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Soft Generalized Topological spaces via Soft Hereditary

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ABSTRACT

The concept of generalized topology was investigated by Csaszar and also modified generalized topology via hereditary. Jyothis and Sunil introduced the concept of Soft Generalized Topological Space (SGTS) and studied soft μ-compactness in SGTS. In this paper, we introduce soft μ-$\widetilde{\mathcal{H}}$ -regular spaces, soft μ-$\widetilde{\mathcal{H}}$ -Normal spaces with a fixed set of parameters and obtain some properties in the light of these notions. We also investigate some properties of these new notions by using soft maps such as soft(μ, η)-continuous map, soft (μ, η)-open map, soft (μ, η) irresolute map in soft generalized topological spaces.Further we introduce $\widetilde{\mathcal{H}}$-submaximal space, $\widetilde{\mathcal{H}}$-extremely disconnected space in soft generalized topological spaces.

KEYWORDS: Soft generalized topology, Hereditary $\widetilde{\mathcal{H}}$, Soft μ-$\widetilde{\mathcal{H}}$ -regular spaces, soft μ-$\widetilde{\mathcal{H}}$ -Normal spaces,$\widetilde{\mathcal{H}}$-submaximal and $\widetilde{\mathcal{H}}$-extremely disconnected.

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1. INTRODUCTION

The idea of generalized topology and hereditary class was introduced andstudied by Csaszar[2,3,4]. A subfamily μ of P(X) is called a generalized topology if $\phi \in \mu$ and union of elements of μ belongs to μ. The elements of μ is said to be μ -open.We saya hereditary class$\mathcal{H}$[3]on (X, μ) is a non-empty collection of subset of X such that$A \subseteq B, B \in \mathcal{H}$ implies $A \in \mathcal{H}$. With respect to the generalizedtopology μ and ahereditary class $\mathcal{H}$, for a subset A of X we define[3]$A_{\mu}^{*}(\mathcal{H})$or simply $A_{\mu}^{*}=\{x \in X : M \cap A \notin \mathcal{H}$ for every$M \in \mu$ such that$\in M\}$. The closure[3]$c_{\mu}^{*}(A)= A \cup A_{\mu}^{*}(\mathcal{H})$.The soft set theory is a rapidly processing field of mathematics. Soft set theory helps derive an effective solution from an uncertain and inadequate data.Molodtsov's[7]introduced the concept Soft Set Theory.
A soft set F_A[7]over the universe U is defined by the set of ordered pairs
$F_A = \{(e, f_A(e)) / e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A:A \rightarrow P(U)$ such that $f_A(e)=\phi$ if $e \notin A$. Here $f_A$is called an approximate function of soft set F_A. The set of all soft sets over U is denoted by S(U).

Jyothis and Sunil [5]introduced the concept of Soft Generalized Topological Space (SGTS). Soft generalized topology is based on soft set theory. Jyothis and Sunil[6] discussed some separation axioms in soft generalized topological space.

The purpose of present paper is with respect to the soft generalized topologyμ or μ_{F_A}defined[5]as collection of soft subsets of F_A satisfying the following properties: (i) $F_{\emptyset} \in \mu$ and (ii) The soft union of any number of soft sets in μ belong to μ, we consider a soft hereditary class $\widetilde{\mathcal{H}} \neq F_{\varphi} \subseteq S(U)$ satisfying; if $F_G \subseteq F_H \in \widetilde{\mathcal{H}}$ implies $F_G \in \widetilde{\mathcal{H}}$.The soft generalized topological space (F_A, μ) with the soft hereditary $\widetilde{\mathcal{H}}$ is called soft hereditary generalized topological space.

Consider U = {1,2,3} be the set of three houses under consideration and E = {e_1(cost), e_2(comfortness), e_3(external facilities)}. A= { e_1(cost), e_2(comfortness)} Let F_{A_1}, F_{A_2} , F_{A_3} be three soft sets representing willingness of the persons who are going to buy, where $F_{A_1}(e_1) = \{2\}$, $F_{A_1}(e_2)=\{1\}$, $F_{A_2}(e_1)=\{2,3\}$, $F_{A_2}(e_2)=\{1, 2\}$, $F_{A_3}(e_1) = \{1,2\}$, $F_{A_3}(e_2) = \{1, 3\}$. Then F_{A_1}, F_{A_2} and F_{A_3} are soft sets over U and $\mu = \{F_{\varphi}, F_{A_1}, F_{A_2}, F_{A_3}, F_A\}$ is the soft generalized topology over U. Let $\widetilde{H}=\{F_{\varphi}, F_{A_4}\}$ be a soft hereditary on F_A, where $F_{A_4}(e_1)=1\}$ $F_{A_4}(e_2)=\varphi$. Then(F_A, μ,$\widetilde{\mathcal{H}}$) is called soft hereditary generalized topological space.

2. PRELIMINARIES

Let μ_{F_A} be the soft generalized topology on F_A; a subset $F_G \subseteq F_A$ is said to be soft μ-open[5] if $F_G \in \mu_{F_A}$ and soft μ-closed if $F_A - F_G \in \mu_{F_A}$. For a subset F_G of F_A, $c_\mu(F_D)$[5] and $i_\mu(F_D)$[5] denotes intersection of all soft μ-closed supersets of F_G and union of all soft μ-open subsets of F_G respectively. Let (F_A, μ) be a SGTS. A subset F_G of F_A is said to be soft μ-regular closed[8](soft pre μ-closed[8], soft αμ-closed[8], soft βμ-closed[8])if $F_G = i_\mu(c_\mu(F_G)$ $(c_\mu(i_\mu(F_D)) \subseteq F_D, c_\mu(i_\mu(c_\mu(F_D))) \subseteq F_D, i_\mu(c_\mu(i_\mu(F_D))) \subseteq F_D)$ respectively. Union of soft μ-regular closed sets is called soft μπ-closed. Note that every soft μ-regular open set is soft μ-open.

Let $\tilde{\mathcal{H}} \neq F_\varphi$ be the hereditary class on soft generalized topological space (F_A, μ). For a subset F_G of F_A, we define ${F_G}_\mu^*(\tilde{\mathcal{H}})$ or simply ${F_G}_\mu^* = \{\alpha \in F_A : F_U \cap F_G \notin \tilde{\mathcal{H}}$ for every $F_U \in \mu$ such that $\alpha \in F_G\}$. The closure $c_\mu^*(F_G) = F_G \cup {F_G}_\mu^*$. A subset F_G of F_A is said to be soft μ*-closed if ${F_G}_\mu^*(\tilde{\mathcal{H}}) \subseteq F_G$, soft μ*-dense if $c_\mu^*(F_G) = F_A$, soft μ*-codense if $F_A - F_G$ is soft μ*-dense.

We define that a subset F_D of F_A is said to be soft be soft $\tilde{\mathcal{H}}$ -regular closed(soft semi-$\tilde{\mathcal{H}}$-open, soft pre $\tilde{\mathcal{H}}$-closed, soft α-$\tilde{\mathcal{H}}$-closed, soft β-$\tilde{\mathcal{H}}$-closed soft β*-$\tilde{\mathcal{H}}$-closed)if $F_G = i_\mu(c_\mu^*(F_G)$ $F_D \subseteq c_\mu^*(i_\mu(F_D)), c_\mu^*(i_\mu(F_D)) \subseteq F_D, F_D \subseteq i_\mu(c_\mu^*(i_\mu(F_D))), F_D \subseteq c_\mu(i_\mu(c_\mu^*(F_D))),$
$F_D \subseteq c_\mu^*(i_\mu(c_\mu^*(F_D)))]$ respectively.

Let (F_A, μ) and (F_B, η) be two SGTS's and $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft function. Then

1. φ_χ is said to be soft (μ, η)-continuous[6] (briefly, soft continuous), if for each soft η-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is a soft μ-open subset of F_A.
2. φ_χ is said to be soft (μ, η)-open[6], if for each soft μ-open subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-open subset of F_B.
3. φ_χ is said to be soft (μ, η)-closed[6], if for each soft μ-closed subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-closed subset of F_B.

3. SOFTμ-$\tilde{\mathcal{H}}$-REGULAR, SOFTμ-$\tilde{\mathcal{H}}$ -NORMAL SPACES.

Definition:3.1 A soft hereditary generalized topological space $(F_A, \mu, \tilde{\mathcal{H}})$ is said to be softμ-H̃-regular if for everysoft μ-closed set F_G and a point $\alpha \notin F_G$, there existsoft μ-open sets F_U and F_V such that $F_G - F_U \in \tilde{H}$, $\alpha \notin F_V$ and $F_U \cap F_V \in \tilde{H}$.

Definition:3.2 Let $(F_A, \mu, \tilde{H})$ be a soft hereditary generalized topological spaces. A space $(F_A, \mu, \tilde{\mathcal{H}})$ is said to be soft μ-$\tilde{\mathcal{H}}$- normal if for every pair of soft μ-closed sets F_G and F_H of F_A, there exist soft μ-open sets F_U and F_V such that $F_G - F_U \in \tilde{\mathcal{H}}$, $F_H - F_V \in \tilde{\mathcal{H}}$, and $F_U \cap F_V \in \tilde{\mathcal{H}}$.

Theorem:3.3 Let (F_A, μ, $\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological space and (F_B, η) be a soft generalized topology. A function φ_χ: (F_A, μ, $\widetilde{\mathcal{H}}$) → (F_B, η) is bijective, soft (μ,η)-continuous and soft (μ, η)-open. If (F_A, μ, $\widetilde{\mathcal{H}}$) is soft μ- $\widetilde{\mathcal{H}}$-regular space, then (F_B, η) is soft η-f($\widetilde{\mathcal{H}}$)-regular space.

Proof: Let α∈F_B and $F_G$be any soft μ -closed set. Since φ_χ is soft (μ, η)-continuous, $\varphi_\chi^{-1}(F_G)$is soft μ -closed subset of F_A. Since (F_A, μ, $\widetilde{\mathcal{H}}$) is soft μ-$\widetilde{\mathcal{H}}$ -regular space, there exists soft μ-open sets F_D and F_E such thatα∈ F_D,$\varphi_\chi^{-1}(F_G)$− F_E∈ H̃and F_D∩F_E∈ H̃.By hypothesis$\varphi_\chi(F_D)$ and $\varphi_\chi(F_E)$ are soft μ-open sets in F_B. Hence (F_G)-φ_χ (F_E)∈$\varphi_\chi(\widetilde{\mathcal{H}})$and φ_χ (F_D)∩φ_χ (F_E)∈ $\varphi_\chi(\widetilde{\mathcal{H}})$. Since the collection $\varphi_\chi(\widetilde{\mathcal{H}})$={$\varphi_\chi$ (F_H): F_H∈ $\widetilde{\mathcal{H}}$} is a soft hereditary class on F_B , (F_B, η) is soft η-$\varphi_\chi(\widetilde{\mathcal{H}})$-regular space.

Theorem:3.4 Let(F_A, μ, $\widetilde{\mathcal{H}}$) be a Soft hereditary generalized topological space.

Then the followings are equivalent:

(a) F_A is a soft μ-$\widetilde{\mathcal{H}}$ -regular space;

(b) for each point α ∈$F_A$and for each soft μ -open neighbourhood F_H of α, thereexists a soft μ-open set F_D of F_A such that $c_\mu{}^*(F_D)$ −F_H∈ H̃

(c) For each pointα ∈F_A and for each soft μ -closed set $F_G$not containing α, thereexists a soft μ-open set F_D of F_A such that $c_\mu{}^*(F_D)$∩ F_G ∈ H̃.

Proof: The equivalency is obvious.

Theorem:3.5 Let (F_A, μ) be a softgeneralized topological space and (F_B, η,$\widetilde{\mathcal{H}}$) be soft hereditary generalized topological space. A function φ_χ: (F_A, μ) → (F_B, η, $\widetilde{\mathcal{H}}$) is injective, soft (μ, η)-closed and soft (μ, η) -continuous. If F_B is soft η-$\widetilde{\mathcal{H}}$-regular space, then F_A is soft μ-φ_χ^{-1}**($\widetilde{\mathcal{H}}$)**-regular space.

Proof: Let α∈F_A and $F_G$be any soft μ-closed set. Since φ_χ is soft (μ,η)-closed, φ_χ (F_G) is soft η-closed. By hypothesis, there exists soft η-open sets F_D and F_E such that φ_χ(α) ∈F_D, $\varphi_\chi(F_G)$− F_E∈ $\widetilde{\mathcal{H}}$and F_D∩F_E∈$\widetilde{\mathcal{H}}$. Sinceφ_χ is soft (μ,η)-continuous and injective, $\varphi_\chi^{-1}(F_D)$and $\varphi_\chi^{-1}(F_E)$ are soft μ-open such that α ∈$\varphi_\chi^{-1}(F_D)$, F_G-$\varphi_\chi^{-1}(F_E)$ ∈$\varphi_\chi^{-1}(\mathcal{H})$. Since, the collection $\varphi_\chi^{-1}(\widetilde{\mathcal{H}})$={$\varphi_\chi^{-1}(F_H)$: F_H∈$\widetilde{\mathcal{H}}$} is a soft hereditary class on F_A,F_A is soft μ-$\varphi_\chi^{-1}(\widetilde{\mathcal{H}})$-regular space.

Theorem :3.6 Let (F_A, μ,$\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological space. If F_A is soft μ -$\widetilde{\mathcal{H}}$-regular space and F_B⊆ F_A is soft μ -closed set, then F_B is soft μ -$\mathcal{H}_{F_B}$-regular space.

Definition: 3.7 Let (F_A, μ, $\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological spaces. A space(F_A, μ, $\widetilde{\mathcal{H}}$) is said to be soft almost(soft quasi, soft ultra) μ-$\widetilde{\mathcal{H}}$-normal if for every pair of soft μ- regular-closed(soft μπ-closed, soft μπ-closed) sets F_G and F_H of F_A, there exist soft μ-open(soft μ-open, soft μ-clopen) sets F_U and F_V such that F_G–F_U∈ $\widetilde{\mathcal{H}}$, F_H – F_V ∈ $\widetilde{\mathcal{H}}$, and F_U∩F_V∈$\widetilde{\mathcal{H}}$.

Definition: 3.8 The soft function φ_χ:(F_A, μ) → (F_B, η) is said to be a is said to be

(i) soft (μ,η) -R pre-closed, if for each soft μ regular-closed subset $F_G$of F_A, the image $\varphi_\chi(F_G)$ isa soft η regular-closed in F_B.

(ii)perfectly(μ, η)-continuous if for each soft η-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is soft μ-clopenin F_A.

Theorem:3.9 Let (F_A, μ,$\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological space and (F_B, η) be a soft generalized topological space. Also let φ_χ:(F_A, μ, $\widetilde{\mathcal{H}}$) → (F_B, η) is soft (μ, η) R-pre-closed and soft (μ, η)-continuous serjective function. If F_B is soft almost η -H̃-normal space, then F_A is soft almost μ-$\varphi_\chi^{-1}(\widetilde{\mathcal{H}})$-normal space.

Proof: The proof is similar to (3.5)

Note that the subspace of soft almost μ-$\widetilde{\mathcal{H}}$-normal space is soft almost μ-$\widetilde{\mathcal{H}}$-normal.

Theorem: 3.10 Let (F_A, μ,$\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological space and (F_B, η) be a soft generalized topological space. Also let φ_χ: (F_A, μ, $\widetilde{\mathcal{H}}$) → (F_B, η) is soft almost (μ, η)-closed and perfectly (μ, η)-continuous injective soft function. If F_B is soft quasi η -$\widetilde{\mathcal{H}}$ -normal space, then F_A is soft quasi ultra μ-$f^{-1}(\widetilde{\mathcal{H}})$- normal space.

The proof is similar to theorem(3.3)

Remark: 3.11 Let (F_A, μ,$\widetilde{\mathcal{H}}$) be a soft hereditary generalized topological space and(F_B, η) be a soft generalized topological space. Also let φ_χ: (F_A, μ, $\widetilde{\mathcal{H}}$) → (F_B, η) is soft almost (μ, η)-pre-closed and soft (μ, η)-continuous surjective function. IfF_B is soft quasi η-H̃-normalspace, then F_A is soft almost μ -$\varphi_\chi^{-1}(\widetilde{\mathcal{H}})$-normal space.

Proof: Sinceevery soft μ-regular-closed set is soft μπ-open the proof is obvious.

4. $\mathcal{H}$-SUBMAXIMAL ANDEXTREMELY DISCONNECTED SPACE.

Definition: 4.1 A soft hereditary generalized topological space (F_A, μ,$\widetilde{\mathcal{H}}$) is said to be

(i) $\widetilde{\mathcal{H}}$-submaximal if every soft μ*-dense set is soft μ- open,

(ii)$\widetilde{\mathcal{H}}$-extremally disconnected if soft- μ*-closure of every soft μ- open set F_G of F_A is soft μ-open.

Theorem:4.2 Let(F_A, μ,$\widetilde{\mathcal{H}}$) be a soft hereditary generalized space. Then F_A is $\widetilde{\mathcal{H}}$-submaximal,iff every soft μ*-codense subset F_G of F_A is soft μ-closed.

Proof: Consider F_G as softμ*-codense subset of F_A. By definition of softμ*-codense, F_A- $F_G$is soft μ*-dense.ThenF_A-F_G is soft μ- open. Thus, $F_G$is soft μ-closed.Simillary we can prove the converse part.

Theorem:4.3 If soft pre-H̃-open set is softsemi-$\widetilde{\mathcal{H}}$-open and every soft α-$\widetilde{\mathcal{H}}$-open set is soft μ-openthen the soft hereditarygeneralized space (F_A,μ,$\widetilde{\mathcal{H}}$) is$\widetilde{\mathcal{H}}$-submaximal space.

Proof: Let F_G be softμ*-dense subset of F_A. So that $c_\mu*(F_G) = F_A$ and hence $F_G$if soft pre$\tilde{H}$-open. By hypothesis F_G soft semi-$\tilde{\mathcal{H}}$ -open. A soft subset F_G is soft α-$\tilde{\mathcal{H}}$ -open if it is soft pre$\tilde{H}$-open and soft semi $\tilde{\mathcal{H}}$-open. Hence F_G is soft α-$\tilde{\mathcal{H}}$ -open. It is given that every soft α-$\tilde{\mathcal{H}}$ -open set is soft μ -open and hence (F_A, μ,$\tilde{\mathcal{H}}$) is$\tilde{\mathcal{H}}$-submaximal space.

Theorem:4.4 If (F_A, μ,$\tilde{\mathcal{H}}$) is$\tilde{\mathcal{H}}$-submaximal space then every soft semi-$\tilde{H}$-open is soft -$\beta^*\tilde{\mathcal{H}}$-open.

Theorem:4.5 Let φ_χ:(F_A, μ) → (F_B, η, $\tilde{\mathcal{H}}$) be a soft μ- open surjective function. If F_A is μ -submaximal, then F_B is $\tilde{\mathcal{H}}$-submaximal.

Proof: Let $F_G \subseteq F_B$be a soft μ*-dense set and $F_A$be μ -submaximal space.Then F_G is soft μ-dense. By hypothesis, $\varphi_\chi^{-1}(F_G)$ is soft μ -dense and hence soft μ- open in F_A. Since $\varphi_\chi$is an open surjectivefunction, $F_G$is soft μ-open. Thus$F_B$is $\tilde{\mathcal{H}}$-submaximal.

Definition:4.6 A soft function φ_χ: (F_A, μ, $\tilde{\mathcal{H}}$) → (F_B,η) is said to be μ*-(μ,η)-continuous(semi $\tilde{\mathcal{H}}$-(μ,η)-continuous, β$\tilde{\mathcal{H}}$-(μ, η)-continuous) if $\varphi_\chi^{-1}(F_G)$ is soft μ*-open(soft semi $\tilde{\mathcal{H}}$-open, soft β$\tilde{\mathcal{H}}$-open) for every soft η-open set $F_{G.}$

Theorem:4.7 Let φ_χ: (F_A, μ, $\tilde{\mathcal{H}}$) → (F_B, η) be a soft function and F_A be $\tilde{\mathcal{H}}$-submaximal and $\tilde{\mathcal{H}}$-extremely disconnected space. Then the following are equivalent.

(i) $\varphi_\chi$is semi$\tilde{\mathcal{H}}$-(μ, η) -continuous

(ii) $\varphi_\chi$isβ$\tilde{\mathcal{H}}$-(μ, η)-continuous

(iii) $\varphi_\chi$isμ*-(μ, η)-continuous.

Proof: Since F_A is $\tilde{\mathcal{H}}$-submaximal and $\tilde{\mathcal{H}}$-extremely disconnected, we havesoft μ*-open sets of (F_A, μ, $\tilde{\mathcal{H}}$) = soft semi$\tilde{\mathcal{H}}$-open sets=soft β$\tilde{\mathcal{H}}$-open sets and hence the proof.

CONCLUSION

Soft topologyserve as a tool for solving uncertainity problems. Apart from theoritical part there are many application oriented problems. Weare concentrating on applicationsof soft topology in computer for my future work.We suggest that the researchers may promote their further works on applications of soft topology.

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