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Ranking Interval valued Fuzzy Numbers

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ABSTRACT:

In this paper, interval valued fuzzy numbers have been defined and a new ranking formula have been proposed. The membership area of the fuzzy numbers are splitted into plane figures and centroid of the centroids of these plane figures are calculated. The ranking formula is calculated by finding the area of this centroid from the origin. The advantage of this paper is that the ranking IVFN by this approach yields better solution when compared with other ranking methods. This approach is illustrated with numerical examples.

KEY WORDS: Triangular fuzzy number, Interval valued fuzzy number, centroid ranking method.

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INTRODUCTION:

Generally, fuzzy numbers are employed to express uncertainty. Type 1 fuzzy set theory is used to deal with imprecision. But it is not always possible for a membership function of the type to assign one point from [0,1]. For a fuzzy number, the degree of the membership is a crisp number whereas the degree of the membership for an interval valued fuzzy number is an interval.

In May 1975, Sambuc¹ presented in his doctoral research, the concept of IVFS named as ϕ fuzzy set. Interval valued fuzzy sets were suggested by Gorzlczany² andTurksen³. Wnag and Li⁴ defined interval valued fuzzy numbers IVFN and gave their extended operations.Stephen Dinagar and Abirami⁶ proposed an analytical method for finding critical path using IVFNS in fuzzy project network. Feng⁷ solved job shop scheduling problem with imprecise processing time as $(\lambda, 1)$ interval valued fuzzy numbers.Abirami and Stephen⁸ developed a new approach on ranking L-R type interval valued fuzzy numbers.

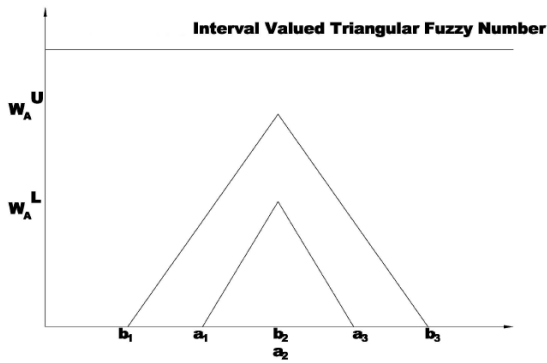
In this paper we define triangular, trapezoidal, pentagonal , hexagonal and octagonal interval valued fuzzy numbers and develop new ranking technique on those fuzzy numbers which are more efficient .

INTERVAL VALUED TRIANGULAR FUZZY NUMBER:

Atriangularfuzzy number A issaidto be an interval valued triangular fuzzy number(IVTFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3$ denoted by $A= \{(a_1, a_2, a_3), (b_1, b_2, b_3); w_A^L, w_A^U\}$, $0 \leq w_A^L \leq w_A^U \leq 1$ if its membership functions are as follows.

$$\mu_A^L(x) = \begin{cases} 0 & x < a_1 \\ \frac{w_A^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ W_A^L - \frac{w_A^L(x-a_3)}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases}$$

$$\mu_A^U(x) = \begin{cases} 0 & x < b_1 \\ \frac{w_A^U(x-b_1)}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ W_A^U - \frac{w_A^U(x-b_3)}{b_3-b_2}, & b_2 \leq x \leq b_3 \\ 0, & x > b_3 \end{cases}$$



Consider the IVTFN $A = \{(a_1, a_2, a_3, w_A^L), (b_1, b_2, b_3, w_A^U)\}$; The centroid of a triangle is considered to be the balancing point of the triangle.

The Centroid of the triangle ABC is $\left(\frac{a_1+a_2+a_3}{3}, \frac{w_A^L}{3}\right)$

Now we define $S(\mu_A^L) = x_0 \cdot y_0 = \left(\frac{a_1+a_2+a_3}{3} * \frac{w_A^L}{3}\right)$

This is the area between the centroid of the centroid and the original point.

Similarly $S(\mu_A^U) = x_0 \cdot y_0 = \left(\frac{b_1+b_2+b_3}{3} * \frac{w_A^U}{3}\right)$

Using the above definitions, the rank of A is defined as follows:

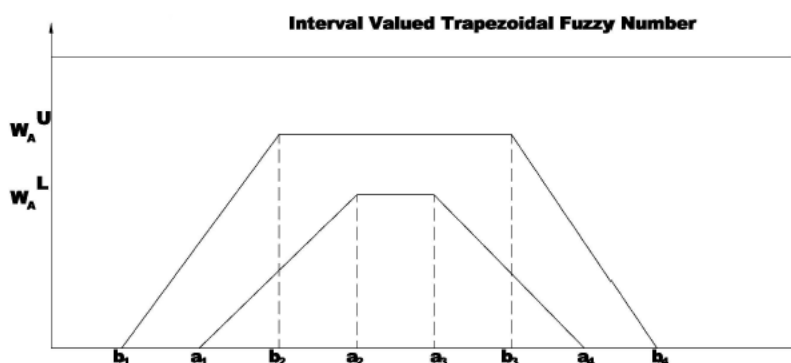
$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U}$$

INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBER

We define a Trapezoidal fuzzy number A to be an *Interval valued* Trapezoidal fuzzy number (IVTrFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ denoted by $A = \{(a_1, a_2, a_3, a_4, w_A^L), (b_1, b_2, b_3, b_4, w_A^U)\}$; $0 \leq w_A^L \leq w_A^U \leq 1$, if its membership functions are as follows.

$$\mu_A^L(x) = \begin{cases} 0 & x < a_1 \\ \frac{w_A^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ w_A^L, & a_2 \leq x \leq a_3 \\ w_A^L - \frac{(w_A^L)(x-a_3)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & x > a_4 \end{cases}$$

$$\mu_A^U(x) = \begin{cases} 0 & x < b_1 \\ \frac{w_A^U(x-b_1)}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ w_A^U, & b_2 \leq x \leq b_3 \\ w_A^U - \frac{(w_A^U)(x-b_3)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ 0, & x > b_4 \end{cases}$$



Consider the IVTrFN $A = \{(a_1, a_2, a_3, a_4, w_A^L), (b_1, b_2, b_3, b_4, w_A^U)\}$. The centroid of a Trapezoid is considered to be the balancing point of the Trapezoid. Divide the membership part of trapezoid into three plane figures namely a triangle, a quadrilateral (kite) and a triangle respectively. Let G_1, G_2, G_3 be the centroids of these three plane figures.

The Centroid of these centroids G_1, G_2, G_3 is considered as the point of reference to define the ranking of generalized Interval valued fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIPFN.

The Centroids of these plane figures are

$$G_1 = \left(\frac{a_1+2a_2}{3}, \frac{w_A^L}{3}\right); \quad G_2 = \left(\frac{a_2+a_3}{2}, \frac{w_A^L}{2}\right) \text{ and } G_3 = \left(\frac{2a_3+a_4}{3}, \frac{w_A^L}{3}\right) \text{ respectively.}$$

Thus G_1, G_2 and G_3 are not collinear and they form a triangle. Thus the centroid of these centroids is

$$G(x_0, y_0) = \left(\frac{(2a_1 + 7a_2 + 7a_3 + 2a_4)}{18}, \frac{7w_A^L}{18}\right)$$

$$\text{Now we define } S(\mu_A^L) = x_0 \cdot y_0 = \frac{(2a_1 + 7a_2 + 7a_3 + 2a_4) \cdot 7w_A^L}{18 \cdot 18}$$

This is the area between the centroid of the centroids and the original point.

Similarly the trapezoid corresponding to the upper membership function is divided into three plane figures. In similar fashion, the centroid of the three plane figures and the centroid of these centroids are evaluated. The centroid of these plane figures are

$G_1 = \left(\frac{b_1+2b_2}{3}, \frac{w_A^U}{3}\right)$; $G_2 = \left(\frac{b_2+b_3}{2}, \frac{w_A^U}{2}\right)$ and $G_3 = \left(\frac{2b_3+b_4}{3}, \frac{w_A^U}{3}\right)$ respectively.

Thus G_1, G_2 and G_3 are not collinear and they form a triangle. Thus the centroid of these centroids is

$$G(x_0, y_0) = \left(\frac{(2b_1+7b_2+7b_3+2b_4)}{18}, \frac{7w_A^U}{18}\right)$$

Now we define $S(\mu_A^U) = x_0 \cdot y_0 = \frac{(2b_1+7b_2+7b_3+2b_4) \cdot 7w_A^U}{18 \cdot 18}$

Using the above definitions, the rank of A is defined as follows:

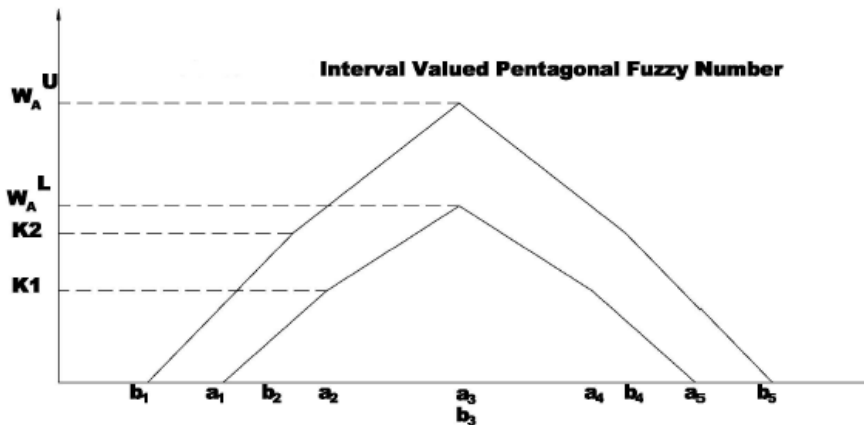
$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U}$$

INTERVAL VALUED PENTAGONAL FUZZY NUMBER

We define a pentagonal fuzzy number A to be an *Interval valued* pentagonal fuzzy number (IVPFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5$ denoted by $A = \{(a_1, a_2, a_3, a_4, a_5, w_A^L), (b_1, b_2, b_3, b_4, b_5, w_A^U)\}$, $0 \leq k_1 \leq k_2 \leq w_A^L \leq w_A^U \leq 1$, if its membership functions are as follows.

$$\mu_A^L(x) = \begin{cases} 0 & x < a_1 \\ K_1 - \frac{K_1(x-a_2)}{a_1-a_2}, & a_1 \leq x \leq a_2 \\ w_A^L + \frac{(K_1-w_A^L)(x-a_3)}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ w_A^L + \frac{(K_1-w_A^L)(x-a_3)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ K_1 - \frac{K_1(x-a_4)}{a_5-a_4}, & a_4 \leq x \leq a_5 \\ 0, & x > a_5 \end{cases}$$

$$\mu_A^U(x) = \begin{cases} 0 & x < b_1 \\ K_2 - \frac{K_2(x-b_2)}{b_1-b_2}, & b_1 \leq x \leq b_2 \\ w_A^L + \frac{(K_2-w_A^L)(x-b_3)}{b_2-b_3}, & b_2 \leq x \leq b_3 \\ w_A^L + \frac{(K_2-w_A^L)(x-b_3)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ K_1 - \frac{K_2(x-b_4)}{b_5-b_4}, & b_4 \leq x \leq b_5 \\ 0, & x > b_5 \end{cases}$$



Consider the IVPFN $A = \{(a_1, a_2, a_3, a_4, a_5, w_A^L), (b_1, b_2, b_3, b_4, b_5, w_A^U)\}$. The centroid of a pentagon is considered to be the balancing point of the pentagon. Divide the lower membership part of pentagon into three plane figures namely a quadrilateral (kite) and two triangles. Let G_1, G_2, G_3 be the centroids of these three plane figures.

The Centroid of these centroids G_1, G_2, G_3 is considered as the point of reference to define the ranking of generalized pentagonal Institutionalistic fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIPFN.

The Centroids of these plane figures are

$$G_1 = \left(\frac{a_1 + a_2 + a_3}{3}, \frac{K_1}{3} \right); \quad G_2 = \left(\frac{a_2 + a_3 + a_4}{3}, \frac{w_A^L + K_1}{3} \right) \text{ and } G_3 = \left(\frac{a_3 + a_4 + a_5}{3}, \frac{K_1}{3} \right) \text{ respectively.}$$

Thus G_1, G_2 and G_3 are not collinear and they form a triangle. Thus the centroid of these centroids is

$$G(x_0, y_0) = \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}, \frac{3K_1 + w_A^L}{9} \right)$$

$$\text{Now we define } S(\mu_A^L) = x_0 \cdot y_0 = \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9} \right) \times \frac{3K_1 + w_A^L}{9}.$$

This is the area between the centroid of the centroids and the original point.

Similarly the pentagon corresponding to the upper membership function is divided into three plane figures. In similar fashion, the centroid of the three plane figures and the centroid of these centroids are evaluated. The centroid of these plane figures are

$$G_1 = \left(\frac{b_1 + b_2 + b_3}{3}, \frac{K_1}{3} \right); \quad G_2 = \left(\frac{b_2 + b_3 + b_4}{3}, \frac{w_A^U + K_1}{3} \right); \quad G_3 = \left(\frac{b_3 + b_4 + b_5}{3}, \frac{K_1}{3} \right).$$

The centroid of these centroids is

$$G'(x_0, y_0) = \left(\frac{b_1 + 2b_2 + 3b_3 + 2b_4 + b_5}{9}, \frac{3K_1 + w_A^U}{9} \right).$$

Now we define $S(\mu_A^U) = x_0.y_0 = \left(\frac{b_1+2b_2+3b_3+2b_4+b_5}{9}\right) \times \frac{3K_1+w_A^U}{9}$

Using the above definitions, the rank of A is defined as follows:

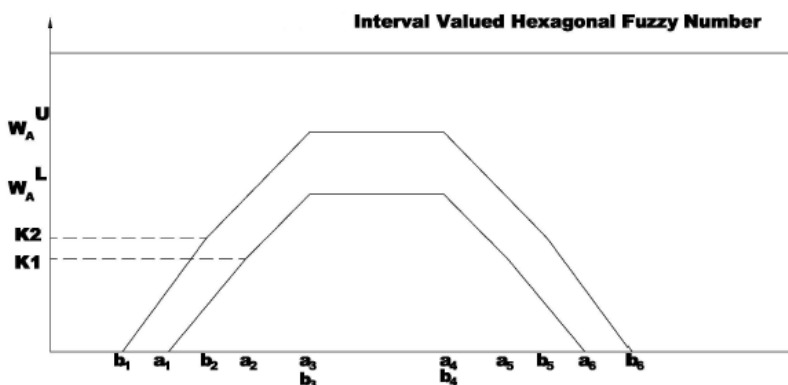
$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U}$$

INTERVAL VALUED HEXAGONAL FUZZY NUMBER

We define hexagonal fuzzy number A to be an interval valued hexagonal fuzzy number (IVHFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5 \leq a_6 \leq b_6$ denoted by $A = \{(a_1, a_2, a_3, a_4, a_5, a_6, w_A^L), (b_1, b_2, b_3, b_4, b_5, b_6, w_A^U)\}$, $0 \leq k_1 \leq k_2 \leq w_A^L \leq w_A^U \leq 1$, if its membership functions are as follows.

$$\mu_A^L(x) = \begin{cases} 0 & x < a_1 \\ \frac{K_1(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ K_1 + \frac{(w_A^L-K_1)(x-a_3)}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ w_A^L & a_3 \leq x \leq a_4 \\ w_A^L + \frac{(K_1-w_A^L)(x-a_4)}{a_4-a_3}, & a_4 \leq x \leq a_5 \\ K_1 - \frac{K_1(x-a_5)}{a_6-a_5}, & a_5 \leq x \leq a_6 \\ 0, & x > a_6 \end{cases}$$

$$\mu_A^U(x) = \begin{cases} 0 & x < b_1 \\ \frac{K_2(x-b_1)}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ K_2 + \frac{(w_A^U-K_2)(x-b_3)}{b_3-b_2}, & b_2 \leq x \leq b_3 \\ w_A^U & b_3 \leq x \leq b_4 \\ w_A^U + \frac{(K_2-w_A^U)(x-b_4)}{b_4-b_3}, & b_4 \leq x \leq b_5 \\ K_2 - \frac{K_2(x-b_5)}{b_6-b_5}, & b_5 \leq x \leq b_6 \\ 0, & x > b_6 \end{cases}$$



Consider the IVHFN $A = \{(a_1, a_2, a_3, a_4, a_5, a_6, w_A^L), (b_1, b_2, b_3, b_4, b_5, b_6, w_A^U)\}$. The centroid of a Hexagon is considered to be the balancing point of the Hexagon. The centroid of Hexagon is the centroid of centroids G_1, G_2, G_3 which is considered as the point of reference to define the ranking of generalized hexagonal Intuitionistic fuzzy numbers. As the centroid of these three plane figures are their balancing points, the centroid of these centroid points is a much better balancing point for a GIHFN.

The Centroids of these plane figures are

$$G_1 = \left(\frac{a_1 + a_2 + a_3}{3}, \frac{k_1}{3} \right), G_3 = \left(\frac{a_4 + a_5 + a_6}{3}, \frac{k_1}{3} \right) \text{ and } G_2 = \left(\frac{2a_1 + 7a_3 + 7a_4 + 2a_5}{18}, \frac{7w_A^L + 4k_1}{18} \right) \text{ respectively.}$$

From the above figure, the centroid of hexagon is calculated as

$$G(x_0, y_0) = \left(\frac{6a_1 + 8a_2 + 13a_3 + 13a_4 + 8a_5 + 6a_6}{54}, \frac{7w_A^L + 16k_1}{54} \right)$$

$$\text{Now we define } S(\mu_A^L) = x_0 \cdot y_0 = \left(\frac{6a_1 + 8a_2 + 13a_3 + 13a_4 + 8a_5 + 6a_6}{54} \right) \times \frac{7w_A^L + 16k_1}{54}$$

This is the area between the centroid of the hexagon and the original point

Similarly

$$\text{we define } S(\mu_A^U) = x_0 \cdot y_0 = \left(\frac{6b_1 + 8b_2 + 13b_3 + 13b_4 + 8b_5 + 6b_6}{54} \right) \times \frac{7w_A^U + 16k_2}{54}$$

Using the above definitions, the rank of A is defined as follows:

$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U}$$

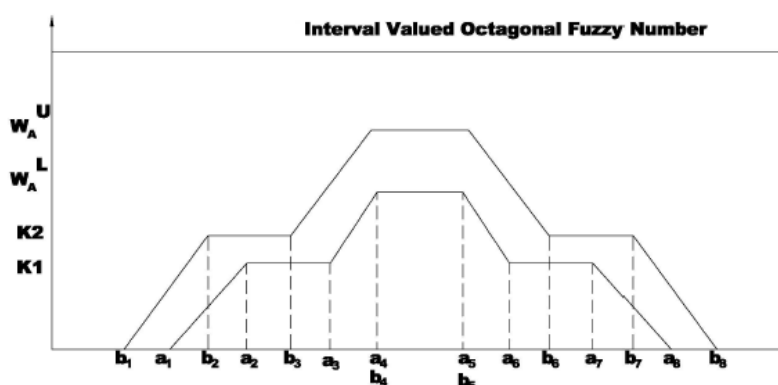
INTERVAL VALUED OCTAGONAL FUZZY NUMBER:

An Octagonal fuzzy number A is said to be a interval valued Octagonal fuzzy number (IVOFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq b_4 \leq a_4 \leq a_5 \leq b_5 \leq a_6 \leq b_6 \leq a_7 \leq b_7 \leq a_8 \leq b_8$ denoted by

$A = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, w_A^L), (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, w_A^U)\}$, $0 \leq k_1 \leq k_2 \leq w_A^L \leq w_A^U \leq 1$, if its membership functions are as follows.

$$\mu_A^L(x) = \begin{cases} 0 & x < a_1 \\ \frac{k_1(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ k_1, & a_2 \leq x \leq a_3 \\ k_1 + \frac{(w_A^L - k_1)(x-a_3)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ w_A^L, & a_4 \leq x \leq a_5 \\ w_A^L + \frac{(k_1 - w_A^L)(x-a_5)}{a_6-a_5}, & a_5 \leq x \leq a_6 \\ K_1, & a_6 \leq x \leq a_7 \\ K_1 - \frac{K_1(x-a_7)}{a_8-a_7}, & a_7 \leq x \leq a_8 \\ 0, & x > a_8 \end{cases}$$

$$\mu_A^U(x) = \begin{cases} 0 & x < b_1 \\ \frac{k_2(x-b_1)}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ k_2, & b_2 \leq x \leq b_3 \\ k_2 + \frac{(w_A^U - K_2)(x-b_3)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ w_A^U & b_4 \leq x \leq b_5 \\ w_A^U + \frac{(K_2 - w_A^U)(x-b_5)}{b_6-b_5}, & b_5 \leq x \leq b_6 \\ K_2, & b_6 \leq x \leq b_7 \\ K_2 - \frac{K_2(x-b_7)}{b_8-b_7}, & b_7 \leq x \leq b_8 \\ 0, & x > b_8 \end{cases}$$



In similar fashion, The centroid of octagon is calculated as

$$G(x_0, y_0) = \left(\frac{(2a_1 + 7a_2 + 9a_3 + 9a_4 + 9a_5 + 9a_6 + 7a_7 + 2a_8)}{54}, \frac{7w_A^L + 18k_1}{54} \right)$$

$$\text{Now we define } S(\mu_A^L) = x_0 \cdot y_0 = \left(\frac{(2a_1 + 7a_2 + 9a_3 + 9a_4 + 9a_5 + 9a_6 + 7a_7 + 2a_8)}{54} \right) \times \frac{7w_A^L + 18k_1}{54}$$

This is the area between the centroid of the centroids and the original point.

Similarly,

$$\text{Now we define } S(\gamma_A^L) = x_0 \cdot y_0 = \left(\frac{(2b_1 + 7b_2 + 9b_3 + 9b_4 + 9b_5 + 9b_6 + 7b_7 + 2b_8)}{54} \right) \times \frac{7w_A^U + 18k_2}{54}$$

Using the above definitions, the rank of A is defined as follows:

$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U}$$

ARITHMETIC OPERATIONS:

If $A = \{(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); w_A^L, w_A^U\}$, and

$B = \{(c_1, c_2, c_3, c_4), (d_1, d_2, d_3, d_4); w_B^L, w_B^U\}$, are two IVTFN then

$$A+B = \{(a_1 + c_1, a_2 + c_2, a_3 + c_3, a_4 + c_4), (b_1 + d_1, b_2 + d_2, b_3 + d_3, b_4 + d_4); w, u\}$$

where $w = \min\{w_A^L, w_B^L\}$, $u = \max\{w_A^U, w_B^U\}$

$$A-B = \{ (a_1 - c_4, a_2 - c_3, a_3 - c_2, a_4 - c_1)(b_1 - d_4, b_2 - d_3, b_3 - d_2, b_4 - d_1); w, u \} \text{ where } w = \min\{w_A^L, w_B^L\}, u = \max\{w_A^U, w_B^U\}$$

$$A * B = \{ (a_1 \cdot R(B), a_2 \cdot R(B), a_3 \cdot R(B), a_4 \cdot R(B))(b_1 \cdot R(B), b_2 \cdot R(B), b_3 \cdot R(B), b_4 \cdot R(B));$$

$$w, u \} \text{ where } w = \min\{w_A^L, w_B^L\}, u = \max\{w_A^U, w_B^U\} \text{ if } R(B) > 0$$

$$A/B = \{ (\frac{a_1}{R(B)}, \frac{a_2}{R(B)}, \frac{a_3}{R(B)}, \frac{a_4}{R(B)})(\frac{b_1}{R(B)}, \frac{b_2}{R(B)}, \frac{b_3}{R(B)}, \frac{b_4}{R(B)}); w, u \} \text{ where } w = \min\{w_A^L, w_B^L\}, u = \max\{w_A^U, w_B^U\}$$

In Similar fashion the arithmetic operations can be defined for other fuzzy numbers.

NUMERICAL ILLUSTRATION.

Consider the Interval valued trapezoidal fuzzy number denoted by $A = \{(a_1, a_2, a_3, a_4; w_A^L), (b_1, b_2, b_3, b_4; w_A^U)\} = \{(2, 4, 5, 7; 1)(1, 3, 6, 8; 1)$

Rank of A by the proposed method:

$$R(A) = \frac{w_A^L S(\mu_A^L) + w_A^U S(\mu_A^U)}{w_A^L + w_A^U} \text{ where}$$

$$S(\mu_A^L) = x_0 \cdot y_0 = \frac{(2a_1 + 7a_2 + 7a_3 + 2a_4)}{18} \cdot \frac{7w_A^L}{18} = 1.75$$

$$S(\mu_A^U) = x_0 \cdot y_0 = \frac{(2b_1 + 7b_2 + 7b_3 + 2b_4)}{18} \cdot \frac{7w_A^U}{18} = 1.75$$

$$R(A) = 1.75$$

Rank of A defined by the existing method in [6]

$$R(A) = \frac{(a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4)}{8} = 4.5$$

CONCLUSION:

This paper proposes interval valued triangular, trapezoidal, Pentagonal, hexagonal and Octagonal fuzzy numbers along with a efficient ranking technique. This centroid based ranking method gives more efficient result and is illustrated with a numerical example.

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