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ON $\text{NANO}\alpha^{s*}$ CLOSED AND $\text{NANO}\alpha^{s*}$ -CONTINUOUS

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ABSTRACT

The purpose of this paper is to define and study a new class of sets called Nano α^{s*} -closed sets in nano topological spaces. Basic properties of nano α^{s*} -closed sets are analyzed. We also used them to introduce the new notions like Nano α^{s*} -closure and Nano α^{s*} -continuous and their relation with already existing well known sets are also investigated.

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KEYWORDS: Nano closed set, Nano α -closed, Nano α^{s*} -closed, Nano α^{s*} -closure, Nano α^{s*} -continuous.

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1 INTRODUCTION:

The notion of α -open sets was introduced by O.Njastad⁷ in 1965. In 1970, Levine² introduced the concept of generalized closed sets as a generalization of closed sets in Topological space. The notion of nano topology was introduced by Lillis Thivagar³ which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. He also introduced the weak forms of Nano open sets namely Nano α -open sets, Nano semi open sets and Nano pre open sets. In this paper we have introduced a new class of sets on nano topological spaces called nano α^{s*} -closed sets and investigate their basic properties. We also discuss their relationship with already existing concepts.

2 PRELIMINARIES:

DEFINITION 2.1⁸

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

- (i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by x .
- (ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.
- (iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

PROPERTY 2.2⁸

If (U, R) is an approximation space and $X, Y \subseteq U$, then

- (i) $L_R(X) \subseteq X \subseteq U_R(X)$
- (ii) $L_R(\emptyset) = U_R(\emptyset)$ and $L_R(U) = U_R(U) = U$
- (iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (v) $L_R(X) \cup L_R(Y) \subseteq L_R(X \cup Y)$

- (vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (ix) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
- (x) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

DEFINITION 2.3³

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property $\tau_R(X)$ satisfies the following axioms:

- (i) U and \emptyset belongs to $\tau_R(X)$.
- (ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- (iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X . $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U . Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

REMARK 2.4⁸

If $\tau_R(X)$ is a Nano topology on U with respect to X , Then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

DEFINITION 2.5³

If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- (i) The Nano interior of the set A is defined as the union of all Nano open sets contained in A and is denoted by $NInt(A)$, $NInt(A)$ is the largest Nano open subsets of A .
- (ii) The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by $NCl(A)$, $NCl(A)$ is the smallest Nano closed sets containing A .

DEFINITION 2.6³

Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

- (i) Nano semiopen if $A \subseteq NCl[NInt(A)]$.
- (ii) Nano semi closed if $NInt[NCl(A)] \subseteq A$.
- (iii) Nano pre open if $A \subseteq NInt[NCl(A)]$.
- (iv) Nano α - open if $A \subseteq NInt[NCl(NInt(A))]$.

The class of all Nano semi open, Nano Semi closed, Nano pre open and Nano α -open is defined by $NSO(U, X)$, $NSF(U, X)$, $NPO(U, X)$ and $\tau_R^\alpha(X)$ resp.

DEFINITION 2.7¹

- (i) A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng-closed) if $NCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.
- (ii) Nano generalized open (briefly Ng-open) if $U \setminus A$ is Ng-closed.

The intersection of all Ng-closed sets containing A is called the Nano g-closure of A and denoted by $NCl^*(A)$ and the Nano g-interior of A is the union of all Nano g-open sets contained in A and is denoted by $NInt^*(A)$.

DEFINITION 2.8⁶

Let $(U, \tau_R(X))$ and $(V, \tau_R^*(Y))$ be two nano topological space . The mapping $f:(U, \tau_R(X)) \rightarrow (V, \tau_R^*(Y))$ is said to Nano α -continuous(Nano continuous) if $f^{-1}(A)$ is Nano α open (Nano open) in U for every Nano open set A in V .

3 NANO α^{s*} -CLOSED SETS

DEFINITION 3.1

A subset A of a Nano topological space $(U, \tau_R(X))$ is called Nano α^{s*} - closed if $NCl^*(NInt(NCl(A))) \subseteq A$. The collection of all Nano α^{s*} -closed set in X is denoted by $N\alpha^{s*}C(X)$.

THEOREM 3.2

Every Nano α -closed set is Nano α^{s*} -closed.

PROOF:

Suppose A is Nano α closed then $NCl(NInt(NCl(A))) \subseteq A$. Now, $NCl^*(NInt(NCl(A))) \subseteq NCl(NInt(NCl(A))) \subseteq A$, it follows that A is Nano α^{s*} -closed.

THEOREM 3.3

Every Nano closed set is Nano α^{s*} -closed.

PROOF:

Since every Nano closed set is Nano α -closed and by Theorem 3.2, the result is clear.

REMARK 3.4

The converse of the above theorem is not true as shown in the following example.

EXAMPLE 3.5

Consider the Nano topological space $(U, \tau_R(X))$ where $U = \{a, b, c, d\}$ $U \setminus R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then $L_R(X) = \{a\}$, $U_R(X) = \{a, b, d\}$ $B_R(X) = \{b, d\}$. Therefore the Nano topology

$\tau_R(X) = \{\emptyset, U, \{a\}, \{a,b,d\}, \{b,d\}\}$. Here $\{a\}$ is Nano α^{s*} -closed but neither Nano α -closed nor Nano closed.

THEOREM 3.6

Arbitrary intersection of Nano α^{s*} -closed set is Nano α^{s*} -closed.

PROOF:

Let $\{F_\alpha\}$ be collection of Nano α^{s*} -closed sets. Then $NCl^*(NInt(NCl(F_\alpha))) \subseteq F_\alpha$ for each α . Now, $NCl^*(NInt(NCl(\cap F_\alpha))) \subseteq NCl^*(NInt(\cap(NCl(F_\alpha)))) \subseteq NCl^*(\cap(NInt(NCl(F_\alpha)))) \subseteq \cap NCl^*(NInt(NCl(F_\alpha))) \subseteq \cap F_\alpha$. Therefore $\cap F_\alpha$ is Nano α^{s*} -closed.

REMARK 3.7

Union of two Nano α^* -closed sets need not be Nano α^* -closed as shown in the following example.

EXAMPLE 3.8

Consider the Nano topological space $(U, \tau_R(X))$ where $U = \{a,b,c,d,e\}$ and $U \setminus R = \{\{a\}, \{c\}, \{b,d\}, \{e\}\}$ and $X = \{a,b\}$ then $L_R(X) = \{a\}$, $U_R(X) = \{a,b,d\}$ and $B_R(X) = \{b,d\}$. Therefore the Nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{a,b,d\}, \{b,d\}\}$. Here $\{a,c\}$ and $\{b,d\}$ are Nano α^{s*} -closed but $\{a,b,c,d\}$ is not Nano α^{s*} -closed.

DEFINITION 3.9

Let A be a subset of $(U, \tau_R(X))$ then Nano α^{s*} - closure of A is defined as the intersection of all Nano α^{s*} -closed set in X containing A . It is denoted by $N\alpha^{s*}Cl(A)$. that is $N\alpha^{s*}Cl(A) = \cap \{F : A \subseteq F \text{ and } F \text{ is Nano } \alpha^{s*}\text{-closed}\}$

THEOREM 3.10

Let A be a subset of X . then A is Nano α^{s*} -closed iff $N\alpha^{s*}Cl(A) = A$.

PROOF:

Suppose A is Nano α^{s*} closed then by definition $N\alpha^{s*}Cl(A) = A$. conversely , suppose $N\alpha^{s*}Cl(A) = A$ by using theorem 3.6 , A is Nano α^{s*} -closed.

THEOREM 3.11

Let A and B be subsets of $(U, \tau_R(X))$, then the following results hold.

- (i) $N\alpha^{s*}Cl(\emptyset) = \emptyset$ and $N\alpha^{s*}Cl(X) = X$.
- (ii) $A \subseteq N\alpha^{s*}Cl(A)$
- (iii) If $A \subseteq B$ then $N\alpha^{s*}Cl(A) \subseteq N\alpha^{s*}Cl(B)$.

$$(iv) A \subseteq N\alpha^{s*}Cl(A) \subseteq N\alpha Cl(A) \subseteq NCl(A)$$

$$(v) N\alpha^{s*}Cl(N\alpha^{s*}Cl(A)) = N\alpha^{s*}Cl(A)$$

PROOF:

(i),(ii) and (iii) follows from Definition. (iv) follows from Theorem 3.2 and Theorem 3.3 and (v) follows from Theorem 3.10 and Definition

THE OREM 3.12

Let A and B be subset of $(U, \tau_R(X))$, then

$$(i) N\alpha^{s*}Cl(A) \cup N\alpha^{s*}Cl(B) \subseteq N\alpha^{s*}Cl(A \cup B)$$

$$(ii) N\alpha^{s*}Cl(A \cap B) \subseteq N\alpha^{s*}Cl(A) \cap N\alpha^{s*}Cl(B)$$

PROOF:

(i) by (iii) of Pre-theorem , $N\alpha^{s*}Cl(A) \subseteq N\alpha^{s*}Cl(A \cup B)$ and $N\alpha^{s*}Cl(B) \subseteq N\alpha^{s*}Cl(A \cup B)$. This implies that $N\alpha^{s*}Cl(A) \cup N\alpha^{s*}Cl(B) \subseteq N\alpha^{s*}Cl(A \cup B)$

(ii) Again by (iii) of Pre-Theorem $N\alpha^{s*}Cl(A \cap B) \subseteq N\alpha^{s*}Cl(A)$ and $N\alpha^{s*}Cl(A \cap B) \subseteq N\alpha^{s*}Cl(B)$. Hence $N\alpha^{s*}Cl(A \cap B) \subseteq N\alpha^{s*}Cl(A) \cap N\alpha^{s*}Cl(B)$

REMARK 3.13

The inclusions in Theorem 3.12 (i) may be strict and equality may also hold.

This can be seen from the following Example.

EXAMPLE 3.14

Consider the Nano topological space $(U, \tau_R(X))$ where $U = \{a, b, c, d, e\}$ $U \setminus R = \{\{a\}, \{c\}, \{b, d\}, \{e\}\}$ and $X = \{a, b\}$ then $L_R(X) = \{a\}$, $U_R(X) = \{a, b, d\}$ $B_R(X) = \{b, d\}$. Therefore the Nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, b, d\}, \{b, d\}\}$.

Let $A = \{a, c\}$ $B = \{b, d\}$ then $N\alpha^{s*}Cl(A) = \{a\}$ $N\alpha^{s*}Cl(B) = \{b, d\}$ and $N\alpha^{s*}Cl(A \cup B) = N\alpha^{s*}Cl(\{a, b, c, d\}) = U$. Therefore $N\alpha^{s*}Cl(A) \cup N\alpha^{s*}Cl(B) \subsetneq N\alpha^{s*}Cl(A \cup B)$

Consider the Nano topological space $(U, \tau_R(X))$ where $U = \{a, b, c, d, e\}$ $U \setminus R = \{\{a\}, \{c\}, \{b, d\}, \{e\}\}$ and $X = \{a, b\}$ then $L_R(X) = \{a\}$, $U_R(X) = \{a, b, d\}$ $B_R(X) = \{b, d\}$. Therefore the Nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, b, d\}, \{b, d\}\}$.

Let $A = \{b\}$ $B = \{c\}$ then $N\alpha^{s*}Cl(A) = \{b, c, d\}$ $N\alpha^{s*}Cl(B) = \{c\}$ and $N\alpha^{s*}Cl(A \cup B) = N\alpha^{s*}Cl(\{b, c\}) = \{b, c, d\}$. Therefore $N\alpha^{s*}Cl(A) \cup N\alpha^{s*}Cl(B) = N\alpha^{s*}Cl(A \cup B)$

REMARK 3. 15

The inclusions in Theorem 3.12(ii) may be strict and equality may also hold. This can be seen from the following Example.

EXAMPLE 3.16

Consider the Nano topological space $(U, \tau_R(X))$ where $U=\{a,b,c,d,e\}$ $U \setminus R = \{\{a\}, \{c\}, \{b,d\}, \{e\}\}$ and $X=\{a,b\}$ then $L_R(X)=\{a\}$, $U_R(X)=\{a,b,d\}$ $B_R(X)=\{b,d\}$. Therefore the Nano topology $\tau_R(X)=\{\emptyset, U, \{a\}, \{a,b,d\}, \{b,d\}\}$.

Let $A=\{a,c\}$ $B=\{b,c\}$ then $N\alpha^{s*}Cl(A)=\{a,c\}$ $N\alpha^{s*}Cl(B)=\{b,c,d\}$ and $N\alpha^{s*}Cl(A \cap B) = N\alpha^{s*}Cl(\{c\}) = \{c\}$.

Therefore $N\alpha^{s*}Cl(A \cap B) = N\alpha^{s*}Cl(A) \cap N\alpha^{s*}Cl(B)$ Let $C=\{c\}$ $D=\{d\}$ then $N\alpha^{s*}Cl(C)=\{c\}$ $N\alpha^{s*}Cl(D)=\{b,c,d\}$ and $N\alpha^{s*}Cl(C \cap D) = N\alpha^{s*}Cl(\emptyset) = \emptyset$.

There fore $N\alpha^{s*}Cl(C \cap D) \subsetneq N\alpha^{s*}Cl(C) \cap N\alpha^{s*}Cl(D)$.

4 NANO α^{s*} CONTINUOUS

DEFINITION 4.1

A subset of a space $(U, \tau_R(X))$ is called Nano α^{s*} open set if $U \setminus A$ is Nano α^{s*} -closed. Let $N\alpha^{s*}O(X)$ denote the collection of all Nano α^{s*} open set in $(U, \tau_R(X))$.

THEOREM 4. 2

A subset A of a space X is Nano α^{s*} open iff $A \subseteq NInt^*(NCl(NInt(A)))$

PROOF:

Let A be a Nano α^{s*} open set then $U \setminus A$ is Nano α^{s*} closed. That is iff $NCl^*(NInt(NCl(U \setminus A))) \subseteq U \setminus A$. that is iff $NInt^*(NCl(NInt(A))) \supseteq A$. note that if $A \subseteq B$ then $U \setminus A \supseteq U \setminus B$ and $NCl^*(U \setminus A) = U \setminus NInt^*A$ and $U \setminus NCl^*A = NInt^*(U \setminus A)$.

THEOREM 4.3

Every Nano open(α -open set) is Nano α^{s*} open.

PROOF:

Suppose A is Nano open (Nano α -open) . then $U \setminus A$ is Nano closed (Nano α closed) . By Theorem 3.2 , $U \setminus A$ is Nano α^{s*} closed. Therefore A is Nano α^{s*} open .

DEFINITION 4.4

Let $(U, \tau_R(X))$ and $(V, \tau_R^*(Y))$ be two nano topological space . The mapping $f:(U, \tau_R(X)) \rightarrow (V, \tau_R^*(Y))$ is said to Nano α^{s*} -continuous if $f^{-1}(A)$ is Nano α^{s*} open in U for every Nano open set A in V.

THEOREM 4.5

Every Nano continuous (Nano α continuous) is Nano α^{s*} -continuous.

PROOF:

Let $f:(U, \tau_R(X)) \rightarrow (V, \tau_R^*(Y))$ be Nano continuous (Nano α continuous). Let V be an open set in Y . then $f^{-1}(V)$ is Nano open (Nano α open in X). By Theorem 4.3 $f^{-1}(V)$ is Nano α^{s*} open in X . Hence f is Nano α^{s*} -continuous.

But the converse is not true in the following Example.

EXAMPLE 4.6

Consider the Nano topological spaces $(U, \tau_R(X))$ and $(V, \tau_R^*(Y))$ where $U = \{a, b, c, d\} = V$, $U \setminus R = \{\{a\}, \{c\}, \{b, d\}\}$, $X = \{a, b\}$ and $V \setminus R^* = \{\{a\}, \{d\}, \{b, c\}\}$, $Y = \{a, d\}$. If $f:(U, \tau_R(X)) \rightarrow (V, \tau_R^*(Y))$ is defined by $f(a)=b$, $f(b)=a$, $f(c)=d$, $f(d)=a$ then f is Nano α^{s*} -continuous but not Nano α continuous and not Nano continuous. Since $f^{-1}(\{a, d\}) = \{b, c, d\}$ is Nano α^{s*} -open but not Nano open (Nano α open).

2.3 CONCLUSION

Thus we introduced Nano α^{s*} -closed and Nano α^{s*} -open sets in topological spaces and discussed their relationship with already existing closed and open sets. The properties of the functions namely Nano α^{s*} -continuous are studied. Further we gave some characterizations for these functions.

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