}} - 2C_{13}$

$C_{18} \quad = \quad -iRe\left(C_1\sqrt{M} - C_2\sqrt{M} + A_2\right) - C_{11}\sqrt{iPr} + C_{12}\sqrt{iPr}$

$C_{19} \quad = \quad C_{16} - C_{14}$