

International Journal of Scientific Research and Reviews

Solution of Integral Equation of Matrix Variable Involving Laguerre Function

Sharma Yogesh

Department of Mathematics, Jodhpur National University, Jodhpur
dryogesh121@rediffmail.com

ABSTRACT

The aim of present paper is to give an Inversion Formula for Fractional integral operator involving laguerre function of Matrix Variable as

$$L_{\nu}^{\gamma}(T) = \frac{\Gamma_p(\gamma + \nu + \frac{p+1}{2})}{\Gamma_p(\gamma + \frac{p+1}{2})} \int_{0 < U < T} |T - U|^{\gamma} {}_1F_1(-\nu, \gamma + \frac{p+1}{2}; -A^{\frac{1}{2}}(T - U)A^{\frac{1}{2}}) f(U) dU$$

$$, \text{Re}(\gamma, \nu) > (p - 1) / 2$$

by successive application of Laplace and inverse Laplace transform of Matrix Variable where $L_{\nu}^{\gamma}(T)$ is know, $f(U)$ is to determined, T and U are real positive, definite and symmetry matrices of order $p \times p$, and $|U| = \det U$, is determinate of U .

All the matrices considered are real positive, definite and symmetric matrices of order $p \times p$. at last we prepare table for different value of function $L_{\nu}^{\gamma}(T)$

KEY WORDS: Laplace transforms, integral equation, matrix variable

Corresponding Author:-

Dr Yogesh sharma
Department of Mathematics
Jodhpur National University
Jodhpur, (Raj.) INDIA
E Mail: dryogesh121@rediffmail.com

INTRODUCTION

1. Laplace and inverse Laplace transforms

Laplace and inverse Laplace transforms of matrices variable are respectively given by equations

$$L_Z[f(Z)] = \int_{T>0} e^{tr(-TZ)} f(T) dT = \Phi(Z) \quad \dots\dots\dots(1.1)$$

And

$$\frac{2^{p(p-1)/2}}{(2\pi i)^{p(p+1)/2}} \int_{\text{Re}(Z)>0} e^{tr(TZ)} \Phi(Z) dZ = \begin{cases} f(T), T > 0 \\ 0, \text{elsewhere} \end{cases} \quad \dots\dots\dots(1.2)$$

AMS (1991) Subject Classification: 44A10, 44A35, 15XX, 26A33

Where $\Phi(Z)$ is complex analytic function and integral is taken over $Z=X+iY$ with X with fixed $X > X_0$ and Y over the space S_p^* , S_p^* is the corresponding space of Z .

For the condition and detail the readers are referred to Mathai¹ and Mathai saxena². Mathai.A.M and H.J. Haubold³

Mathai¹ generalized the Convolution theorem for Laplace transform for scalar variable to the Matrix Variables follows:

let $g_1(T)$ and $g_2(T)$ be Laplace transform of $f_1(X)$ and $f_2(X)$ respectively; then

$$L_T \left[\int_{0 < U < X} f_1(X-U) f_2(U) dU \right] = g_1(T) g_2(T) \quad \dots\dots\dots(1.3)$$

Where $\int_{0 < U < X}^x$ means that the integral is over $U > 0, X > 0, X-U > 0$ and 0 denotes the null matrix of order $p \times p$.

Sharma^{4,5,6} obtain the solution of Integral Transform of Matrix Variable involving gauss hyper geometric Function, generalized stieltjes transform of Matrix variable.

The aim of present paper to give inversion formula for integral operator of Matrix variable using Laplace transform (1.1) and inverse Laplace transform (1.2).

2. INTEGRAL EQUATION OF MATRIX VARIABLE

Consider the convolution fractional integral operator involving Jacobi Polynomials

$$L_v^\gamma(T) = \frac{\Gamma_p(\gamma + \nu + \frac{p+1}{2})}{\Gamma_p(\gamma + \frac{p+1}{2})} \int_{0 < U < T} |T - U|^\gamma {}_1F_1(-\nu, \gamma + \frac{p+1}{2}; -A^{\frac{1}{2}}(T - U)A^{\frac{1}{2}}) f(U) dU$$

, Re(γ, ν) > ($p - 1$)/2.....(2.1)

Apply Laplace Transform L_X (1.1); to both side of (2.1); we have

$$L_X[L_v^\gamma(T)] = \frac{\Gamma_p(\gamma + \nu + \frac{p+1}{2})}{\Gamma_p(\gamma + \frac{p+1}{2})} L_X \left[\int_{0 < U < T} |T - U|^\gamma {}_1F_1(-\nu, \gamma + \frac{p+1}{2}; -A^{\frac{1}{2}}(T - U)A^{\frac{1}{2}}) f(U) dU \right]$$

, Re(γ, ν) > ($p - 1$)/2
.....(2.2)

Using, Convolution theorem given by Mathai¹, page259(5.1.13)of Laplace transform

$$L_X[L_v^\gamma(T)] = \Gamma_p(\gamma + \nu + \frac{p+1}{2}) \left| I + A^{\frac{1}{2}} X^{-1} A^{\frac{1}{2}} \right|^\nu |X|^{-\gamma - \frac{(p+1)}{2}} L_X[f(U)]$$

.....(2.3)

$$\frac{L_X[g_v^\gamma(T)]}{\Gamma_p(\gamma + \nu + \frac{p+1}{2})} |A + X|^{-\nu} |X|^{\nu + \gamma + \frac{(p+1)}{2}} = L_X[f(U)]$$

.....(2.4)

Taking Inverse Laplace Transform; we get

$$f(U) = L_U^{-1} \left[\frac{L_X[g_v^\gamma(T)]}{\Gamma_p(\gamma + \nu + \frac{p+1}{2})} |A + X|^{-\nu} |X|^{\nu + \gamma + \frac{(p+1)}{2}} \right]$$

.....(2.5)

Verification In order to verify (2.5) if we take

$$L_v^\gamma(T) = \text{etr}(-A^{\frac{1}{2}} T A^{\frac{1}{2}}) |T|^{-\nu - \frac{p+1}{2}}, T > 0$$

.....(2.6)

Then first we take Laplace Transform of (2.6) is given as

$$L_X[\text{etr}(-A^{\frac{1}{2}} T A^{\frac{1}{2}}) |T|^{-\nu - \frac{p+1}{2}}] = \Gamma_p(-\nu) |A + X|^\nu$$

.....(2.7)

Which is easily obtain by using Laplace transform and the inverse Laplace transform L_U^{-1} of (2.7) can be obtained by

$$L_U^{-1} \left[\frac{|X|^{v+r+\frac{p+1}{2}} \Gamma_p(-v) L_X [L_v^\gamma(T)]}{\Gamma_p(\gamma+v+\frac{p+1}{2})} \right]$$

$$= \frac{|U|^{-v-r-(p+1)} \Gamma_p(-v)}{\Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(-\gamma-v-\frac{(p+1)}{2})}$$

.....(2.8)

With $L_v^\gamma(T)$ as in (2.6), our solution of (2.1) is then

$$f(U) = \frac{|U|^{-v-r-(p+1)} \Gamma_p(-v)}{\Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(-\gamma-v-\frac{(p+1)}{2})}, U = U^1 > 0, \text{Re}(v) > 0$$

.....(2.9)

Putting f(U) given by (2.9) in equation (2.1), we get

$$L_v^\gamma(T) = \frac{\Gamma_p(\gamma+v+\frac{p+1}{2})}{\Gamma_p(\gamma+\frac{p+1}{2})} \int_{0 < U < T} |T-U|^\gamma {}_1F_1(-v, \gamma+\frac{p+1}{2}; -A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}}) \frac{|U|^{-v-r-(p+1)} \Gamma_p(-v)}{\Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(-\gamma-v-\frac{(p+1)}{2})} dU$$

using lemma 1 given by Mathai [1, page 293 (5.2.32)], we get

$$= \frac{\Gamma_p(\gamma+v+\frac{p+1}{2})}{\Gamma_p(\gamma+\frac{p+1}{2})} \int_{0 < U < T} |U|^\gamma {}_1F_1(-v, \gamma+\frac{p+1}{2}; -A^{\frac{1}{2}}UA^{\frac{1}{2}}) \frac{|T-U|^{-v-r-(p+1)} \Gamma_p(-v)}{\Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(-\gamma-v-\frac{(p+1)}{2})} dU$$

, $\text{Re}(\gamma, v) > (p-1)/2$
.....(2.10)

Using Mathai and Saxena², after the transformations

$U = T^{\frac{1}{2}}VT^{\frac{1}{2}}, dU = |T|^{\frac{1+p}{2}} dV$ for Fixed T and the Respective Region is $0 < U < T, 0 < V < I$, we have

$$= \frac{\Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(\gamma+\frac{p+1}{2}) \Gamma_p(-v)}{\Gamma_p(\gamma+\frac{p+1}{2}) \Gamma_p(\gamma+v+\frac{p+1}{2}) \Gamma_p(-v)} \text{etr}(-A^{\frac{1}{2}}TA^{\frac{1}{2}}) |T|^{-v-\frac{p+1}{2}}$$

.....(2.11)

We have

$$L_v^\gamma(T) = \text{etr}(-A^{\frac{1}{2}}TA^{\frac{1}{2}}) |T|^{-v-\frac{p+1}{2}}, T > 0$$

.....(2.12)

Particular Case: if we take Matrix of order 1×1 i.e if we take $p=1$ we get,

$$f(u) = \frac{|U|^{-v-r-2} \Gamma_p(-v)}{\Gamma_p(\gamma+v+1)\Gamma_p(-\gamma-v-1)}, U = U^1 > 0, \text{Re}(v) > 0$$

... ..(2.13)

Gives inverse of

$$L_v^\gamma(t) = \frac{\Gamma_p(\gamma+v+\frac{p+1}{2})}{\Gamma_p(\gamma+\frac{p+1}{2})} \int_{0 < U < T} |t-u|^\gamma {}_1F_1(-v, \gamma+\frac{p+1}{2}; -a^{\frac{1}{2}}(t-u)a^{\frac{1}{2}}) f(U) dU$$

, $\text{Re}(\gamma, v) > (p-1)/2 \dots$
.....(2.14)

TABLE FOR DIFFERENT MATRIX VARIABLE

S. N	$L_v^\gamma(T) = \frac{\Gamma_p(\gamma+v+\frac{p+1}{2})}{\Gamma_p(\gamma+\frac{p+1}{2})}$ $\int_{0 < U < T} T-U ^\gamma {}_1F_1(-v, \gamma+\frac{p+1}{2}; -A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}}) f(U) dU$	$f(U) = L_U^{-1} \left[\frac{L_X[g_v^\gamma(T)]}{\Gamma_p(\gamma+v+\frac{p+1}{2})} A+X ^{-v} X ^{v+\gamma+\frac{(p+1)}{2}} \right]$
1	$\text{etr}(-A^{\frac{1}{2}}TA^{\frac{1}{2}}) T ^{-v-\frac{p+1}{2}}$	$\frac{ U ^{-v-r-(p+1)} \Gamma_p(-v)}{\Gamma_p(\gamma+v+\frac{p+1}{2})\Gamma_p(-\gamma-v-\frac{(p+1)}{2})}, U = U^1 > 0, \text{Re}(v)$
2	$\text{etr}(-A^{\frac{1}{2}}TA^{\frac{1}{2}}) T ^{-\beta-\frac{p+1}{2}}$	$\frac{ U ^{-\beta-r-(p+1)} \Gamma_p(-\beta)}{\Gamma_p(\gamma+\beta+\frac{p+1}{2})\Gamma_p(-\gamma-\beta-\frac{(p+1)}{2})}, U = U^1 > 0, \text{Re}(\beta)$
3	$L_v^\gamma(T)$	$f(U)$
4	$ T ^{v-\frac{p+1}{2}}$	$\frac{ U ^{-v-r-(p+1)} \Gamma_p(v) {}_1F_1(v; v+\gamma+\frac{p+1}{2}; -AU)}{\Gamma_p(\gamma+v+\frac{p+1}{2})\Gamma_p(-\gamma-v-\frac{(p+1)}{2})},$ $R(\Gamma_p(\gamma+v+\frac{p+1}{2})\Gamma_p(-\gamma-v-\frac{(p+1)}{2})) > 0$
5	$ T ^{v-\frac{p+1}{2}} {}_1F_1(\beta, v; -AU)$	$\frac{ U ^{-2v-r-(p+1)} \Gamma_p(v) {}_1F_1(v+\beta; 2v+\gamma+\frac{p+1}{2}; -AU)}{\Gamma_p(\gamma+v+\frac{p+1}{2})\Gamma_p(-\gamma-2v-\frac{(p+1)}{2})}$ $\Gamma_p(\gamma+v+\frac{p+1}{2})\Gamma_p(-\gamma-2v-\frac{(p+1)}{2})$

REFERENCES

1. Mathai A.M., Jacobians of matrix transformations and function of matrix argument, **world scientific publishing co pvt ltd**; Singapore. 1997;
2. Mathai A.M and saxena R.K, The H –function with applications in statistics and other disciplines, **john –wiley&sons, new York**. 1978;
3. Mathai A.M and H.J., Haubold, *Special Functions for Applied Scientists*, Springer, New York, 2008; 464+xxv.
4. Sharma Yogesh, **Generalized stieltjes transform of matrix variable:**, **Indian academy of mathematics**, 2005; 27(2): 289-293.
5. Sharma Yogesh, **Integral Transform of Matrix Variable involving gauss hyper geometric Function**. **Vijnana Parished Anusandhan Patrika, Allahabad**. 2010; 53 (3) July, 47-51
6. Sharma Yogesh, **Distribution Properties of Legendre Polynomials of matrix Variable** **Indian academy of Mathematics**. (communicated) 2013;